

NA-72-803

THEORETICAL PREDICTION OF AIRPLANE  
STABILITY DERIVATIVES AT  
SUBCRITICAL SPEEDS

By J. Tulinus, W. Clever, A. Niemann,  
K. Dunn, and B. Gaither

(NASA-CR-132681) THEORETICAL PREDICTION OF  
AIRPLANE STABILITY DERIVATIVES AT  
SUBCRITICAL SPEEDS (North American Rockwell  
Corp.) 598 p HC \$13.25 CSCI 01C

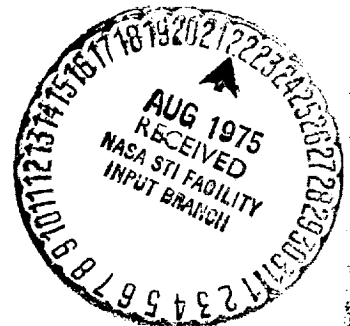
N75-29089

G3/05 Unclas  
31994

Prepared under Contract NAS1-10828  
NORTH AMERICAN ROCKWELL  
Los Angeles, California

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION





# CONTENTS

	Page
SUMMARY. . . . .	iv
INTRODUCTION . . . . .	v
SYMBOLS. . . . .	vii
THEORETICAL DEVELOPMENT. . . . .	1
Configuration Representation. . . . .	1
Discrete Influence Equations. . . . .	12
Constrained Influence Equations . . . . .	19
Surface Velocities and Pressures. . . . .	33
Section and Total Loads and Moments . . . . .	40
COMPUTER PROGRAM RESULTS . . . . .	57
Airfoil Section Velocities. . . . .	57
Section Induced Drag Due to Thickness . . . . .	57
Section Induced Drag Due to Lift. . . . .	67
Sphere Surface Velocity . . . . .	70
X-15 Wing-Fuselage-Horizontal Tail-Vertical Tail . . . . .	70
COMPUTER PROGRAM USAGE . . . . .	77
Program Setup . . . . .	77
Input Format. . . . .	80
Input Description . . . . .	96
Sample Input. . . . .	108
Sample Output . . . . .	113
CONCLUSIONS. . . . .	129
APPENDIX A . . . . .	130
Numerical Procedures. . . . .	130
APPENDIX B . . . . .	139
Skewed Source-Vortex Lattice Influence Equations. . . . .	139
APPENDIX C . . . . .	146
Quadrilateral Vortex Influence Equations. . . . .	146
APPENDIX D . . . . .	153
Woodward's Distributed Panel Influence Equations . . . . .	153
APPENDIX E . . . . .	256
Wagner's Lifting Surface Equations. . . . .	256
APPENDIX F. . . . .	300
Potential Form Drag . . . . .	300
APPENDIX G . . . . .	313
Computer Program Listing . . . . .	313
REFERENCES . . . . .	582





# LIST OF ILLUSTRATIONS

Figure		Page
1	Arbitrary lifting body. . . . .	1
2	Body subpanel defined by points 1, 4, 5, and 8 with subdivision points 2, 3, 6, and 7 . . . . .	2
3	Thick lifting panel . . . . .	8
4	Karman-Trefftz airfoil pressure distribution at 0.0 degree angle of attack. . . . .	58
5	Karman-Trefftz airfoil pressure distribution at 1.0 degree angle of attack. . . . .	59
6	Swept wing alone pressure distribution at 24.5 percent semi-span, 0.0 degree angle of attack . . . . .	60
7	Swept wing alone pressure distribution at 65.3 percent semi-span, 0.0 degree angle of attack . . . . .	61
8	Swept wing alone pressure distribution at 89.8 percent semi-span, 0.0 degree angle of attack . . . . .	62
9	Swept wing alone pressure distribution at 24.5 percent semi-span, 4.2 degrees angle of attack. . . . .	63
10	Swept wing alone pressure distribution at 65.3 percent semi-span, 4.2 degrees angle of attack. . . . .	64
11	Swept wing alone pressure distribution at 89.8 percent semi-span, 4.2 degrees angle of attack. . . . .	65
12	Spanwise distribution of potential form drag due to thickness . . . . .	66
13	Spanwise distribution of potential form drag due to lift. . . . .	69
14	Velocity ratio around a sphere. . . . .	71
15	X-15 Configuration. . . . .	72
16	Normal load distribution on X-15 fuselage at five degrees angle of attack . . . . .	73
17	Unit span load distribution on X-15 wing. . . . .	74
18	Unit span load distribution on X-15 horizontal tail . . . . .	75
19	Chordwise pressure distribution on X-15 wing at 17.8 percent semi-span at five degrees angle of attack . . . . .	76
20	Deck setup for use with NASA/Langley data array format. . . . .	78
21	Deck setup for use with NR data array format. . . . .	79
22	Control card deck sequence for use with NR data array format. . . . .	80
C-1	Velocity induced by finite vortex segment . . . . .	146

## SUMMARY

This report describes the theoretical development and application of an analysis for predicting the major static and rotary stability derivatives for a complete airplane. The analysis utilizes potential flow theory to compute the surface flow fields and pressures on any configuration that can be synthesized from arbitrary lifting bodies and nonplanar thick lifting panels. The pressures are integrated to obtain section and total configuration loads and moments due side slip, angle of attack, pitching motion, rolling motion, yawing motion, and control surface deflection. Subcritical compressibility is accounted for by means of the Gothert similarity rule.

Within the scope of predicting the total configuration stability derivatives it was necessary to study the problem of computing the spanwise variation of potential form drag due to panel lift and thickness. Included in appendix F is a solution to this problem. Also, in solving the potential form drag problem the work of Woodward and Wagner was thoroughly analyzed. A complete derivation of Woodward's influence equations is given in appendix D and a comprehensive review of Wagner's lifting surface theory is included in appendix E.

## INTRODUCTION

In order to develop a procedure for predicting total configuration stability derivatives, the perturbation flow due to the vehicle had to be represented by a grid of efficient aerodynamic finite elements general enough to satisfy the boundary conditions over the surfaces of diverse shapes and also produce the correct resultant forces and moments. The quadrilateral vortex was selected to represent the perturbation velocity due to the bodies because of its numerical efficiency, net force producing capability, limited range of influence, and relationship to the horseshoe vortex which has demonstrated amazing accuracy in predicting loads on wings of arbitrary shape.

The nonplanar thick lifting panels are divided into two sections, (1) the outboard section which is defined by a locus of chord lines, and (2) the root section which is a transition region from the outboard section to the juncture of the panel and a body. If the panel is attached to another panel there is no root section. The perturbation velocity due to panel lift is represented by quadrilateral vortices in the root section and skewed horseshoe vortices in the outboard section. The perturbation velocity due to panel thickness is represented by a source lattice. The panel aerodynamic finite elements were selected because of their numerical efficiency and proven accuracy in predicting flow fields over wings of general shape.

The panel singularities are placed on a mean surface instead of the actual external surface of the panel to maintain computing efficiency. This will sacrifice surface pressure accuracy at the juncture between two panels or a panel and a body, but for this type of general analysis the savings in computer time makes the compromise practical. Second order corrections to account for the interference between lift and thickness and to account for blunt leading edge airfoil sections are included.

The source and vortex lattice influence equations are formulated in terms of the same quantities, which allows the perturbation velocity due to lift and thickness to be computed simultaneously and thereby save computing effort. Also, due to the limited range of significant influence of the quadrilateral vortex the influence of any quadrilateral vortex is only computed at those points within a given area of influence. This can save considerable computing time in developing the aerodynamic influence matrix.

Computer time is also saved by reducing the number of unknowns by transforming the aerodynamic influence matrix by constraint matrices. The constraint matrices constrain the body and panel vorticity, thereby reducing the number of unknowns from that of the number of vortex elements to the number of constraint functions. This is an option in the program and can be

applied in just the longitudinal direction, lateral direction, both directions, or not at all.

The pressures are integrated by numerical means and the panel section drag is computed by means of the Kutta-Joukowski theorem. The total configuration induced drag is computed by means of a Trefftz plane analysis.

# LIST OF SYMBOLS

$AR$	aspect ratio
$A_R$	reference area
$[A_x], [A_y], [A_z]$	aerodynamic influence matrix components
$a_{B_i}$	body constraint function coefficient
$a_{P_K}$	panel constraint function coefficient
$b$	panel span
$c$	chord or aerodynamic coefficient
$\bar{c}$	reference chord
$c_S$	chord line between $(X_J, Y_J, Z_J)$ and $(X_R, Y_R, Z_R)$
$c_f$	trailing edge flap chord
$c_{\bar{K}}$	leading edge flap chord
$C_P$	pressure coefficient
$[E]$	two dimensional influence matrix
$\hat{h}$	unit vector tangent to control surface hinge line
$h$	body height
$\hat{i}, \hat{j}, \hat{k}$	unit vectors in the x, y, and z directions, respectively
$K$	quadrilateral vortex strength
$M_\infty$	reference mach number
$\bar{\bar{N}}$	equivalent incompressible surface normal unit vector

$[\bar{N}_x], [\bar{N}_y], [\bar{N}_z]$	equivalent incompressible surface normal unit vector component matrices
$[N_x], [N_y], [N_z]$	actual surface normal unit vector component matrices
$\bar{N}_p$	panel surface normal unit vector
$p$	roll rate
$q$	pitch rate
$r$	yaw rate
$R_{c_x}, R_{c_y}, R_{c_z}$	subarea centroid position vector components
$R_B$	body radius
$[R]$	transformation matrix between quadrilateral and horseshoe vortex strengths
$\bar{R}$	general position vector
$S_B$	body circumferential distance
$[S_x], [S_y], [S_z]$	source influence matrix components
$\bar{\bar{T}}_M$	equivalent incompressible surface longitudinal tangent unit vector
$[\bar{T}_{M_x}], [\bar{T}_{M_y}], [\bar{T}_{M_z}]$	equivalent incompressible surface longitudinal tangent unit vector component matrices
$\bar{\bar{T}}_T$	equivalent incompressible surface lateral tangent unit vector
$[\bar{T}_{T_x}], [\bar{T}_{T_y}], [\bar{T}_{T_z}]$	equivalent incompressible surface lateral tangent unit vector component matrices
$\vec{T}_p$	panel surface lateral tangent unit vector
$[T_{B_i}]$	body aerodynamic constraint transformation matrix

$[T_{P_K}]$

panel aerodynamic constraint transformation matrix

$[T_{M_x}], [T_{M_y}], [T_{M_z}]$

actual surface longitudinal tangent unit vector component matrices

$[T_{T_x}], [T_{T_y}], [T_{T_z}]$

actual surface lateral tangent unit vector component matrices

U

total velocity in x direction

V

total velocity in y direction

$V_x, V_y, V_z$

onset flow components

$V_\infty$

reference velocity

W

total velocity in z direction

w

body width

X, Y, Z

global coordinates

$X_B, Y_B, Z_B$

body coordinates

$X_P, Y_P, Z_P$

panel coordinates

$X_J, Y_J, Z_J$

points along intersection of panel root section and outboard section

$X_R, Y_R, Z_R$

points along intersection of panel root section and a body

$X_{C.G.}, Y_{C.G.}, Z_{C.G.}$

center of gravity position vector components

$X_k$

trailing edge of leading edge control surface

$X_f$

leading edge of trailing edge control surface

$X_h$

hinge line location

$Y_{E_M}$

body multiplication factor in y direction

$Z_{E_M}$

body multiplication factor in z direction

$z_c$  Perpendicular distance between chord line and mean camber line

$z_t$  airfoil thickness

#### GREEK SYMBOLS

$\alpha$  angle of attack

$\beta$   $\sqrt{1-M_\infty^2}$  or angle of yaw

$\gamma$  vorticity or ratio of specific heats

$\delta$  control surface deflection

$\Gamma$  horseshoe vortex strength

$\Delta A_x, \Delta A_y, \Delta A_z$  directed subareas

$\Delta Y_B$  displacement of body cross-section in y direction

$\Delta Z_B$  displacement of body cross-section in z direction

$\vec{\Delta S}$  incremental vector tangent to constant percent chord line

$\Delta Y$  panel surface increment in y direction

$\Delta Z$  panel surface increment in z direction

$\epsilon$  panel twist

$\eta$  fraction of lateral or circumferential distance

$\theta_B$  body polar coordinate

$\theta$  general lateral polar coordinate

$\Lambda$  sweep

$\rho$  density



$z$	source strength
$\phi$	general longitudinal polar coordinate

#### SUBSCRIPTS

AVG.	average
B	body
C	camber
$C_x, C_y, C_z$	centroid components
C.P.	center of pressure
C.G.	center of gravity
E	edge
f	trailing edge control surface or final
h	hinge line
i	summation index or induced
j	summation index
J	root section and outboard panel juncture
k	summation index
L	lower surface or lift
L.E.	leading edge
m	longitudinal direction
MAX.	maximum
M	longitudinal subpanel number
n	lateral subpanel number
O	origin

P	panel
r	summation index
R	panel-body juncture
S	lateral or circumferential direction
T	lateral direction
t	airfoil thickness
T.E.	trailing edge
u	upper surface or number of longitudinal constraint function
w	number of lateral constraint function
X, Y, Z	directions
$\eta$	lateral or circumferential direction
$\infty$	infinity

## THEORETICAL DEVELOPMENT

### Configuration Representation

The theory discussed in this report is capable of predicting the surface pressures and integrated loads on any configuration which can be synthesized from lifting bodies and thick lifting panels of arbitrary shape. These two basic elements can be attached along longitudinal subpanel edges in order to represent complete airplane configurations of arbitrary shape.

**Lifting Body.** - The arbitrary lifting body, as shown in figure 1, can be of the solid or flow-through type. The body external surface is divided into a grid, which represents the edges of the body subpanels. The body bound vortex lines are placed at the quarter chord point of the subpanels and the fixed trailing vortex lines along the longitudinal edges of the subpanels. The trailing vortex lines are shed from the aft end of the body and extend in the X direction to infinity unless the free wake option of the program is used, in which case the locations of the trailing vortices aft of the body are determined such that they are force-free. If the body is closed to a point at the aft end, all of the body trailing vortices aft of the body cancel and are, therefore, neglected.

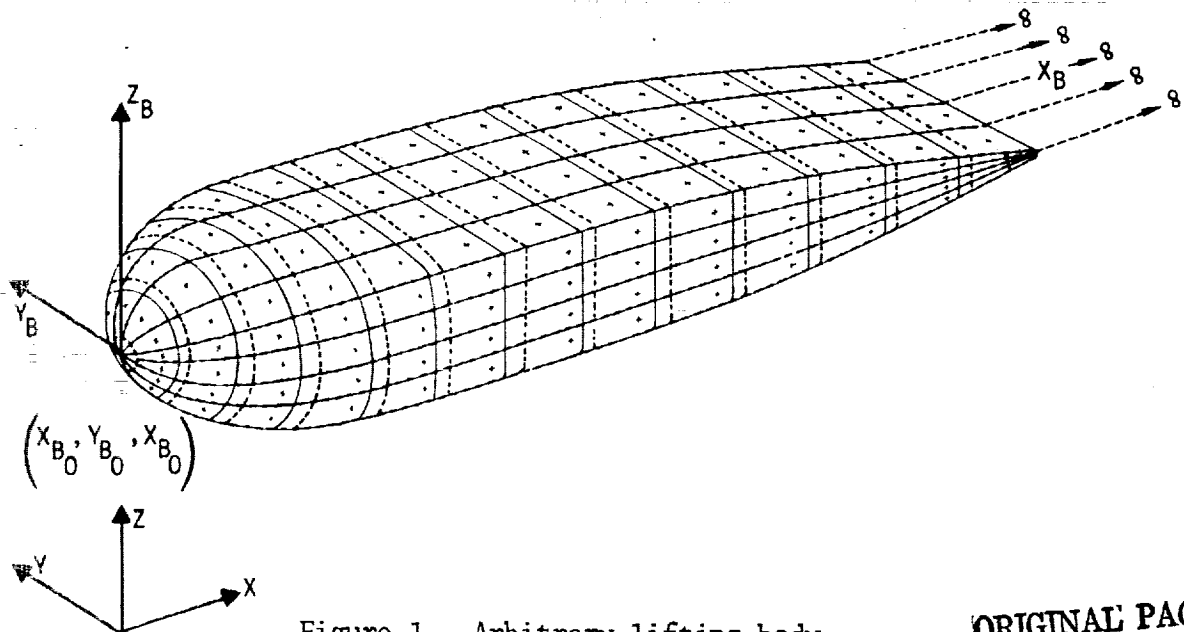


Figure 1.- Arbitrary lifting body.

ORIGINAL PAGE IS  
OF POOR QUALITY

The body vortex strengths are assumed constant around closed paths defined by the bound vortex lines of two subpanels, adjacent to each other in the longitudinal direction, and that portion of the longitudinal edges of these adjacent panels between the two bound vortex lines. The contribution from the vortex loop, or quadrilateral vortex, is defined as positive when the Biot Savart line integral is taken in the clockwise direction by an observer looking at the external surfaces of the two subpanels. The body subpanels and quadrilateral vortices are shown in figure 1 along with the location of the body control points.

The body control points are located at the three-quarter chord of the body subpanels. At these points the total flow is summed and forced to be a minimum in the direction normal to the body surface. If the body vorticity is not constrained by functions with unknown coefficients, a control point is placed at each subpanel and a discrete solution for the unknown body vortex strengths is obtained. If the body vorticity is constrained, control points are placed at as many subpanels as is necessary to obtain a good representation of the body shape and to insure that there is sufficient control of the constraint functions. In this case the unknown coefficients of the constraint functions are determined by the method of least squares.

The body subpanels are further subdivided in the lateral direction so that the vortex grid is mapped to the body surface more accurately. This also allows the use of only one quadrilateral vortex in the lateral direction for the case of a body of revolution in a uniform flow at zero angle of attack. In this case the two side edges of the subpanel are coincident, and therefore, the fixed trailing vortices cancel leaving a longitudinal distribution of ring vortices located at the quarter chord of each subpanel.

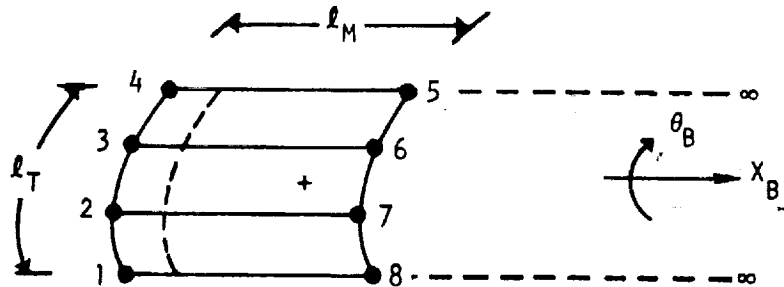


Figure 2.- Body subpanel defined by points 1, 4, 5, and 8 with subdivision points 2, 3, 6, and 7.

The coordinates of the body subpanel subdivisions, as shown in figure 2, are used to compute unit vectors tangent to the body surface in both the longitudinal and lateral directions, unit vectors normal to the body surface, directed subareas to be used in the integration of the body surface pressures, and the centroids of the subareas to compute moments about the configuration center of gravity.

The unit vector tangent to the equivalent incompressible body subpanel in the longitudinal direction is given by,

$$\bar{\bar{T}}_M = \bar{T}_{M_X} \hat{i} + \bar{T}_{M_Y} \hat{j} + \bar{T}_{M_Z} \hat{k} \quad (1)$$

$$\bar{T}_{M_X} = \frac{X_7 - X_2 + X_6 - X_3}{\sqrt{(X_7 - X_2)^2 + \beta^2(Y_7 - Y_2)^2 + \beta^2(Z_7 - Z_2)^2} + \sqrt{(X_6 - X_3)^2 + \beta^2(Y_6 - Y_3)^2 + \beta^2(Z_6 - Z_3)^2}} \quad (2)$$

$$\bar{T}_{M_Y} = \frac{\beta(Y_7 - Y_2 + Y_6 - Y_3)}{\sqrt{(X_7 - X_2)^2 + \beta^2(Y_7 - Y_2)^2 + \beta^2(Z_7 - Z_2)^2} + \sqrt{(X_6 - X_3)^2 + \beta^2(Y_6 - Y_3)^2 + \beta^2(Z_6 - Z_3)^2}} \quad (3)$$

$$\bar{T}_{M_Z} = \frac{\beta(Z_7 - Z_2 + Z_6 - Z_3)}{\sqrt{(X_7 - X_2)^2 + \beta^2(Y_7 - Y_2)^2 + \beta^2(Z_7 - Z_2)^2} + \sqrt{(X_6 - X_3)^2 + \beta^2(Y_6 - Y_3)^2 + \beta^2(Z_6 - Z_3)^2}} \quad (4)$$

The unit vector tangent to the equivalent incompressible body subpanel in the lateral direction is given by,

$$\bar{\bar{T}}_T = \bar{T}_{T_X} \hat{i} + \bar{T}_{T_Y} \hat{j} + \bar{T}_{T_Z} \hat{k} \quad (5)$$

$$\bar{T}_{T_X} = \frac{X_3 - X_2 + X_6 - X_7}{\sqrt{(X_3 - X_2)^2 + \beta^2(Y_3 - Y_2)^2 + \beta^2(Z_3 - Z_2)^2} + \sqrt{(X_6 - X_7)^2 + \beta^2(Y_6 - Y_7)^2 + \beta^2(Z_6 - Z_7)^2}} \quad (6)$$

ORIGINAL PAGE IS  
OF POOR QUALITY

$$\bar{T}_{T_Y} = \frac{\beta(Y_3 - Y_2 + Y_6 - Y_7)}{\sqrt{(X_3 - X_2)^2 + \beta^2(Y_3 - Y_2)^2 + \beta^2(Z_3 - Z_2)^2} + \sqrt{(X_6 - X_7)^2 + \beta^2(Y_6 - Y_7)^2 + \beta^2(Z_6 - Z_7)^2}} \quad (7)$$

$$\bar{T}_{T_Z} = \frac{\beta(Z_3 - Z_2 + Z_6 - Z_7)}{\sqrt{(X_3 - X_2)^2 + \beta^2(Y_3 - Y_2)^2 + \beta^2(Z_3 - Z_2)^2} + \sqrt{(X_6 - X_7)^2 + \beta^2(Y_6 - Y_7)^2 + \beta^2(Z_6 - Z_7)^2}} \quad (8)$$

The unit vector normal to the equivalent incompressible body subpanel is given by,

$$\bar{\bar{N}} = \bar{N}_X \hat{i} + \bar{N}_Y \hat{j} + \bar{N}_Z \hat{k} \quad (9)$$

$$\bar{N}_X = \frac{\bar{T}_{M_Y} \bar{T}_{T_Z} - \bar{T}_{M_Z} \bar{T}_{T_Y}}{\sqrt{(\bar{T}_{M_Y} \bar{T}_{T_Z} - \bar{T}_{M_Z} \bar{T}_{T_Y})^2 + (\bar{T}_{T_X} \bar{T}_{M_Z} - \bar{T}_{T_Z} \bar{T}_{M_X})^2 + (\bar{T}_{M_X} \bar{T}_{T_Y} - \bar{T}_{M_Y} \bar{T}_{T_X})^2}} \quad (10)$$

$$\bar{N}_Y = \frac{\bar{T}_{T_X} \bar{T}_{M_Z} - \bar{T}_{T_Z} \bar{T}_{M_X}}{\sqrt{(\bar{T}_{M_Y} \bar{T}_{T_Z} - \bar{T}_{M_Z} \bar{T}_{T_Y})^2 + (\bar{T}_{T_X} \bar{T}_{M_Z} - \bar{T}_{T_Z} \bar{T}_{M_X})^2 + (\bar{T}_{M_X} \bar{T}_{T_Y} - \bar{T}_{M_Y} \bar{T}_{T_X})^2}} \quad (11)$$

$$\bar{N}_Z = \frac{\bar{T}_{M_X} \bar{T}_{T_Y} - \bar{T}_{M_Y} \bar{T}_{T_X}}{\sqrt{(\bar{T}_{M_Y} \bar{T}_{T_Z} - \bar{T}_{M_Z} \bar{T}_{T_Y})^2 + (\bar{T}_{T_X} \bar{T}_{M_Z} - \bar{T}_{T_Z} \bar{T}_{M_X})^2 + (\bar{T}_{M_X} \bar{T}_{T_Y} - \bar{T}_{M_Y} \bar{T}_{T_X})^2}} \quad (12)$$

The directed subareas for the middle lateral subdivision for the actual body are given by;

$$\Delta A_X = \frac{1}{2} [(Y_6 - Y_2) (Z_3 - Z_7) - (Z_6 - Z_2) (Y_3 - Y_7)] \quad (13)$$

$$\Delta A_Y = \frac{1}{2} [(X_3 - X_7) (Z_6 - Z_2) - (Z_3 - Z_7) (X_6 - X_2)] \quad (14)$$

$$\Delta A_Z = \frac{1}{2} [(X_6 - X_2) (Y_3 - Y_7) - (Y_6 - Y_2) (X_3 - X_7)] \quad (15)$$

The components of the position vector to the centroid of this subdivision are given by,

$$R_{C_X} = \frac{1}{4} (X_2 + X_3 + X_6 + X_7) \quad (16)$$

$$R_{C_Y} = \frac{1}{4} (Y_2 + Y_3 + Y_6 + Y_7) \quad (17)$$

$$R_{C_Z} = \frac{1}{4} (Z_2 + Z_3 + Z_6 + Z_7) \quad (18)$$

Analogous directed subarea and centroidal position vector expressions are obtained for the other lateral subdivisions.

The body description is defined independent of the subpanel grid and can be input to the program in a number of different ways. Both cartesian and polar coordinates can be used to describe the body cross-sections at a set of chord stations. After the basic cross-sections have been described they can be scaled and then translated in planes perpendicular to the body chord or mean camber line. This provides a means of inputting the correct side and top views

of an arbitrary body with a minimum of input data to obtain a preliminary estimate of the loads on the body. It also facilitates in inputting the exact shape of some bodies which can be represented by longitudinal segments, over which the cross-sections are mathematically similar. The actual arbitrary shape can be input directly without the use of the scaling and translation options if it is more convenient.

If polar coordinates are used to describe the body cross-sections, the lateral location of body subpanel side edges or fixed trailing vortex lines are also given in terms of angles measured from the local section  $Z_B$  axis, in a plane parallel to the  $(Y_B-Z_B)$  plane, at an independent set of chordwise stations. These subpanel lateral edges are either specified at a given set of angles or defined to be at equally spaced angle locations. The subpanel longitudinal edges are specified at given  $X_B$  stations, evenly spaced in terms of  $X_B$ , or evenly spaced in terms of  $\phi_B$ , where  $\phi_B = \cos^{-1} (1 - 2 X_B/C_B)$ .

The following procedure is used to determine the coordinates  $X_{BE}$ ,  $(Y_{BE} - \Delta Y_{BE})/(Y_{BM})$ ,  $(Z_{BE} - \Delta Z_{BE})/(Z_{BM})$  of the subpanel corners.

1.  $R_{B_i}$  versus  $\theta_{B_i}$  is input at  $X_{B_i}$ .
2.  $\theta_{BE}$  (subpanel lateral edge location) versus  $X_{B_\theta}$  is given as input
3. Interpolation on  $\theta_{BE}$  versus  $X_{B_\theta}$  is done to obtain  $\theta_{BE}$  versus  $X_{BE}$  (subpanel longitudinal edge location) and  $X_{B_i}$ .
4. Interpolation on  $R_{B_i}$  versus  $\theta_{B_i}$  is done at each  $X_{B_i}$  to obtain  $R_B$  versus  $X_{B_i}$  at each  $\theta_{BE}$
5. Interpolation on  $R_B$  versus  $X_{B_i}$  is done to obtain  $R_{BE}$  versus  $X_{BE}$  at each  $\theta_{BE}$ .
6.  $X_{BE}$ ,  $(Y_{BE} - \Delta Y_{BE})/(Y_{BM})$ ,  $(Z_{BE} - \Delta Z_{BE})/(Z_{BM})$  are computed from  $X_{BE}$ ,  $R_{BE}$ , and  $\theta_{BE}$ .

If the subpanels are subdivided the coordinates for the corners of these subdivision are computed using the same procedure. There is always an odd number of subdivisions in both the longitudinal and lateral directions.



If cartesian coordinates are used to describe the body cross-sections, the lateral location of the subpanel side edges are defined by the percent circumferential length  $\eta_{B_i} = S_{B_i}/S_{B_{MAX_E}}$  at independent chordwise stations.

Tables of  $(Y_{B_i} - \Delta Y_{B_i})/Y_{B_M}$  and  $(Z_{B_i} - \Delta Z_{B_i})/Z_{B_M}$  versus percent circumferential length  $\eta_{B_i} = S_{B_i}/S_{B_{MAX_i}}$  are developed at the input longitudinal stations  $X_{B_i}$ . The same procedure that was used to obtain  $X_{B_E}$ ,  $(Y_{B_E} - \Delta Y_{B_E})/Y_{B_M}$ , and  $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$  when the body was defined by polar coordinates is also used for this case, except that  $R_{B_i}$  is replaced by  $(Y_{B_i} - \Delta Y_{B_i})/Y_{B_M}$  and  $(Z_{B_i} - \Delta Z_{B_i})/Z_{B_M}$ , and  $\theta_{B_i}$  is replaced by  $\eta_{B_i} = S_{B_i}/S_{B_{MAX_i}}$ . Also, step 6 is unnecessary. The procedure is cycled through twice, first for  $(Y_{B_E} - \Delta Y_{B_E})/Y_{B_M}$  and then for  $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$ .

After  $X_{B_E}$ ,  $(Y_{B_E} - \Delta Y_{B_E})/Y_{B_M}$ , and  $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$  have been computed,  $Y_{B_E}$  is determined by multiplying  $(Y_{B_E} - \Delta Y_{B_E})/Y_{B_M}$  by the multiplication factor  $Y_{B_M}$  and then adding the translation increment  $\Delta Y_{B_E}$ .  $Z_{B_E}$  is determined by multiplying  $(Z_{B_E} - \Delta Z_{B_E})/Z_{B_M}$  by the multiplication factor  $Z_{B_M}$  and then adding the translation increment  $\Delta Z_{B_E}$ .

Thick lifting panel. - The thick lifting panel, as shown in figure 3, can be warped in any manner laterally, have an arbitrary distribution of chord length, thickness, twist, and camber. It can be used to represent a wing, canard, fin, pylon, horizontal or vertical tail, or be wrapped around to represent a flow through nacelle. The panels can be attached to other panels, such as in the case of a pylon on a wing, or attached to bodies. The panels can have plain leading or trailing edge flaps, ailerons, rudders, or elevators. These control surfaces can be of the full or partial span type and their hinge lines are not restricted to constant percent chord lines.

The panel is divided into two sections; (1) the outboard section where the subpanel longitudinal edges are assumed to be straight lines in the X direction, and (2) the root section which is a transition region from the outboard section to the intersection of the panel and a body. The line of intersection between the panel and a body does not have to be a straight line. Therefore the subpanel longitudinal edge lines will change in shape from that of the line of intersection at the side of the body to a straight line in the X direction at the outboard section. If the panel is not attached to a body, there is no root section.

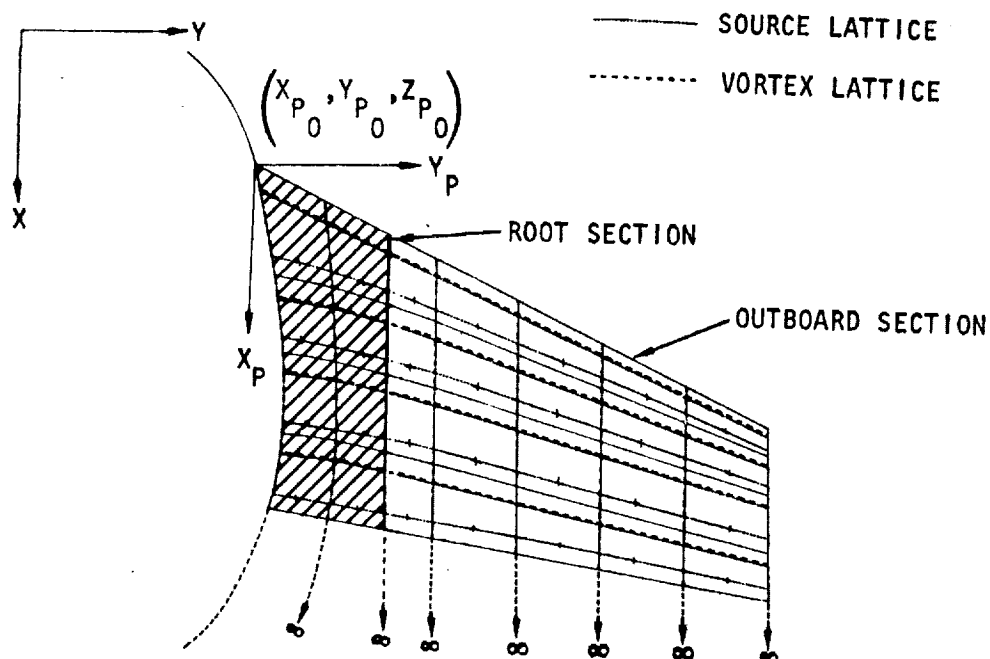


Figure 3.- Thick lifting panel.

The root section and outboard section can have any distribution of sub-panel lengths and widths. The subpanel lateral edges are specified as a list of percent chord stations, evenly spaced in terms of percent chord, or evenly spaced in terms of  $\phi$ , where

$$\phi = \cos^{-1} \left[ 1 - 2 \left( \frac{X - X_{LE}}{C} \right) \right] .$$

The subpanel side edges are specified as a list of percent of surface semi-span  $\eta$ , evenly spaced in terms of percent of surface semi-span, or evenly spaced in terms of  $\theta$ , where  $\theta = \cos^{-1}\eta$ .

The percent of surface semi-span  $\eta$  is defined as the percent of length of the line projected into the (y-Z) plane by the panel leading edge. A constant  $\eta$  line is in general curved in the root section. In the outboard section, lines of constant  $\eta$  are straight and in the X direction. The curved constant  $\eta$  lines associated with the longitudinal edges of subpanels in the root section are computed such that the sweep of the leading and trailing edges of the subpanels vary linearly from that of the sweep of the panel at the leading edge to that of the sweep of the panel at the trailing edge. Also, the corner points of the subpanels are equally spaced in the lateral direction along lines of constant percent chord. The chord at a lateral station in the root section is the length of the curved constant  $\eta$  line at that station. If the lateral distance along a constant percent chord line between two corner points is defined by  $|\Delta \vec{S}|$ , the unit vector tangent to the leading or trailing edge of a subpanel at any percent chord station is given by;

$$\left( \frac{\Delta \vec{S}}{|\Delta \vec{S}|} \right)_K = \frac{\Delta \vec{S}_{L.E.K}}{|\Delta \vec{S}_{L.E.K}|} + \left( \frac{\Delta \vec{S}_{T.E.K}}{|\Delta \vec{S}_{T.E.K}|} - \frac{\Delta \vec{S}_{L.E.K}}{|\Delta \vec{S}_{L.E.K}|} \right) \text{ (percent chord)} \quad (19)$$

where

$$|\Delta \vec{S}_K| = \frac{c_s^2}{K_J \sum_{K=1} \left( \frac{\Delta \vec{S}}{|\Delta \vec{S}|} \right)_K \cdot \vec{c}_s} \quad (20)$$

ORIGINAL PAGE IS  
OF POOR QUALITY

and  $K_J$  equals the number of subpanels in the lateral direction in the root section.

$$\vec{C}_S = (X_J - X_R) \hat{i} + (Y_J - Y_R) \hat{j} + (Z_J - Z_R) \hat{k} \quad (21)$$

where  $(X_R, Y_R, Z_R)$  and  $(X_J, Y_J, Z_J)$  are the points on the curved constant percent chord line at the line of intersection of the panel and the body and at the juncture of the root and outboard sections, respectively. The equally spaced corner points along a constant percent chord line are then computed by

$$R(X, Y, Z)_\eta = R(X_R, Y_R, Z_R) + \sum_{K=1}^{K=K_\eta} \left( \frac{\Delta \bar{S}}{|\Delta \bar{S}|} \right)_K |\Delta \bar{S}_K| \quad (22)$$

where  $K_\eta$  is the number of subpanels, in the lateral direction, the constant  $\eta$  line is located from the line of intersection of the panel and the body. Since constant  $\eta$  lines in the outboard section are straight and in the  $X$  direction the subpanel corner points are defined directly from the input list of subpanel edge locations in terms of percent of chord and lateral  $\eta$  station.

The coordinates of the leading edge point  $(X_{L.E.}, Y_{L.E.}, Z_{L.E.})_K$  associated with a constant  $\eta_K$  line are obtained from the panel perimeter description by converting the leading edge input points to tables of  $X_{L.E.i}$  versus  $\eta$ ,  $Y_{L.E.i}$  versus  $\eta$ , and  $Z_{L.E.i}$  versus  $\eta$ , and then interpolating in these tables to determine the leading edge coordinates at a desired lateral  $\eta_K$  station. Unit vectors tangent and normal to the panel at any  $\eta_K$  station can also be determined from these tables by evaluating the  $\Delta Y_{L.E.K}$  and  $\Delta Z_{L.E.K}$  about the lateral station  $\eta_K$ . The unit vectors are then defined by;

$$\vec{N}_{P_K} = - \frac{\Delta Z_{L.E.K}}{\sqrt{\Delta Y_{L.E.K}^2 + \Delta Z_{L.E.K}^2}} \hat{j} + \frac{\Delta Y_{L.E.K}}{\sqrt{\Delta Y_{L.E.K}^2 + \Delta Z_{L.E.K}^2}} \hat{k} \quad (23)$$

$$\vec{T}_{PK} = \frac{\Delta Y_{L.E.K}}{\sqrt{\Delta Y_{L.E.K}^2 + \Delta Z_{L.E.K}^2}} \hat{j} + \frac{\Delta Z_{L.E.K}}{\sqrt{\Delta Y_{L.E.K}^2 + \Delta Z_{L.E.K}^2}} \hat{k} \quad (24)$$

With the coordinates of the subpanel corner points known, the coordinates and sweep of the vortex and source lattices used to represent the perturbation velocities due to lift and thickness, respectively, can be defined. The bound vortex lines and control points are placed at the quarter and three-quarter chord points of the subpanels, respectively. The fixed trailing vortices are placed along the subpanel side edges. In the root section the vortex lattice is a quadrilateral system, where as, in the outboard section the vortex lattice is a skewed horseshoe system. The skewed source lines are placed at both the quarter and three-quarter chord points of the subpanels.

The section geometry is input in terms of a percent thickness, percent camber, and twist. Where the percent thickness and camber are based on the local chord and measured in the  $\vec{N}_{PK}$  direction. Twist is defined in the plane described by the unit vectors  $\hat{i}$  and  $\vec{N}_{PK}$ . The X component of the unit vector normal to the section mean camber line and in the plane defined by  $\hat{i}$  and  $\vec{N}_{PK}$  is then given by;

$$\vec{N}_{PX} = \beta \left( -\frac{dz_C}{dx} + \epsilon + \tan \delta \right) \quad (25)$$

where  $z_C$  is the perpendicular distance between the section chord line and the mean camber line,  $\epsilon$  is the angle of twist, and  $\delta$  the deflection of any control surface. All three of these quantities are functions of both percent chord and  $\eta$ . The thickness is defined in the same manner as camber.

The trailing vortices aft of the trailing edge of the outboard section are straight lines in the X direction going off to infinity. The trailing vortices aft of the trailing edge of the root section lie along curved constant  $\eta$  lines to the end of the body and then go off to infinity in the X direction. These curved constant  $\eta$  lines are determined in the same manner as the constant  $\eta$  lines in the root section. If the free wake option of the program is utilized the location of the panel free trailing vortices are iterated for such that the wake is force free.

## Discrete Influence Equations

The perturbation velocity due to the arbitrary lifting bodies is represented by quadrilateral vortices on the external surface of the bodies. The perturbation velocity due to lift on the panels is represented by quadrilateral vortices in the root section of the panel and by skewed horseshoe vortices in the outboard section of the panel. The thickness is represented by skewed source lines in both the root and outboard sections of the panel. All the panel singularities are placed on the panel chordal surface. The source strengths  $\Sigma$  are defined by the change in thickness over that portion of the subpanel it represents, so that;

$$\frac{\Sigma}{V_{\infty}} = \frac{2 \beta \Delta Z_t}{\sqrt{1 + (\tan \Lambda)^2 / \beta^2}} \frac{\bar{V}_X}{V_{\infty}} \quad (26)$$

where  $\Lambda$  is the sweep of the source line and  $\bar{V}_X$  is the total onset velocity in the X direction. The quadrilateral vortex strengths  $K$  and the skewed horseshoe vortex strengths  $\Gamma$  must be solved for utilizing the boundary condition that a minimum of flow passes through the external surface of the bodies and the chordal surface of the thick lifting panels at a finite number of control points. In order to satisfy this boundary condition the total flow due to all singularities and onset flow is summed at each control point and the scalar product of this sum and the surface unit normal is minimized. This results in a set of linear aerodynamic influence equations which are solved for the unknown vortex strengths by means of Householder's method, described in appendix A.

The influence equations for the  $j^{\text{th}}$  equivalent incompressible body are given by;

$$\begin{aligned}
 & \sum_{i=1}^{N_B} \left[ \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} A_{X_{B_j B_i}} \\ A_{Y_{B_j B_i}} \\ A_{Z_{B_j B_i}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} A_{X_{B_j P_K}} \\ A_{Y_{B_j P_K}} \\ A_{Z_{B_j P_K}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} S_{X_{B_j P_K}} \\ S_{Y_{B_j P_K}} \\ S_{Z_{B_j P_K}} \end{bmatrix} \right] \left\{ \frac{K}{V_\infty} \right\}_{B_i} \\
 & + \sum_{K=1}^{N_P} \left[ \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} A_{X_{B_j P_K}} \\ A_{Y_{B_j P_K}} \\ A_{Z_{B_j P_K}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} S_{X_{B_j P_K}} \\ S_{Y_{B_j P_K}} \\ S_{Z_{B_j P_K}} \end{bmatrix} \right] \left\{ \frac{\Gamma}{V_\infty} \right\}_{P_K} \\
 & + \sum_{K=1}^{N_P} \left[ \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} S_{X_{B_j P_K}} \\ S_{Y_{B_j P_K}} \\ S_{Z_{B_j P_K}} \end{bmatrix} \right] \left\{ \frac{\Sigma}{V_\infty} \right\}_{P_K} \\
 & + \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \left\{ \frac{\bar{V}_X}{V_\infty} \right\} + \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \left\{ \frac{\bar{V}_Y}{V_\infty} \right\} + \begin{bmatrix} \bar{N}_{X_{B_j}} \\ \bar{N}_{Y_{B_j}} \\ \bar{N}_{Z_{B_j}} \end{bmatrix} \left\{ \frac{\bar{V}_Z}{V_\infty} \right\} = \{e\}_{B_j} \quad (27)
 \end{aligned}$$

where

$\bar{N}_{X_{B_j}}$ ,  $\bar{N}_{Y_{B_j}}$ , and  $\bar{N}_{Z_{B_j}}$  are the components of the  $j^{\text{th}}$  equivalent incompressible body surface unit normal vectors,

$A_{X_{B_j B_i}}$ ,  $A_{Y_{B_j B_i}}$ , and  $A_{Z_{B_j B_i}}$  are the components of the perturbation velocity induced by the  $i^{\text{th}}$  equivalent incompressible body unit strength vortices onto the  $j^{\text{th}}$  equivalent incompressible body,

$A_{X_{B_j P_K}}$ ,  $A_{Y_{B_j P_K}}$ , and  $A_{Z_{B_j P_K}}$  are the components of the perturbation velocity induced by the  $k^{\text{th}}$  equivalent incompressible panel unit strength vortices onto the  $j^{\text{th}}$  equivalent incompressible body, and

$S_{X_{B_j P_K}}$ ,  $S_{Y_{B_j P_K}}$ , and  $S_{Z_{B_j P_K}}$  are the components of the perturbation velocity induced by the  $k^{\text{th}}$  equivalent incompressible panel unit strength sources onto the  $j^{\text{th}}$  equivalent incompressible body.

Also,

$\left\{ \frac{\bar{V}_X}{V_\infty} \right\}$ ,  $\left\{ \frac{\bar{V}_Y}{V_\infty} \right\}$ , and  $\left\{ \frac{\bar{V}_Z}{V_\infty} \right\}$  are the components of the onset flow divided by the reference velocity  $V_\infty$ ,

$\{K\}_{B_i}$  are the quadrilateral vortex strengths on the  $i^{\text{th}}$  equivalent incompressible body, and

$\{K\}_{P_K}$ ,  $\{\Gamma\}_{P_K}$ , and  $\{\Sigma\}_{P_K}$  are the root section quadrilateral vortex strengths, the outboard section skewed vortex strengths, and the skewed source strengths on the  $k^{\text{th}}$  equivalent incompressible panel, respectively. The vector  $\{e\}_{B_j}$  is the flow through the surface of the  $j^{\text{th}}$  equivalent incompressible body at the control points, for a discrete body solution this vector is zero.

The influence equations for the  $j^{\text{th}}$  equivalent incompressible panel are given by;

$$\begin{aligned}
 & \sum_{i=1}^{N_B} \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \begin{matrix} A_{X_{P_j B_i}} \\ A_{Y_{P_j B_i}} \\ A_{Z_{P_j B_i}} \end{matrix} \right] + \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \begin{matrix} A_{X_{P_j P_K}} \\ A_{Y_{P_j P_K}} \\ A_{Z_{P_j P_K}} \end{matrix} \right] \left\{ \frac{K}{V_\infty} \right\}_{B_i} \\
 & + \sum_{K=1}^{N_P} \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \begin{matrix} A_{X_{P_j P_K}} \\ A_{Y_{P_j P_K}} \\ A_{Z_{P_j P_K}} \end{matrix} \right] \left\{ \frac{K}{V_\infty} \right\}_{P_K} \\
 & + \sum_{K=1}^{N_P} \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \begin{matrix} S_{X_{P_j P_K}} \\ S_{Y_{P_j P_K}} \\ S_{Z_{P_j P_K}} \end{matrix} \right] \left\{ \frac{\Sigma}{V_\infty} \right\}_{P_K} \\
 & + \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left\{ \frac{\bar{V}_X}{V_\infty} \right\} + \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left\{ \frac{\bar{V}_Y}{V_\infty} \right\} + \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left\{ \frac{\bar{V}_Z}{V_\infty} \right\} = \{e\}_{P_j}
 \end{aligned}
 \tag{28}$$



Where

$\bar{N}_{X_{P_j}}$ ,  $N_{Y_{P_j}}$ , and  $N_{Z_{P_j}}$  are the components of the  $j^{\text{th}}$  equivalent incompressible panel mean camber surface unit normal vectors,

$A_{X_{P_j B_i}}$ ,  $A_{Y_{P_j B_i}}$ , and  $A_{Z_{P_j B_i}}$  are the components of the perturbation velocity induced by the  $i^{\text{th}}$  equivalent incompressible body unit strength vortices onto the  $j^{\text{th}}$  equivalent incompressible panel,

$A_{X_{P_j P_k}}$ ,  $A_{Y_{P_j P_k}}$ , and  $A_{Z_{P_j P_k}}$  are the components of the perturbation velocity induced by the  $k^{\text{th}}$  equivalent incompressible panel unit strength vortices onto the  $j^{\text{th}}$  equivalent incompressible panel, and

$S_{X_{P_j P_k}}$ ,  $S_{Y_{P_j P_k}}$ , and  $S_{Z_{P_j P_k}}$  are the components of perturbation velocity induced by the  $k^{\text{th}}$  equivalent incompressible panel unit strength sources onto the  $j^{\text{th}}$  equivalent incompressible panel. The vector  $\{c\}_{pj}$  is the flow through the mean camber surface of the  $j^{\text{th}}$  equivalent incompressible panel at the control points. This vector is also zero for a discrete panel solution.

The onset flow velocity ratios

$\frac{\bar{V}_X}{V_\infty}$ ,  $\frac{\bar{V}_Y}{V_\infty}$ , and  $\frac{\bar{V}_Z}{V_\infty}$  are given by the following expressions,

$$\frac{\bar{V}_X}{V_\infty} = 1 - q^* \frac{2\beta (Z - Z_{C.G.})}{\bar{c}} - \gamma^* \frac{2\beta (Y - Y_{C.G.})}{b} \quad (29)$$

$$\frac{\bar{V}_Y}{V_\infty} = -\beta\beta - P^* \frac{2\beta (Z - Z_{C.G.})}{b} + \gamma^* \frac{2(X - X_{C.G.})}{b} \quad (30)$$

$$\frac{\bar{V}_Z}{V_\infty} = \alpha\beta + P^* \frac{2\beta (Y - Y_{C.G.})}{b} + q^* \frac{2(X - X_{C.G.})}{\bar{c}} \quad (31)$$

Where  $P^*$ ,  $q^*$ , and  $\gamma^*$  are the nondimensional roll, pitch, and yaw rates, respectively. These are defined as;

$$P^* = P / \frac{2V_\infty}{b}, \quad q^* = q / \frac{2V_\infty}{b}, \quad \text{and} \quad \gamma^* = \gamma / \frac{2V_\infty}{b} \quad (32)$$

The above onset flow equations assume that angle of attack  $\alpha$  and angle of yaw  $\beta$  are small. The coordinates  $X_{C.G.}$ ,  $Y_{C.G.}$ , and  $Z_{C.G.}$  define the location of the center of gravity.

Equations (27) and (28) can be combined into a single matrix aerodynamic influence equation. For a completely discrete type solution the influence equation is,

$$\begin{bmatrix} [A]_{B_j B_i} & [A]_{B_j P_k} \\ [A]_{P_j B_i} & [A]_{P_j P_k} \end{bmatrix} \begin{bmatrix} \left\{ \frac{K}{V_\infty} \right\}_{B_i} \\ \left\{ \frac{K}{V_\infty} \right\}_{P_k} \\ \left\{ \frac{\Gamma}{V_\infty} \right\}_{P_k} \end{bmatrix} = \begin{bmatrix} \left\{ \nabla S \right\}_{B_j} \\ \left\{ \nabla S \right\}_{P_j} \end{bmatrix} \quad (33)$$

where

$$B_j = B_1, B_2, \dots, B_{N_B}$$

$$B_i = B_1, B_2, \dots, B_{N_B}$$

$$P_j = P_1, P_2, \dots, P_{N_P}$$

$$P_k = P_1, P_2, \dots, P_{N_P}$$

$$N_B = \text{Number of bodies}$$

and

$$N_P = \text{Number of panels}$$

The matrices

$$[A]_{B_j B_i}, [A]_{B_j P_k}, [A]_{P_j B_i}, \text{ and } [A]_{P_j P_k}$$

are defined as;

$$[A]_{B_j B_i} = \begin{bmatrix} \bar{N}_{X_{B_j}} \end{bmatrix} \begin{bmatrix} A_{X_{B_j B_i}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Y_{B_j}} \end{bmatrix} \begin{bmatrix} A_{Y_{B_j B_i}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} A_{Z_{B_j B_i}} \end{bmatrix} \quad (34)$$

$$[A]_{B_j P_k} = \begin{bmatrix} \bar{N}_{X_{B_j}} \end{bmatrix} \begin{bmatrix} A_{X_{B_j P_k}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Y_{B_j}} \end{bmatrix} \begin{bmatrix} A_{Y_{B_j P_k}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} A_{Z_{B_j P_k}} \end{bmatrix} \quad (35)$$

$$[A]_{P_j B_i} = \begin{bmatrix} \bar{N}_{X_{P_j}} \end{bmatrix} \begin{bmatrix} A_{X_{P_j B_i}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Y_{P_j}} \end{bmatrix} \begin{bmatrix} A_{Y_{P_j B_i}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Z_{P_j}} \end{bmatrix} \begin{bmatrix} A_{Z_{P_j B_i}} \end{bmatrix} \quad (36)$$

$$[A]_{P_j P_k} = \begin{bmatrix} \bar{N}_{X_{P_j}} \end{bmatrix} \begin{bmatrix} A_{X_{P_j P_k}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Y_{P_j}} \end{bmatrix} \begin{bmatrix} A_{Y_{P_j P_k}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Z_{P_j}} \end{bmatrix} \begin{bmatrix} A_{Z_{P_j P_k}} \end{bmatrix} \quad (37)$$

The known quantities in the influence equation  $\{\nabla S\}_{B_j}$  and  $\{\nabla S\}_{P_j}$  are defined as;

$$\begin{aligned} \{\nabla S\}_{B_j} = & - \sum_{k=1}^{N_p} \left[ \begin{bmatrix} \bar{N}_{X_{B_j}} \end{bmatrix} \begin{bmatrix} S_{X_{B_j P_k}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Y_{B_j}} \end{bmatrix} \begin{bmatrix} S_{Y_{B_j P_k}} \end{bmatrix} \right. \\ & + \left. \begin{bmatrix} \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} S_{Z_{B_j P_k}} \end{bmatrix} \right] \left\{ \frac{\Sigma}{V_\infty} \right\}_{P_k} - \begin{bmatrix} \bar{N}_{X_{B_j}} \end{bmatrix} \left\{ \frac{\bar{V}_X}{V_\infty} \right\} \\ & - \begin{bmatrix} \bar{N}_{Y_{B_j}} \end{bmatrix} \left\{ \frac{\bar{V}_Y}{V_\infty} \right\} - \begin{bmatrix} \bar{N}_{Z_{B_j}} \end{bmatrix} \left\{ \frac{\bar{V}_Z}{V_\infty} \right\} \end{aligned} \quad (38)$$

$$\begin{aligned}
\left[ \nabla S \right]_{P_j} = & - \sum_{k=1}^{N_P} \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \begin{matrix} S_{X_{P_j P_k}} \\ S_{Y_{P_j P_k}} \\ S_{Z_{P_j P_k}} \end{matrix} \right] + \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \begin{matrix} \bar{V}_X \\ \bar{V}_Y \\ \bar{V}_Z \end{matrix} \right] \\
& + \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \begin{matrix} S_{X_{P_j P_k}} \\ S_{Y_{P_j P_k}} \\ S_{Z_{P_j P_k}} \end{matrix} \right] \left[ \frac{\Sigma}{V_\infty} \right]_{P_k} - \left[ \begin{matrix} \bar{N}_{X_{P_j}} \\ \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \frac{\bar{V}_X}{V_\infty} \right] \\
& - \left[ \begin{matrix} \bar{N}_{Y_{P_j}} \\ \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \frac{V_Y}{V_\infty} \right] - \left[ \begin{matrix} \bar{N}_{Z_{P_j}} \end{matrix} \right] \left[ \frac{V_Z}{V_\infty} \right] \quad (39)
\end{aligned}$$

The elements of the above matrices are computed using the influence equations derived in Appendices B and C. Each element is associated with the influence of a singularity on a control point. The singularities and control points are ordered such that all of the longitudinal stations for the first lateral station are cycled through first and then all of the longitudinal stations for the second lateral station. This process is continued until all stations have been cycled through. The longitudinal stations start at the leading edge of the panel, the nose of a solid body, and the tail end of the inside surface of a flow through body. The lateral stations start at the inside edge of the panel and go toward the tip of the panel. The lateral stations on the body start at the top of the body and progress in a clockwise direction when looking at the body from the tail to the nose.

All bodies are cycled through first and then the panels. Both the bodies and the panels are cycled in the order that they are input. The columns of the influence matrices are associated with singularities and the rows with control points. The elements of the matrices and the submatrices of the combined matrix influence equation are sequenced, for both the columns and the rows, in the same order that the singularities, points, bodies, and panels are cycled.

The influences of quadrilateral vortices in the panel root section and on the body surface are computed by equations derived in Appendix C. The influences of skewed horseshoe vortices on the panel outboard section are computed by equations derived in Appendix B. The influences of skewed source lines on the panel are computed by equations derived in Appendix B.

If the force free wake option of the program is used the influences of the free trailing vortices are computed using the influence equations in Appendix C. The locations of the force free trailing vortices are computed using the following iteration procedure.

1. A solution for the vortex strengths will be obtained first by assuming the location of the free vortices to be in the longitudinal direction and to be straight except in a thick lifting panel-body juncture region.
2. The total velocities  $U$ ,  $V$ , and  $W$  in the  $X$ ,  $Y$ , and  $Z$  directions, respectively, will be computed at the midpoint of each free trailing vortex division.
3. The actual mean camber surfaces of the thick lifting panels will be computed so that the correct relationship between fixed vortices and control points on the thick lifting panels and the free trailing vortices is maintained.
4. The ratios  $V/U$  and  $W/U$  are integrated in the  $X$  direction from the edges of the actual thick lifting panel mean camber surfaces and body aft ends where the free vortices are assumed to be shed, in order to obtain their new locations.
5. The influence of the free vortices on the actual thick lifting panel mean camber and body surfaces is computed and the difference between this influence and that from the free vortices at their previous location is added to the influence matrices for the complete configuration.
6. A new solution for the vortex strengths is determined and a new set of total velocities  $U$ ,  $V$ , and  $W$  along the new free vortex lines is computed using quadrilateral vortices and source lines on the actual thick lifting panel mean camber surfaces and quadrilateral vortices on the body surfaces.

The above procedure is iterated between steps (4) and (6) until the surface pressures converge. When the influence matrix of any individual body or panel is no longer significantly changed due to a new positioning of the free vortices, the calculation of the perturbation to that matrix is terminated.

#### Constrained Influence Equations

The vortex strengths on any of the bodies or panels can be constrained by representing the vorticity by a finite series. This uncouples the number of unknowns, equations, and vortices used to represent the perturbation velocity. The series used must be capable of producing the type of perturbation velocity needed to satisfy the boundary conditions at the control points to an acceptable degree. If such a series can be found for a given body or panel the number of equations or control points can be reduced from that of the number of

vortices or subpanels to that necessary to describe the geometry. This substantially reduces the computer time, since in general the number of vortices needed to represent the perturbation velocity is far more than the number of control points needed to represent the geometry of the body or panel.

This reduction in the number of required control points results primarily in reducing the computational effort necessary to set up the aerodynamic influence equations. A further reduction in computer time can be realized in the solution of the influence equations, if constraint functions are used, since the number of unknowns is reduced from that of the number of vortices to the number of terms in the constraint series. This results in a system of equations which is overdetermined and is solved by the method of least squares. In this process the vectors  $\{e\}_{B_j}$  and  $\{e\}_{p_j}$  are minimized. The body constraint equation is given by;

$$\left\{ \frac{K}{V_\infty} \right\}_{B_i} = \begin{bmatrix} T_{B_i} \end{bmatrix} \left\{ a_{B_i} \right\} \quad (40)$$

where

$$\begin{bmatrix} T_{B_i} \end{bmatrix} = \begin{bmatrix} R \end{bmatrix} \begin{bmatrix} \bar{l}_{M_{mn}} \end{bmatrix} \begin{bmatrix} \left( \frac{\gamma}{V_\infty} \right)_{mn} \end{bmatrix} \quad (41)$$

The matrix  $[R]$  is a transformation matrix used to obtain the quadrilateral vortex strengths  $K$  from the bound horseshoe vortex strengths  $\Gamma_b$ .

$$\begin{bmatrix} R \end{bmatrix} = \begin{bmatrix} [R]_n \end{bmatrix} \quad (42)$$

where  $[R]_n$  is made up of ones along the diagonal and in the lower triangle. The upper triangle is filled with zeros.

The indices  $m$  and  $n$  refer to longitudinal and lateral subpanel locations on the equivalent incompressible body, respectively. The vorticity ratio  $(\gamma/V_\infty)_{mn}$  is the value of the vorticity series at the  $m$ th subpanel from the nose of the body and the  $n$ th subpanel in the lateral direction around the body. The longitudinal length  $\bar{l}_{M_{mn}}$  is given by;

$$\bar{l}'_{M_{mn}} = \frac{1}{2} \left[ \frac{3}{4} \bar{l}_{M_{(m-1)n}} + \bar{l}_{M_{mn}} + \frac{1}{4} \bar{l}_{M_{(m+1)n}} \right] \quad (43)$$

where  $\bar{l}_{mn}$  is the length of the  $m^{\text{th}}$  subpanel from the nose of the equivalent incompressible body at the  $n^{\text{th}}$  lateral station.

The vorticity series  $(\gamma/V_\infty)_{mn}$  is the product of a longitudinal series and a lateral series. The elements of the matrix  $\{a_{B,i}\}$  are the unknown coefficients associated with the terms in  $(\gamma/V_\infty)_{mn}$  produced by the product of the longitudinal and lateral series. The terms in  $(\gamma/V_\infty)_{mn}$  are ordered such that the products of all of the longitudinal constraint functions and the first lateral constraint function are first, then the products of all of the longitudinal constraint functions and the second lateral constraint functions are second, and etcetera. This process is continued until all combinations of longitudinal and lateral constraint functions have been cycled through.

Both the longitudinal and lateral constraint functions are defined over segments of the body. The same functions are used in all segments. The origin of the segment is designated by the subscript o and the end by f. The longitudinal and lateral constraint function segments are given in the data input array.

The longitudinal constraint functions for the body are defined as;

$$\frac{\gamma}{V_\infty} (X_B/C_B)_1 = 1 \quad (44)$$

$$\frac{\gamma}{V_\infty} (X_B/C_B)_2 = 1 / \left\{ 1 + \left[ Y_B \left( \frac{\bar{N}_{X_B}}{\bar{N}_{Y_B}} \right) + Z_B \left( \frac{\bar{N}_{X_B}}{\bar{N}_{Z_B}} \right) \right]^2 / (Y_B^2 + Z_B^2) \right\}^{1/2} \quad (45)$$

$$\frac{\gamma}{V_\infty} (\phi_B)_3 = \cot \phi_{B/2} \quad (46)$$

$$\frac{\gamma}{V_\infty} (\phi_B)_4 = \cot (\pi/2 - \phi_{B/2}) \quad (47)$$

$$\frac{\gamma}{V_\infty} (\phi_B)_5 = \sin \left[ \pi \left( \frac{\phi_B - \phi_o}{\phi_f - \phi_o} \right) \right] \quad (48)$$

$$\frac{\gamma}{V_\infty} (\phi_B)_6 = \cos \left[ \pi \left( \frac{\phi_B - \phi_o}{\phi_f - \phi_o} \right) \right] \quad (49)$$

ORIGINAL PAGE IS  
OF POOR QUALITY

$$\frac{\gamma}{V_{\infty}} (X_B/C_B)_7 = \left[ \frac{\left(\frac{X_B}{C_B}\right)_B - \left(\frac{X_B}{C_B}\right)_0}{\left(\frac{X_B}{C_B}\right)_f - \left(\frac{X_B}{C_B}\right)_0} \right] \quad (50)$$

$$\frac{\gamma}{V_{\infty}} (\phi_B)_8 = \sin \left[ 2\pi \left( \frac{\phi_B - \phi_0}{\phi_f - \phi_0} \right) \right] \quad (51)$$

$$\frac{\gamma}{V_{\infty}} (\phi_B)_9 = \cos \left[ 2\pi \left( \frac{\phi_B - \phi_0}{\phi_f - \phi_0} \right) \right] \quad (52)$$

$$\frac{\gamma}{V_{\infty}} (X_B/C_B)_{10} = \left[ \frac{\left(\frac{X_B}{C_B}\right)_B - \left(\frac{X_B}{C_B}\right)_0}{\left(\frac{X_B}{C_B}\right)_f - \left(\frac{X_B}{C_B}\right)_0} \right]^2 \quad (53)$$

•  
•  
•

$$\frac{\gamma}{V_{\infty}} (X_B/C_B)_{3270} = L_{S_1} (X_B/C_B) \quad (54)$$

$$\frac{\gamma}{V_{\infty}} (X_B/C_B)_{3320} = L_{S_2} (X_B/C_B)$$

$$\frac{\gamma}{V_{\infty}} (X_B/C_B)_{3370} = L_{S_2} (X_B/C_B) \quad (55)$$



The functions to be used on the body are designated by their number in the above sequence. The last three functions listed above are special functions and are input at the locations in the data array, designated by their subscripts. The independent variable  $\theta_B$  is given by,  $\theta_B = \cos^{-1} [1 - 2(X_B/C_B)]$

The lateral constraint functions for the body are defined as;

$$\frac{\gamma}{V_\infty} (\theta_B)_1 = 1 \quad (57)$$

$$\frac{\gamma}{V_\infty} (\theta_B)_2 = \bar{N}_{Z_B} / \left[ \bar{N}_{Z_B}^2 + \bar{N}_{Y_B}^2 \right]^{1/2} \quad (58)$$

$$\frac{\gamma}{V_\infty} (\theta_B)_3 = \bar{N}_{Y_B} / \left[ \bar{N}_{Z_B}^2 + \bar{N}_{Y_B}^2 \right]^{1/2} \quad (59)$$

$$\frac{\gamma}{V_\infty} (\theta_B)_4 = \sin \left[ \pi \left( \frac{\theta_B - \theta_o}{\theta_f - \theta_o} \right) \right] \quad (60)$$

$$\frac{\gamma}{V_\infty} (\theta_B)_5 = \cos \left[ \pi \left( \frac{\theta_B - \theta_o}{\theta_f - \theta_o} \right) \right]$$

$$\frac{\gamma}{V_\infty} (\theta_B)_6 = \left[ \pi \left( \frac{\theta_B - \theta_o}{\theta_f - \theta_o} \right) \right] \text{ or } \left[ \frac{\eta_B - \eta_o}{\eta_f - \eta_o} \right] \quad (61)$$

$$\frac{\gamma}{V_{\infty}} (\theta_B)_7 = \sin \left[ 2\pi \left( \frac{\theta_B^- - \theta_o}{\theta_f^- - \theta_o} \right) \right] \quad (62)$$

$$\frac{\gamma}{V_{\infty}} (\theta_B)_8 = \cos \left[ 2\pi \left( \frac{\theta_B^- - \theta_o}{\theta_f^- - \theta_o} \right) \right] \quad (63)$$

$$\frac{\gamma}{V_{\infty}} (\theta_B)_9 = \left[ \pi \left( \frac{\theta_B^- - \theta_o}{\theta_f^- - \theta_o} \right) \right]^2 \quad \text{or} \quad \left[ \frac{\eta_B^- - \eta_o}{\eta_f^- - \eta_o} \right]^2 \quad (64)$$

•  
•  
•

Here again, the lateral functions to be used on the body are designated by their number in the above sequence. If no constraint functions are specified the program will do a discrete solution.

The panel constraint equation is given by;

$$\begin{Bmatrix} \frac{\kappa}{V_{\infty}} \\ \frac{\Gamma}{V_{\infty}} \end{Bmatrix}_{P_K} = \begin{bmatrix} T_{P_K} \end{bmatrix} \begin{Bmatrix} a_{P_K} \end{Bmatrix} \quad (65)$$

The  $[T_{P_K}]$  transformation matrix condenses all rows of the discrete aerodynamic influence matrix by the following procedure.

1. That portion of the row which is associated with the vortices on panel  $P_K$  is divided into the elements due to each lateral station. A new temporary matrix  $[A_T]$  is developed with all of the elements due to the vortices at the first lateral station in row one, all of the elements due to the vortices at the second lateral station in row two, and etcetera.

2. This temporary matrix is then premultiplied by the spanwise constraint function transformation matrix  $[T(\eta)]$  and postmultiplied by the chordwise constraint function transformation matrix  $[T(NC)]$ .
3. The transformed temporary matrix  $[\bar{A}_T] = [T(\eta)][A_T][T(X/C)]$  is then opened up into a row again and replaces the old row in the original discrete aerodynamic influence matrix. The new row is formed by placing all of the elements from the first row of  $[\bar{A}_T]$  in the portion of the row, from the discrete matrix, due to the  $P_K$  panel first, then by placing all of the elements from the second row of  $[\bar{A}_T]$  next to those elements from the first row of  $[\bar{A}_T]$ , and etcetera. The result will be the reduction of the number of elements in a given row due to the  $P_K$  panel from that of the number of vortices or subpanels on panel  $P_K$  to the number of chordwise constant functions times the number of spanwise constraint functions.

If only the chordwise constraint functions are used, then the number of elements in the row due to the  $P_K$  panel will be reduced from the number of vortices on the  $P_K$  panel to the number of chordwise constraint functions times the number of subpanels in the lateral direction. On panels with freestream edges, at span stations other than the tip, only the chordwise functions should be used, since the available spanwise constraint functions are not sufficient to produce the necessary perturbation velocity to satisfy the boundary conditions on a panel of this type.

The above described transformation constrains the skewed horseshoe vortex strengths by the following series developed in appendices F. and G. of reference (26).

$$\begin{aligned}
 \frac{\Gamma}{V_\infty}(\eta)_m = & \sqrt{1-\eta^2} \sum_{w=1}^{N_w - P_w} \left\{ \left( \frac{\Gamma'_{om}}{V_\infty} \right) a_{ow} + \sum_{f=1}^{N_f} \left( \frac{\Gamma'_{fm}}{V_\infty} \right) a_{fw} + \sum_{k=1}^{N_k} \left( \frac{\Gamma'_{km}}{V_\infty} \right) a_{kw} \right. \\
 & \left. + \sum_{n=1}^{N_u-1} \left( \frac{\Gamma'_{nm}}{V_\infty} \right) a_{nw} \right\} \eta^w + \sum_{w=(N_w - P_w + 1)}^{N_w} \left\{ \left( \frac{\Gamma'_{om}}{V_\infty} \right) a_{ow} \right. \\
 & \left. + \sum_{f=1}^{N_f} \left( \frac{\Gamma'_{fm}}{V_\infty} \right) a_{fw} + \sum_{k=1}^{N_k} \left( \frac{\Gamma'_{km}}{V_\infty} \right) a_{kw} + \sum_{n=1}^{N_u-1} \left( \frac{\Gamma'_{nm}}{V_\infty} \right) a_{nw} \right\} P_w(\eta)_s
 \end{aligned} \tag{66}$$

Where the indices  $m, f, k, o, n,$  and  $w$  indicate the number of the subpanel aft of the panel leading edge  $\Gamma/V_\infty$  is defined on, the number of the trailing edge flap, the number of the leading edge flap, the first term of the Birnbaum series, the number of the sine term in the Birnbaum series, and the number of the spanwise constraint function, respectively. The quantities  $N_u, N_f, N_k, P_w,$  and  $N_w$  are the number of terms in the Birnbaum series, the number of trailing edge flaps with unique hinge line locations, the number of leading edge flaps with unique hinge line locations, the number of special spanwise constraint functions, and the total number of spanwise constraint functions, respectively.

The columns of the matrix  $[T(X/C)]$  are made up of the  $(\Gamma'/V_\infty)_m$  values, where  $m$  indicates the number of the element in the column. There is one column for each set of  $(\Gamma'/V_\infty)_m$ 's associated with a unique chordwise constraint function. The  $[T(X/C)]$  matrix is filled such that the  $(\Gamma'_{om}/V_\infty)$  values are in column one, then the  $(\Gamma'_{n}/V_\infty)$  values are in the next set of  $N_u-1$  columns, then the  $(\Gamma'_{fm}/V_\infty)$  values are in the next set of  $N_f$  columns, and then the  $(\Gamma'_{km}/V_\infty)$  values are in the next set of  $N_k$  columns.

The  $[T(X/C)]$  matrix is solved here in the same manner as is shown in appendix G. of reference (26). The  $(\Gamma'/V_\infty)_m$  values are solved for such that the same downwash is obtained at the three quarter chord point of each subpanel, except at the last subpanel, due to a distribution of discrete vortices as would be obtained by integrating the vorticity distribution in the Blot-Savart integral for each chordwise constraint function. The additional condition, that the sum of the discrete vortex strengths must be equal to the integral of the vorticity distribution is also used. These conditions result in the following matrix equation which is solved for  $[T(x/c)]$ .

$$[E] [T(x/c)] = [W] \quad (67)$$

The matrix  $[E]$  is the two-dimensional discrete vortex influence matrix and is defined as follows.

$$[E] = \begin{bmatrix} \frac{1}{[(x/c)_j - (x/c)_i]} \\ \vdots \\ 1, 1, \dots, 1 \end{bmatrix} \quad (68)$$

ORIGINAL PAGE IS  
OF POOR QUALITY

Where  $0 \leq (x/c) \leq 1.0$  and the indices  $j$  and  $i$  as used here indicate the location of the three quarter and quarter chord points, respectively.

There is one column in the  $[w]$  matrix for each unique chordwise constraint function. In these columns is the downwash due to evaluating the vorticity for each chordwise constraint function in the two-dimensional Biot-Savart integral. The columns in  $[w]$  are ordered such that the column due to a given chordwise constraint function is in the same location in  $[w]$  as its corresponding  $(\Gamma'/V_\infty)_m$  column is in the  $[T(x/c)]$  matrix. The five basic types of columns in  $[w]$  are defined as follows.

Due to the first term of the Birnbaum series;

$$\left\{ w \right\}_{\cot\phi/2} = \begin{Bmatrix} 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \\ \text{---} \\ 1/2 \end{Bmatrix} \quad (69)$$

Due to the second term of the Birnbaum series;

$$\left\{ w \right\}_{\sin\phi} = \begin{Bmatrix} -\cos \phi_1 \\ -\cos \phi_2 \\ \cdot \\ \cdot \\ \cdot \\ -\cos \phi_{(N_1-1)} \\ \text{---} \\ 1/4 \end{Bmatrix} \quad (70)$$

Due to the third and higher order terms of the Birnbaum series;

$$\left\{ w \right\}_{\sin(n\phi)} = \left\{ \begin{array}{c} -\cos n \phi_1 \\ -\cos n \phi_2 \\ \vdots \\ -\cos n \phi_{(N_i-1)} \\ \hline 0 \end{array} \right\} \quad (71)$$

where  $\phi_j = \cos^{-1} [1 - 2 (x/c)_j]$

Due to the trailing edge flap term;

$$\left\{ w \right\}_{\log \left| \frac{\sin 1/2(\phi + \phi_f)}{\sin 1/2(\phi - \phi_f)} \right|} = \left\{ \begin{array}{c} w_f(\phi_1) \\ w_f(\phi_2) \\ \vdots \\ w_f(\phi_{N_i-1}) \\ \hline 1/2 \sin \phi_f \end{array} \right\} \quad (72)$$

where  $w_f(\phi_j) = -(\pi - \phi_f)$  for  $0 \leq \phi_j < \phi_f$ ,  $w_f(\phi_j) = \phi_f$  for  $\phi_f \leq \phi_j \leq \pi$ ,  
is the polar coordinate at the flap leading edge, and  $N_i$  equals the number  
of subpanels per chord.

due to the leading edge flap term;

$$\left\{ w \right\} = \begin{Bmatrix} w_k(\phi_1) \\ w_k(\phi_2) \\ \vdots \\ w_k \left[ \phi_{(N_i-1)} \right] \\ \vdots \\ 1/2 \sin \phi_k \end{Bmatrix} \quad (73)$$

$$\log \left| \frac{\sin 1/2(\phi + \phi_k)}{\sin 1/2(\phi - \phi_k)} \right|$$

where  $w_k(\phi_j) = -(\pi - \phi_k)$  for  $0 \leq \phi_j \leq \phi_k$ ,  $w_k(\phi_j) = \phi_k$  for  $\phi_k < \phi_j \leq \pi$ , and  $\phi_k$  is the polar coordinate at the flap leading edge.

In the root section the  $[T(X/C)]$  matrix is transformed to  $[\bar{T}(X/C)]$ , where each column of  $[T(X/C)]$  is transformed as follows;

$$\bar{T}(X/C)_m = \sum_{i=1}^m T(X/C)_i \quad (74)$$

where  $i$  and  $m$  indicate the number of the element in the column in the  $[T(X/C)]$  and  $[\bar{T}(X/C)]$  matrices, respectively.

The rows of the  $[T(\eta)]$  matrix are made up of the spanwise constraint functions. Each element in a row is equal to the value of the constraint function at the lateral subpanel which has the same lateral index as the number of the column in the  $[T(\eta)]$  matrix. There are  $N_W$  rows in the  $[T(\eta)]$  matrix, with the standard functions described by  $\eta^W \sqrt{1 - \eta^2}$  in the first  $N_W - P_W$  rows, and then the special  $P_W(\eta)$ s functions in the last  $P_W$  rows.

The special spanwise constraint functions are needed to account for flow induced by discontinuities in the sweep of the constant percent chord lines, for body-panel juncture induced flow, and for partial semi-span flaps. There are two basic special spanwise constraint functions, (1) polygonal functions which account for the discontinuities in the sweep of the constant percent chord lines and the body-panel junctures, and (2) flap functions which account for the partial semi-span flaps. These special functions are derived in Appendix F of Reference (26).

If the range of influences inboard and outboard of a discontinuity in the sweep of the constant percent chord line are defined as  $\Delta\eta_i$  and  $\Delta\eta_o$ , respectively, and the discontinuity station  $\eta_b$ , then the following expressions define the special spanwise polygonal constraint function.

For  $0 < |\eta_b| < \Delta\eta_i$

$$P(\theta)_S = (1 - \eta_b / \Delta\eta_i) M(\theta, \frac{\pi}{2}) + \frac{1}{\Delta\eta_i} P(\theta, \frac{\pi}{2}) - \left[ \frac{(\Delta\eta_i + \Delta\eta_o)(1 - \eta_b)}{\Delta\eta_i \Delta\eta_o} \right] P(\theta, \theta_b) \\ + \left[ \frac{\Delta\eta_i + \Delta\eta_o}{\Delta\eta_i \Delta\eta_o} - \frac{1}{\Delta\eta_i} \right] [1 - \Delta\eta_o - \eta_b] P(\theta, \cos^{-1}(\eta_b + \Delta\eta_o)) \quad (75)$$

For  $\Delta\eta_i \leq |\eta_b| < (1 - \Delta\eta_o)$

$$P(\theta)_S = (1 - \eta_b + \Delta\eta_i) (1 / \Delta\eta_i) P(\theta, \cos^{-1}(\eta_b - \Delta\eta_i)) \\ - \left[ \frac{\Delta\eta_o + \Delta\eta_i}{\Delta\eta_o \Delta\eta_i} \right] [1 - \eta_b] P(\theta, \theta_b) \\ + (1 - \eta_b - \Delta\eta_o) (1 / \Delta\eta_o) P(\theta, \cos^{-1}(\eta_b + \Delta\eta_o)) \quad (76)$$

And for  $(1 - \Delta\eta_o) \leq |\eta_b| < 1.0$

$$P(\phi)_S = (1 - \eta_b + \Delta\eta_i) (1 / \Delta\eta_i) P(\theta, \cos^{-1}(\eta_b - \Delta\eta_i)) \\ - [1 + (1 - \eta_b) / \Delta\eta_i] P(\theta, \theta_b) \quad (77)$$



Where

$$P(\theta, \theta^*) = \frac{1}{2\pi(1 - \cos \theta^*)} \left\{ (\cos \theta^* - \cos \theta) \text{LOG}_e \left| \frac{\sin 1/2 (\theta^* - \theta)}{\sin 1/2 (\theta^* + \theta)} \right| \right. \\ \left. + (\cos \theta^* + \cos \theta)^2 \text{LOG}_e \left| \frac{\cos 1/2 (\theta^* + \theta)}{\cos 1/2 (\theta^* - \theta)} \right| \right. \\ \left. + (4 \theta^* \cos \theta^* - 2 \sin \theta^*) \sin \theta \right\} \quad (78)$$

And

$$\theta^* = \cos^{-1} \eta$$

The special spanwise flap constraint function is given by;

$$P(\theta)_S = M(\theta, \theta_i) - M(\theta, \theta_o) \quad (79)$$

Where  $\theta_i = \cos^{-1} \eta_i$ ,  $\theta_o = \cos^{-1} \eta_o$ ,  $\eta_i$  is the inboard station where the control surface begins, and  $\eta_o$  is the outboard station where the control surface ends.

$$M(\theta, \theta^*) = \frac{1}{\pi} \left\{ (\cos \theta^* - \cos \theta) \text{LOG}_e \left| \frac{\sin 1/2 (\theta^* - \theta)}{\sin 1/2 (\theta^* + \theta)} \right| \right. \\ \left. + (\cos \theta^* + \cos \theta) \text{LOG}_e \left| \frac{\cos 1/2 (\theta^* + \theta)}{\cos 1/2 (\theta^* - \theta)} \right| + 2 \theta^* \sin \theta \right\} \quad (80)$$

The discrete aerodynamic influence equation (33) is transformed by substituting equations (40) and (65) into equation (33).

$$\begin{bmatrix} [A]_{B_j B_i} & [A]_{B_j P_K} \\ [A]_{P_j B_i} & [A]_{P_j P_K} \end{bmatrix} \begin{bmatrix} [T_{B_i}] [0] \\ [0] [T_{P_K}] \end{bmatrix} \begin{bmatrix} |a|_{B_i} \\ |a|_{P_K} \end{bmatrix} = \begin{bmatrix} |\nabla S|_{B_j} \\ |\nabla S|_{P_j} \end{bmatrix} + \begin{bmatrix} |e|_{B_j} \\ |e|_{P_j} \end{bmatrix} \quad (81)$$

The constrained aerodynamic influence equation is then given by;

$$\begin{bmatrix} [A]_{B_j B_i} & [A]_{B_j P_K} \\ [A]_{P_j B_i} & [A]_{P_j P_K} \end{bmatrix} \begin{bmatrix} |a|_{B_i} \\ |a|_{P_K} \end{bmatrix} = \begin{bmatrix} |\nabla S|_{B_j} \\ |\nabla S|_{P_j} \end{bmatrix} + \begin{bmatrix} |e|_{B_j} \\ |e|_{P_j} \end{bmatrix} \quad (82)$$

Where the arrays  $|a|_{B_i}$  and  $|a|_{P_K}$  are solved for such that the sum of the squares of the elements of the arrays  $|e|_{B_j}$  and  $|e|_{P_j}$  are a minimum.

## Surface Velocities and Pressures

The velocity tangent to the surface of the  $j^{\text{th}}$  body in the longitudinal direction is given by;

$$\begin{aligned}
 \left( \frac{V_M}{V_\infty} \right)_{mn} \Big|_{B_j} = & \sum_{i=1}^{N_B} \left[ \frac{1}{\beta^2} \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \begin{bmatrix} A_{XB_j B_i} \\ A_{YB_j B_i} \\ A_{ZB_j B_i} \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \begin{bmatrix} A_{XB_j P_K} \\ A_{YB_j P_K} \\ A_{ZB_j P_K} \end{bmatrix} \right] \left( \frac{K}{V_\infty} \right)_{B_i} \\
 & + \sum_{K=1}^{N_P} \left[ \frac{1}{\beta^2} \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \begin{bmatrix} S_{XB_j P_K} \\ S_{YB_j P_K} \\ S_{ZB_j P_K} \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \begin{bmatrix} S_{XB_j P_K} \\ S_{YB_j P_K} \\ S_{ZB_j P_K} \end{bmatrix} \right] \left( \frac{K}{V_\infty} \right)_{P_K} \\
 & + \sum_{K=1}^{N_P} \left[ \frac{1}{\beta^2} \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \begin{bmatrix} S_{XB_j P_K} \\ S_{YB_j P_K} \\ S_{ZB_j P_K} \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \begin{bmatrix} S_{XB_j P_K} \\ S_{YB_j P_K} \\ S_{ZB_j P_K} \end{bmatrix} \right] \left( \frac{Z}{V_\infty} \right)_{P_K} \\
 & + \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \left( \frac{V_X}{V_\infty} \right) + \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \left( \frac{V_Y}{V_\infty} \right) + \begin{bmatrix} T_{MX_{B_j}} \\ T_{MY_{B_j}} \\ T_{MZ_{B_j}} \end{bmatrix} \left( \frac{V_Z}{V_\infty} \right) \\
 & + \left\{ \left( \frac{\Delta V_M}{V_\infty} \right)_{mn} / \beta \sqrt{\beta^2 \bar{T}_{M_X}^2 + \bar{T}_{M_Y}^2 + \bar{T}_{M_Z}^2} \right\}_{B_j}
 \end{aligned} \tag{83}$$

Where

$$\begin{aligned}
 \left( \frac{\Delta V_M}{V_\infty} \right)_{mn} = & \left[ \frac{\Gamma_{b_{mn}}}{V_\infty} / \left( \frac{3}{4} \bar{I}_{M(m-1)n} + \bar{I}_{M_{mn}} + \frac{1}{4} \bar{I}_{M(m+1)n} \right) \right] \left[ \frac{\bar{I}_{M_{mn}} + \bar{I}_{M(m+1)n}}{3\bar{I}_{M_{mn}} + \bar{I}_{M(m+1)n}} \right] \\
 & + \left[ \frac{\Gamma_{b_{(m+1)n}}}{V_\infty} / \left( \frac{3}{4} \bar{I}_{M_{mn}} + \bar{I}_{M(m+1)n} + \frac{1}{4} \bar{I}_{M(m+2)n} \right) \right] \left[ \frac{2\bar{I}_{M_{mn}}}{3\bar{I}_{M_{mn}} + \bar{I}_{M(m+1)n}} \right]
 \end{aligned} \tag{84}$$

And

$$\frac{\Gamma_{b_{mn}}}{V_\infty} = \frac{K_{mn}}{V_\infty} - \frac{K_{(m-1)n}}{V_\infty} \tag{85}$$

$$\frac{\Gamma_b (m+1)N}{V_\infty} = \frac{K (m+1)N}{V_\infty} - \frac{K_{mN}}{V_\infty} \quad (86)$$

The onset flow ratios  $V_X/V_\infty$ ,  $V_Y/V_\infty$ , and  $V_Z/V_\infty$  are given by the following expressions.

$$\frac{V_X}{V_\infty} = 1 - q^* \frac{2(Z - Z_{C.G.})}{\bar{c}} - \gamma^* \frac{2(Y - Y_{C.G.})}{b} \quad (87)$$

$$\frac{V_Y}{V_\infty} = \beta + p^* \frac{2(Z - Z_{C.G.})}{b} + \gamma^* \frac{2(X - X_{C.G.})}{\bar{b}} \quad (88)$$

$$\frac{V_Z}{V_\infty} = \alpha + p^* \frac{2(Y - Y_{C.G.})}{b} + q^* \frac{(X - X_{C.G.})}{\bar{c}} \quad (89)$$

The components of the unit vectors tangent to the actual body subpanels in the longitudinal direction are given by;

$$T_{M_X} = \beta \bar{T}_{M_X} / \sqrt{\beta^2 \bar{T}_{M_X}^2 + \bar{T}_{M_Y}^2 + \bar{T}_{M_Z}^2} \quad (90)$$

$$T_{M_Y} = \bar{T}_{M_Y} / \sqrt{\beta^2 \bar{T}_{M_X}^2 + \bar{T}_{M_Y}^2 + \bar{T}_{M_Z}^2} \quad (91)$$

$$T_{M_Z} = \bar{T}_{M_Z} / \sqrt{\beta^2 \bar{T}_{M_X}^2 + \bar{T}_{M_Y}^2 + \bar{T}_{M_Z}^2} \quad (92)$$

and in the lateral direction by

$$T_{T_X} = \beta \bar{T}_{T_X} / \sqrt{\beta^2 \bar{T}_{T_X}^2 + \bar{T}_{T_Y}^2 + \bar{T}_{T_Z}^2}$$

$$T_{T_Y} = \bar{T}_{T_Y} / \sqrt{\beta^2 \bar{T}_{T_X}^2 + \bar{T}_{T_Y}^2 + \bar{T}_{T_Z}^2}$$

$$T_{T_Z} = \bar{T}_{T_Z} / \sqrt{\beta^2 \bar{T}_{T_X}^2 + \bar{T}_{T_Y}^2 + \bar{T}_{T_Z}^2}$$

The velocity tangent to the surface of the  $j^{\text{th}}$  body in the lateral direction is given by;

$$\begin{aligned}
 \left\{ \left( \frac{V_T}{V_\infty} \right)_{mn} \right\}_{B_j} &= \sum_{i=1}^{N_B} \left[ \frac{1}{\beta^2} \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \begin{bmatrix} A_{XB_j B_i} \\ A_{YB_j B_i} \\ A_{ZB_j B_i} \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \begin{bmatrix} A_{XB_j B_i} \\ A_{YB_j B_i} \\ A_{ZB_j B_i} \end{bmatrix} \right] \left\{ \frac{K}{V_\infty} \right\}_{B_i} \\
 &+ \sum_{K=1}^{N_P} \left[ \frac{1}{\beta^2} \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \begin{bmatrix} A_{XB_j P_K} \\ A_{YB_j P_K} \\ A_{ZB_j P_K} \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \begin{bmatrix} A_{XB_j P_K} \\ A_{YB_j P_K} \\ A_{ZB_j P_K} \end{bmatrix} \right] \left\{ \frac{K}{V_\infty} \right\}_{P_K} \\
 &+ \sum_{K=1}^{N_P} \left[ \frac{1}{\beta^2} \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \begin{bmatrix} S_{XB_j P_K} \\ S_{YB_j P_K} \\ S_{ZB_j P_K} \end{bmatrix} + \frac{1}{\beta} \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \begin{bmatrix} S_{XB_j P_K} \\ S_{YB_j P_K} \\ S_{ZB_j P_K} \end{bmatrix} \right] \left\{ \frac{Z}{V_\infty} \right\}_{P_K} \\
 &+ \left\{ \left( \frac{\Delta V_T}{V_\infty} \right)_{mn} / \beta \sqrt{\beta^2 \bar{T}_{TX}^2 + \bar{T}_{TY}^2 + \bar{T}_{TZ}^2} \right\}_{B_j} \\
 &+ \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \left\{ \frac{V_X}{V_\infty} \right\} + \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \left\{ \frac{V_Y}{V_\infty} \right\} + \begin{bmatrix} T_{TX_{B_j}} \\ T_{TY_{B_j}} \\ T_{TZ_{B_j}} \end{bmatrix} \left\{ \frac{V_Z}{V_\infty} \right\}
 \end{aligned} \tag{93}$$

Where

$$\left( \frac{\Delta V_T}{V_\infty} \right)_{mn} = -\frac{1}{2} \left[ \frac{\Gamma_{T_{m(n - \Delta n/2)}}}{V_\infty} / \left( \bar{T}_{T_{mn}} + \bar{T}_{T_{m(N-1)}} \right) + \frac{\Gamma_{T_{m(n + \Delta n/2)}}}{V_\infty} / \left( \bar{T}_{T_{mn}} + \bar{T}_{T_{m(n+1)}} \right) \right] \tag{94}$$

And

$$\frac{\Gamma_{T_m(n - \Delta n/2)}}{V_\infty} = \frac{K_{m(n-1)}}{V_\infty} - \frac{K_{mn}}{V_\infty} - \sum_{r=1}^{N_a} \left[ \frac{K_{m(n - \Delta n/2)}}{V_\infty} \right]_{P_r} \quad (95)$$

$$\frac{\Gamma_{T_m(n + \Delta n/2)}}{V_\infty} = \frac{K_{mn}}{V_\infty} - \frac{K_{m(n+1)}}{V_\infty} - \sum_{r=1}^{N_a} \left[ \frac{K_{m(n + \Delta n/2)}}{V_\infty} \right]_{P_r} \quad (96)$$

Where

$$\sum_{r=1}^{N_a} \left[ \frac{K_{m(n - \Delta n/2)}}{V_\infty} \right] \quad \text{and} \quad \sum_{r=1}^{N_a} \left[ \frac{K_{m(n + \Delta n/2)}}{V_\infty} \right]_{P_r}$$

are the contributions from the trailing legs of the panel vortices along the juncture line of the  $j$ th body and  $r$ th panel to be attached at the  $(n - \Delta n/2)$  and  $(n + \Delta n/2)$  lateral stations, respectively.  $N_a$  is the total number of panels attached at any one point along the line.

The velocity tangent to the surface of the  $j$ th panel in the longitudinal direction is given by;

$$\begin{aligned} \left[ \left( \frac{V_M}{V_\infty} \right)_{mn} \right]_{P_j} &= \left[ \frac{1}{[1 + (1 + \tan^2 \Lambda)_{mn} (dz_T/dx \pm dz_c/dx)^2]^{1/2}} \right] \left\{ \sum_{i=1}^{N_B} \frac{1}{\beta^2} \left[ A_{X_{P_j B_i}} \right] \left[ \frac{K}{V_\infty} \right]_{B_i} \right. \\ &\quad + \sum_{k=1}^{N_P} \frac{1}{\beta^2} \left[ \left[ A_{X_{T_j P_k}} \right] \left[ \frac{K}{V_\infty} \right]_{P_k} + \left[ S_{X_{P_j P_k}} \right] \left[ \frac{\Sigma}{V_\infty} \right]_{P_k} \right] \\ &\quad + \frac{1}{\beta^2} \left[ \left( \frac{\Delta V_M}{V_\infty} \right)_{mn} \right]_{P_j} \left[ \left[ 1 \right] + \left[ \sqrt{1 + \tan^2 \Lambda_{mn}} \right] \left[ S_{X_{P_j P_j}} \right] \left[ \left[ \frac{\Sigma}{V_\infty} \right]_{P_j} - \left[ \frac{\Sigma}{V_\infty} \right]_{P_j} \right] \right] \\ &\quad \left. + \left[ \frac{V_X}{V_\infty} \right] \right\} \quad (97) \end{aligned}$$

Where

$$\frac{\Sigma'}{V_\infty} = \left[ \frac{(Z_T/C)}{(X/C) (1 - X/C)} \right] \left[ \frac{\bar{V}_X/V_\infty}{\sqrt{1 + \frac{\tan^2 \Lambda}{\beta^2}}} \right] \frac{\bar{\ell}_M}{2} \quad (98)$$

and where  $(\Delta V_M/V_\infty)_{mn}$  is computed using equation (84) and  $X/C$  is the local percent chord.

The velocity tangent to the surface of the  $j^{\text{th}}$  panel in the lateral direction is given by;

$$\begin{aligned} \left\{ \left( \frac{V_T}{V_\infty} \right)_{mn} \right\}_{P_j} &= \left[ \frac{1}{|1 + (1 + \tan^2 \Lambda)_{mn} (dz_t/dx + dz_c/dx)^2|^{1/2}} \right] \left\{ \sum_{i=1}^{N_B} \frac{1}{\beta} \left[ \begin{matrix} T_{Y P_j} \\ A_{Y P_j B_i} \end{matrix} \right] \right. \\ &\quad + \left[ \begin{matrix} T_{Z P_j} \\ A_{Z P_j B_i} \end{matrix} \right] \left\{ \frac{K}{V_\infty} \right\}_{B_i} + \sum_{K=1}^{N_P} \frac{1}{\beta} \left[ \left[ \begin{matrix} T_{Y P_j} \\ A_{Y P_j P_K} \end{matrix} \right] + \left[ \begin{matrix} T_{Z P_j} \\ A_{Z P_j P_K} \end{matrix} \right] \right] \left\{ \frac{K}{V_\infty} \right\}_{P_K} \\ &\quad + \sum_{K=1}^{N_P} \frac{1}{\beta} \left[ \left[ \begin{matrix} T_{Y P_j} \\ S_{Y P_j P_K} \end{matrix} \right] + \left[ \begin{matrix} T_{Z P_j} \\ S_{Z P_j P_K} \end{matrix} \right] \right] \left\{ \frac{\Sigma}{V_\infty} \right\}_{P_K} + \left[ \begin{matrix} T_{Y P_j} \\ V_Y/V_\infty \end{matrix} \right] \\ &\quad + \left[ \begin{matrix} T_{Z P_j} \\ V_Z/V_\infty \end{matrix} \right] + \frac{1}{\beta} \left[ \left( \frac{\Delta V_T}{V_\infty} \right)_{mn} \right] \left[ 1 + \sqrt{1 + \tan^2 \Lambda_{mn}} \right] \left[ \begin{matrix} T_{Y P_j} \\ S_{Y P_j P_K} \end{matrix} \right] \\ &\quad \left. + \left[ \begin{matrix} T_{Z P_j} \\ S_{Z P_j P_K} \end{matrix} \right] \right] \left[ \left\{ \frac{\Sigma}{V_\infty} \right\}_{P_j} - \left\{ \frac{\Sigma'}{V_\infty} \right\}_{P_j} \right] \right\} \end{aligned} \quad (99)$$

where

$$\left( \frac{\Delta V_T}{V_\infty} \right)_{mn} = \frac{1}{2} \left[ \frac{\frac{\Gamma_{T_m} \left( n - \frac{\Delta n}{2} \right)}{V_\infty}}{\left( \bar{\ell}_{T_{mn}} + \bar{\ell}_{T_{m(n-1)}} \right)} + \frac{\frac{\Gamma_{T_m} \left( n + \frac{\Delta n}{2} \right)}{V_\infty}}{\left( \bar{\ell}_{T_{mN}} + \bar{\ell}_{T_{m(n+1)}} \right)} \right] + \left[ \left( \frac{\Delta V_M}{V_\infty} \right) \text{TAN } \Lambda \right]_{mn} \quad (100)$$

and where

$$\frac{\Gamma_{T_m} \left( n - \frac{\Delta n}{2} \right)}{V_\infty}$$

and

$$\frac{\Gamma_{T_m} \left( n + \frac{\Delta n}{2} \right)}{V_\infty}$$

are computed the same as for the body except

$$\sum_{r=1}^{N_a} \left[ \frac{K_m \left( n - \frac{\Delta n}{2} \right)}{V_\infty} \right]_{P_r}$$



and

$$\sum_{r=1}^{N_a} \left[ \frac{K_m n + \left(\frac{\Delta n}{2}\right)}{V_\infty} \right]_{P_r}$$

represent the contributions from the trailing legs of the panel vortices along the juncture line of the  $j$ th panel and the  $r$ th panel to be attached at the lateral stations,

$$\left(n - \frac{\Delta n}{2}\right) \text{ and } \left(n + \frac{\Delta n}{2}\right), \text{ respectively.}$$

The surface pressure coefficients at each of the control points on the bodies and panels are then computed using the following expression.

$$C_{p_{mn}} = \frac{2}{\gamma M_\infty^2} \left[ \left\{ 1 + \frac{\gamma-1}{2} M_\infty^2 \left[ 1 - \left( \frac{V_M}{V_\infty} \right)_{mn}^2 - \left( \frac{V_T}{V_\infty} \right)_{mn}^2 \right] \right\}^{\gamma/(\gamma-1)} \right] \quad (101)$$

where  $\gamma$  is the ratio of specific heats.

Note, in equations (97) and (100) the top sign is used to compute the velocity on the upper surface and the bottom sign the lower surface in those terms which have a plus and minus in front.

## Section and Total Loads and Moments

The section loads are computed in coefficient form on each of the bodies by interpolating for the surface pressure coefficients at the centroid of the subareas, defined by the corner points of the divisions of the body subpanels, and then using the following equations to sum the product of the pressure coefficients and directed subareas.

$$\left( \frac{C_X W}{W_{AVG}} \right)_K = - \frac{1}{A_Z \Delta \left( \frac{X_B}{C_B} \right)_K} \sum_i C_{P_{iK}} \Delta A_{X_{iK}} \quad (102)$$

$$\left( \frac{C_Y h}{h_{AVE}} \right)_K = - \frac{1}{A_Y \Delta \left( \frac{X_B}{C_B} \right)_K} \sum_i C_{P_{iK}} \Delta A_{Y_{iK}} \quad (103)$$

$$\left( \frac{C_Z W}{W_{AVG}} \right)_K = - \frac{1}{A_Z \Delta \left( \frac{X_B}{C_B} \right)_K} \sum_i C_{P_{iK}} \Delta A_{Z_{iK}} \quad (104)$$

where  $i$  is indexed over all subareas in the longitudinal segment  $\Delta(X_B/C_B)_K$ .

$$A_X = \frac{1}{2} \sum_K \sum_i |\Delta A_{X_{iK}}| \quad (105)$$

$$A_Y = \frac{1}{2} \sum_K \sum_i |\Delta A_{Y_{iK}}| \quad (106)$$

$$A_Z = \frac{1}{2} \sum_K \sum_i |\Delta A_{Z_{iK}}| \quad (107)$$

Also;

$$W_{AVG} = \frac{A_Z}{C_B} \quad (108)$$

and

$$h_{AVG} = \frac{A_Y}{C_B} \quad (109)$$

The total loads on a body are then obtained by summing in the longitudinal direction.

$$C_{X_{B_j}} = \frac{A_Z}{A_{B_j}} \sum_K \left( \frac{C_X W}{W_{AVG}} \right)_K \Delta \left( \frac{X_B}{C_B} \right)_K \quad (110)$$

$$C_{Y_{B_j}} = \frac{A_Y}{A_{B_j}} \sum_K \left( \frac{C_Y h}{h_{AVG}} \right)_K \Delta \left( \frac{X_B}{C_B} \right)_K \quad (111)$$

$$C_{Z_{B_j}} = \frac{A_Z}{A_{B_j}} \sum_K \left( \frac{C_Z W}{W_{AVG}} \right)_K \Delta \left( \frac{X_B}{C_B} \right)_K \quad (112)$$

The total moments on a body are summed about the center of gravity.

$$C_{M_{X_{B_j}}} = -\frac{1}{A_{B_j} \bar{C}} \sum_K \sum_i \left[ (Y_{iK} - Y_{C.G.}) \Delta A_{Z_{iK}} - (Z_{iK} - Z_{C.G.}) \Delta A_{Y_{iK}} \right] C_{P_{iK}} \quad (113)$$

$$C_{M_{Y_{B_j}}} = -\frac{1}{A_{B_j} \bar{C}} \sum_K \sum_i \left[ (Z_{iK} - Z_{C.G.}) \Delta A_{X_{iK}} - (X_{iK} - X_{C.G.}) \Delta A_{Z_{iK}} \right] C_{P_{iK}} \quad (114)$$

$$C_{M_{Z_{B_j}}} = -\frac{1}{A_{B_j} \bar{C}} \sum_K \sum_i \left[ (X_{iK} - X_{C.G.}) \Delta A_{Y_{iK}} - (Y_{iK} - Y_{C.G.}) \Delta A_{X_{iK}} \right] C_{P_{iK}} \quad (115)$$

The body center of pressure position vector components divided by  $\bar{C}$  are given by;

$$\left(\frac{X}{\bar{C}}\right)_{C.P.} = \left(\frac{X}{\bar{C}}\right)_{C.G.} + \frac{C_{Y_{B_j}} C_{M_{Z_{B_j}}} - C_{Z_{B_j}} C_{M_{Y_{B_j}}}}{C_{X_{B_j}}^2 + C_{Y_{B_j}}^2 + C_{Z_{B_j}}^2} \quad (116)$$

$$\left(\frac{Y}{\bar{C}}\right)_{C.P.} = \left(\frac{Y}{\bar{C}}\right)_{C.G.} + \frac{C_{Z_{B_j}} C_{M_{X_{B_j}}} - C_{X_{B_j}} C_{M_{Z_{B_j}}}}{C_{X_{B_j}}^2 + C_{Y_{B_j}}^2 + C_{Z_{B_j}}^2} \quad (117)$$

$$\left(\frac{z}{\bar{c}}\right)_{C.P.} = \left(\frac{z}{\bar{c}}\right)_{C.G.} + \frac{C_{X_{B_j}} C_{M_{Y_{B_j}}} - C_{Y_{B_j}} C_{M_{X_{B_j}}}}{C_{X_{B_j}}^2 + C_{Y_{B_j}}^2 + C_{Z_{B_j}}^2} \quad (118)$$

The panel section loads, moments, and centers of pressure relative to the leading edge are obtained by numerically evaluating the following integrals.

$$\frac{C_X C}{C_{AVG}} = \frac{C b_S AR}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right] \sin \phi \, d\phi - \left( \frac{C_T C}{C_{AVG}} \right) \quad (119)$$

$$\frac{C_Y C}{C_{AVG}} = \frac{C b_S AR N_{Y_{P_j}}}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} (C_{P_L} - C_{P_U}) \sin \phi \, d\phi \quad (120)$$

$$\frac{C_Z C}{C_{AVG}} = \frac{C b_S AR N_{Z_{P_j}}}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} (C_{P_L} - C_{P_U}) \sin \phi \, d\phi \quad (121)$$

where

$$b_S = \sum_{N=1}^{N_N} \sqrt{\Delta Y_{L.E.,N}^2 + \Delta Z_{L.E.,N}^2} \quad (122)$$

$$\Delta S = \sqrt{\Delta Y_{mN}^2 + \Delta Z_{mN}^2} = \text{local panel width} \quad (123)$$

$$\Delta S_{L.E.} = \sqrt{\Delta Y_{L.E.N}^2 + \Delta Z_{L.E.N}^2} = \text{panel width at leading edge.} \quad (124)$$

and  $\phi = \cos^{-1} [1 - 2 (X/C)]$  where  $(X/C)$  is the local percent chord.

Also;

$$\tan \alpha = \frac{\tan \delta \mp \frac{dz_t}{dX} - \frac{dz_c}{dX} + \epsilon}{1 - \left( \mp \frac{dz_t}{dX} - \frac{dz_c}{dX} + \epsilon \right) \tan \delta} \quad (125)$$

where the top sign is used with the upper surface and the bottom sign is used with the lower surface. The control surface angle is equal to  $-\delta_K$  along the leading edge flap and equal to  $\delta_f$  along a trailing control surface.  $\delta$  is equal to zero at other points on the chord.

The section normal force coefficient is obtained by taking the following scalar product.

$$\frac{C_N C}{C_{AVG}} = \frac{C_Y C}{C_{AVG}} N_{Y_{P_j}} + \frac{C_Z C}{C_{AVG}} N_{Z_{P_j}} \quad (126)$$

The section moment coefficients about the leading edge are computed as follows.

$$\frac{C_{M_{L.E.X}} C}{C_{AVG}} = \frac{AR b_S}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Y - Y_{L.E.}) N_{Z_{P_j}} - (Z - Z_{L.E.}) N_{Y_{P_j}} \right] [C_{P_L} - C_{P_U}] \sin \phi \, d\phi \quad (127)$$

$$\frac{C_{M_{L.E.Y}} C}{C_{AVG}} = \frac{AR b_S}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Z - Z_{L.E.}) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) - (X - X_{L.E.}) N_{Z_{P_j}} (C_{P_L} - C_{P_U}) \right] \sin \phi \, d\phi \quad (128)$$

$$\frac{C_{M.L.E.}^C}{C_{AVG}} = \frac{AR b_S}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (X - X_{L.E.}) N_{Y_{P_j}} (C_{P_L} - C_{P_U}) - (Y - Y_{L.E.}) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) \right] \sin \phi \, d\phi \quad (129)$$

The section center of pressure relative to the leading edge is given by;

$$\left(\frac{X}{C}\right)_{C.P.} = \frac{\left(\frac{C_Y C}{C_{AVG}}\right) \left(\frac{C_{M.L.E.Z}^C}{C_{AVG}}\right) - \left(\frac{C_Z C}{C_{AVG}}\right) \left(\frac{C_{M.L.E.Y}^C}{C_{AVG}}\right)}{\left(\frac{C_X C}{C_{AVG}}\right)^2 + \left(\frac{C_Y C}{C_{AVG}}\right)^2 + \left(\frac{C_Z C}{C_{AVG}}\right)^2} \quad (130)$$

$$\left(\frac{Y}{C}\right)_{C.P.} = \frac{\left(\frac{C_Z C}{C_{AVG}}\right) \left(\frac{C_{M.L.E.X}^C}{C_{AVG}}\right) - \left(\frac{C_X C}{C_{AVG}}\right) \left(\frac{C_{M.L.E.Z}^C}{C_{AVG}}\right)}{\left(\frac{C_X C}{C_{AVG}}\right)^2 + \left(\frac{C_Y C}{C_{AVG}}\right)^2 + \left(\frac{C_Z C}{C_{AVG}}\right)^2} \quad (131)$$

$$\left(\frac{Z}{C}\right)_{C.P.} = \frac{\left(\frac{C_X C}{C_{AVG}}\right) \left(\frac{C_{M.L.E.Y}^C}{C_{AVG}}\right) - \left(\frac{C_Y C}{C_{AVG}}\right) \left(\frac{C_{M.L.E.X}^C}{C_{AVG}}\right)}{\left(\frac{C_X C}{C_{AVG}}\right)^2 + \left(\frac{C_Y C}{C_{AVG}}\right)^2 + \left(\frac{C_Z C}{C_{AVG}}\right)^2} \quad (132)$$

The section zero percent suction drag coefficient

$$\frac{C_{d_{T=0}}^C}{C_{AVG}},$$

the section induced drag due to lift coefficient

$$\frac{C_{d_{L_i}} C}{C_{AVG}},$$

the section leading edge thrust coefficient  $C_T C/C_{AVG}$ , and the section induced drag due to thickness coefficient

$$\frac{C_{d_{T_i}} C}{C_{AVG}}$$

are derived in appendix C.

The panel total force coefficients are obtained by numerically integrating the section force coefficients in the spanwise direction.

$$C_{X_{P_j}} = \int_0^{\pi/2} \frac{C_X C}{C_{AVG}} \sin \theta \, d\theta \quad (133)$$

$$C_{Y_{P_j}} = \int_0^{\pi/2} \frac{C_Y C}{C_{AVG}} \sin \theta \, d\theta \quad (134)$$

$$C_{Z_{P_j}} = \int_0^{\pi/2} \frac{C_Z C}{C_{AVG}} \sin \theta \, d\theta \quad (135)$$

where

$$\theta = \cos^{-1} \eta \quad (136)$$



The panel total moment coefficients about the center of gravity are numerically integrated as follows.

$$C_{M_{X_{P_j}}} = \frac{AR b_S}{2 b^2 C} \int_0^{\pi/2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Y-Y_{C.G.}) N_{Z_{P_j}} - (Z-Z_{C.G.}) N_{Y_{P_j}} \right] (C_{P_L} - C_{P_U}) \sin \phi \sin \theta d\phi d\theta \quad (137)$$

$$C_{M_{Y_{P_j}}} = \frac{AR b_S}{2 b^2 C} \int_0^{\pi/2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Z-Z_{C.G.}) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) - (X-X_{C.G.}) N_{Z_{P_j}} (C_{P_L} - C_{P_U}) \right] \sin \phi d\phi \cdot (Z-Z_{C.G.}) \left( \frac{C_T C}{C_{AVG}} \right) \sin \theta d\theta \quad (138)$$

$$C_{M_{Z_{P_j}}} = \frac{AR b_S}{2 b^2 C} \int_0^{\pi/2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (X-X_{C.G.}) N_{Y_{P_j}} (C_{P_L} - C_{P_U}) - (Y-Y_{C.G.}) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) \right] \sin \phi d\phi + (Y-Y_{C.G.}) \left( \frac{C_T C}{C_{AVG}} \right) \sin \theta d\theta \quad (139)$$

The panel total zero suction drag, near field induced drag due to lift, leading edge thrust, and near field induced drag due to thickness are given by:

$$C_{D_{T=0_{P_j}}} = \int_0^{\pi/2} \frac{C_{d_{T=0}} C}{C_{AVG}} \sin \theta d\theta \quad (140)$$

$$C_{D_{L_i_{P_j}}} = \int_0^{\pi/2} \frac{C_{d_{L_i}} C}{C_{AVG}} \sin \theta d\theta \quad (141)$$

$$C_{T_{P_j}} = \int_0^{\pi/2} \frac{C_T C}{C_{AVG}} \sin \theta d\theta \quad (142)$$

$$C_{D_{T_i P_j}} = \int_0^{\pi/2} \frac{C_{d_{T_i}}^C}{C_{AVG}} \sin \theta \, d\theta \quad (143)$$

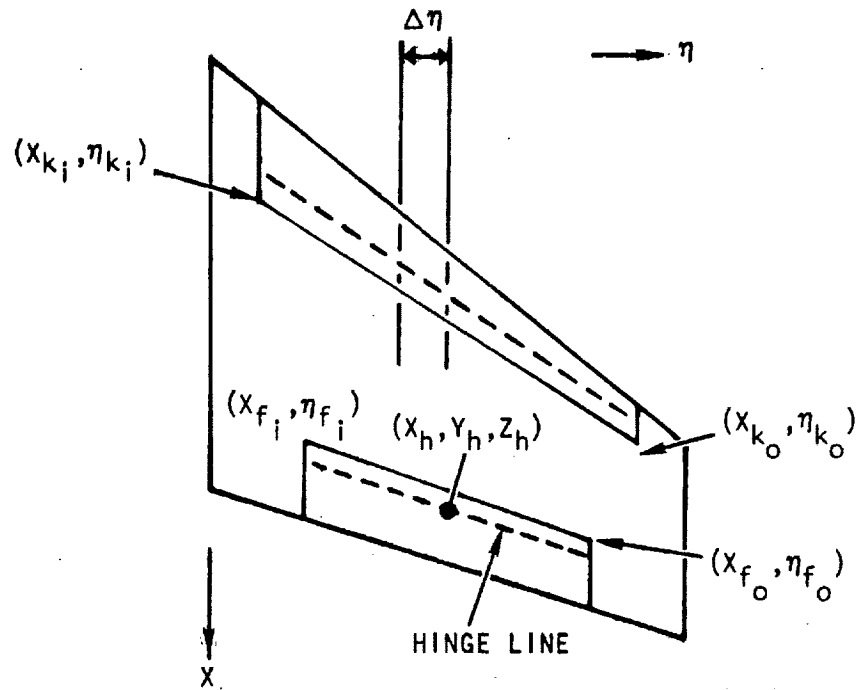
The panel center of pressure position vector components divided by  $\bar{C}$  are computed as follows.

$$\left(\frac{\bar{X}}{\bar{C}}\right)_{C.P.} = \left(\frac{\bar{X}}{\bar{C}}\right)_{C.G.} + \frac{C_{Y_{P_j}} C_{M_{Z_{P_j}}} - C_{Z_{P_j}} C_{M_{Y_{P_j}}}}{C_{X_{P_j}}^2 + C_{Y_{P_j}}^2 + C_{Z_{P_j}}^2} \quad (144)$$

$$\left(\frac{\bar{Y}}{\bar{C}}\right)_{C.P.} = \left(\frac{\bar{Y}}{\bar{C}}\right)_{C.G.} + \frac{C_{Z_{P_j}} C_{M_{X_{P_j}}} - C_{X_{P_j}} C_{M_{Z_{P_j}}}}{C_{X_{P_j}}^2 + C_{Y_{P_j}}^2 + C_{Z_{P_j}}^2} \quad (145)$$

$$\left(\frac{\bar{Z}}{\bar{C}}\right)_{C.P.} = \left(\frac{\bar{Z}}{\bar{C}}\right)_{C.G.} + \frac{C_{X_{P_j}} C_{M_{Y_{P_j}}} - C_{Y_{P_j}} C_{M_{X_{P_j}}}}{C_{X_{P_j}}^2 + C_{Y_{P_j}}^2 + C_{Z_{P_j}}^2} \quad (146)$$

The section loads, moments, and center of pressure for the control surfaces are computed as follows.



The equation for the trailing edge of leading edge control surface;

$$x_k = x_{k_i} + \left( \frac{x_{k_o} - x_{k_i}}{\eta_{k_o} - \eta_{k_i}} \right) (\eta - \eta_{k_i}) \quad (147)$$

The equation for the leading edge of trailing edge control surface;

$$x_f = x_{f_i} + \left( \frac{x_{f_o} - x_{f_i}}{\eta_{f_o} - \eta_{f_i}} \right) (\eta - \eta_{f_i}) \quad (148)$$

The equation for a control surface hinge line;

$$x_h = x_{h_i} + \left( \frac{x_{h_o} - x_{h_i}}{\eta_o - \eta_i} \right) (\eta - \eta_i) \quad (149)$$

The unit vector in the direction of the hinge line is given by;

$$\hat{h} = \left\{ \left[ X_h \left( \eta + \frac{\Delta\eta}{2} \right) - X_h \left( \eta - \frac{\Delta\eta}{2} \right) \right] \hat{i} + \left[ Y_h \left( \eta + \frac{\Delta\eta}{2} \right) - Y_h \left( \eta - \frac{\Delta\eta}{2} \right) \right] \hat{j} + \left[ Z_h \left( \eta + \frac{\Delta\eta}{2} \right) - Z_h \left( \eta - \frac{\Delta\eta}{2} \right) \right] \hat{k} \right\} / \left\{ \left[ X_h \left( \eta + \frac{\Delta\eta}{2} \right) - X_h \left( \eta - \frac{\Delta\eta}{2} \right) \right]^2 + \left[ Y_h \left( \eta + \frac{\Delta\eta}{2} \right) - Y_h \left( \eta - \frac{\Delta\eta}{2} \right) \right]^2 + \left[ Z_h \left( \eta + \frac{\Delta\eta}{2} \right) - Z_h \left( \eta - \frac{\Delta\eta}{2} \right) \right]^2 \right\}^{1/2} \quad (150)$$

The section loads on a leading edge control surface are obtained by numerically evaluating the following integrals

$$\frac{C_{h_x} C}{C_{AVG}} = \frac{C_{x^{AR}} b_s}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right) \sin \phi_k d\phi_k - \left( \frac{C_T C}{C_{AVG}} \right) \quad (151)$$

$$\frac{C_{h_y} C}{C_{AVG}} = \frac{C_{k^{AR}} b_s N_{Y_{P_j}}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} - C_{P_U} \right) \sin \phi_k d\phi_k \quad (152)$$

$$\frac{C_{h_y} C}{C_{AVG}} = \frac{C_{k^{AR}} b_s N_{Z_{P_j}}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} - C_{P_U} \right) \sin \phi_k d\phi_k \quad (153)$$

where

$$\phi_k = \cos^{-1} \left[ 1 - 2 \left( \frac{C}{C_k} \right) \left( \frac{X}{C} \right) \right] \quad (154)$$

and  $C_k$  is the chord of the leading edge control surface.  $(X/C)$  is the local percent chord of the panel.

The section loads on a trailing edge control surface are obtained by numerically evaluating the following integrals.

$$\frac{C_{h_x} C}{C_{AVG}} = \frac{C_f^{AR} b_s}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right) \sin \phi_f d\phi_f \quad (155)$$

$$\frac{C_{h_y} C}{C_{AVG}} = \frac{C_f^{AR} b_s N_{Y_{P_j}}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} - C_{P_U} \right) \sin \phi_f d\phi_f \quad (156)$$

$$\frac{C_{h_z} C}{C_{AVG}} = \frac{C_f^{AR} b_s N_{Z_{P_j}}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} - C_{P_U} \right) \sin \phi_f d\phi_f \quad (157)$$

where

$$\phi_f = \cos^{-1} \left[ 1 - 2 \left( \frac{C}{C_f} \right) \left( \frac{X + C_f - C}{C} \right) \right] \quad (158)$$

and  $C_f$  is the chord of the trailing edge control surface.

The section normal load on the control surface is given by;

$$\frac{C_{h_N}^C}{C_{AVG}} = \left( \frac{C_{h_Y}^C}{C_{AVG}} \right) N_{Y_{P_j}} + \left( \frac{C_{h_Z}^C}{C_{AVG}} \right) N_{Z_{P_j}} \quad (159)$$

The section moments about the hinge line for the leading edge control surface are obtained by numerically evaluating the following integrals.

$$\frac{C_{M_{h_X}}^C}{C_{AVG}} = \frac{C_k AR b_s}{2b^2 \bar{C}} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Y - Y_h) N_{Z_{P_j}} - (Z - Z_h) N_{Y_{P_j}} \right] (C_{P_L} - C_{P_U}) \sin \phi_k d\phi_k \quad (160)$$

$$\frac{C_{M_{h_Y}}^C}{C_{AVG}} = \frac{C_k AR b_s}{2b^2 \bar{C}} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Z - Z_h) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) - (X - X_h) N_{Z_{P_j}} (C_{P_L} - C_{P_U}) \right] \sin \phi_k d\phi_k - \left( \frac{Z_{L.E.} - Z_h}{\bar{C}} \right) \left( \frac{C_T C}{C_{AVG}} \right) \quad (161)$$

$$\frac{C_{M_{h_Z}}^C}{C_{AVG}} = \frac{C_k AR b_s}{2b^2 \bar{C}} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (X - X_h) N_{Y_{P_j}} (C_{P_L} - C_{P_U}) - (Y - Y_h) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) \right] \sin \phi_k d\phi_k - \left( \frac{Y_{L.E.} - Y_h}{\bar{C}} \right) \left( \frac{C_T C}{C_{AVG}} \right) \quad (162)$$

The section moments about the hinge line for the trailing edge control surface are obtained by numerically evaluating the following integrals.

$$\frac{C_{M_{h_X}}^C}{C_{AVG}} = \frac{C_f AR b_s}{2b^2 \bar{C}} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Y - Y_h) N_{Z_{P_j}} - (Z - Z_h) N_{Y_{P_j}} \right] (C_{P_L} - C_{P_U}) \sin \phi_f d\phi_f \quad (163)$$

$$\frac{C_{M_{h_y}}^C}{C_{AVG}} = \frac{C_f AR b_s}{2b^2 \bar{C}} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Z - Z_h) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) - (X - X_h) N_{Z_{P_j}} (C_{P_L} - C_{P_U}) \right] \sin \phi_f d\phi_f \quad (164)$$

$$\frac{C_{M_{h_z}}^C}{C_{AVG}} = \frac{C_f AR b_s}{2b^2 \bar{C}} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (X - X_h) N_{Y_{P_j}} (C_{P_L} - C_{P_U}) - (Y - Y_h) (C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U) \right] \sin \phi_f d\phi_f \quad (165)$$

The section hinge moment is computed by the following equation.

$$\frac{C_{M_h}^C}{C_{AVG}} = \left( \frac{C_{M_{h_x}}^C}{C_{AVG}} \right) h_x + \left( \frac{C_{M_{h_y}}^C}{C_{AVG}} \right) h_y + \left( \frac{C_{M_{h_z}}^C}{C_{AVG}} \right) h_z \quad (166)$$

The section center of pressure, due to the control surface loading, relative to the leading edge is given by the following expressions.

$$\left( \frac{X}{\bar{C}} \right)_{C.P.} = \left( \frac{X}{\bar{C}} \right)_h + \frac{\bar{C}}{\left( \frac{C_{h_x}^C}{C_{AVG}} \right)^2 + \left( \frac{C_{h_y}^C}{C_{AVG}} \right)^2 + \left( \frac{C_{h_z}^C}{C_{AVG}} \right)^2} \frac{\left( \frac{C_{h_y}^C}{C_{AVG}} \right) \left( \frac{C_{M_{h_z}}^C}{C_{AVG}} \right) - \left( \frac{C_{h_z}^C}{C_{AVG}} \right) \left( \frac{C_{M_{h_y}}^C}{C_{AVG}} \right)}{\left( \frac{C_{h_x}^C}{C_{AVG}} \right)^2 + \left( \frac{C_{h_y}^C}{C_{AVG}} \right)^2 + \left( \frac{C_{h_z}^C}{C_{AVG}} \right)^2} \quad (167)$$

$$\left(\frac{Y}{C}\right)_{C.P.} = \left(\frac{Y}{C}\right)_h + \frac{\bar{C}}{C} \frac{\left(\frac{C_{h_z} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_x}}^C}{C_{AVG}}\right) - \left(\frac{C_{h_x} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_z}}^C}{C_{AVG}}\right)}{\left(\frac{C_{h_x} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_y} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_z} C}{C_{AVG}}\right)^2} \quad (168)$$

$$\left(\frac{Z}{C}\right)_{C.P.} = \left(\frac{Z}{C}\right)_h + \frac{\bar{C}}{C} \frac{\left(\frac{C_{h_x} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_y}}^C}{C_{AVG}}\right) - \left(\frac{C_{h_y} C}{C_{AVG}}\right)\left(\frac{C_{M_{h_x}}^C}{C_{AVG}}\right)}{\left(\frac{C_{h_x} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_y} C}{C_{AVG}}\right)^2 + \left(\frac{C_{h_z} C}{C_{AVG}}\right)^2} \quad (169)$$

The total loads and hinge moment on the control surface are given by;

$$C_{h_X} = \int_{\eta_i}^0 \frac{C_{h_x} C}{C_{AVG}} d\eta \quad (170)$$

$$C_{h_Y} = \int_{\eta_i}^0 \frac{C_{h_y} C}{C_{AVG}} d\eta \quad (171)$$

$$C_{h_Z} = \int_{\eta_i}^0 \frac{C_{h_z} C}{C_{AVG}} d\eta \quad (172)$$



and

$$C_{M_h} = \int_{\eta_i}^{\eta_o} \frac{C_{M_h} C}{C_{AVG}} d\eta \quad (173)$$

The total loads and moments for the complete configuration are then given by the following equations.

$$C_X = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} C_{X_{Bj}} + \sum_{j=1}^{N_P} \frac{A_{Pj}}{A_R} F_{Sj} C_{X_{Pj}} \quad (174)$$

$$C_Y = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} C_{Y_{Bj}} + \sum_{j=1}^{N_P} \frac{A_{Pj}}{A_R} F_{Sj} C_{Y_{Pj}} \quad (175)$$

$$C_Z = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} C_{Z_{Bj}} + \sum_{j=1}^{N_P} \frac{A_{Pj}}{A_R} F_{Sj} C_{Z_{Pj}} \quad (176)$$

$$C_{M_X} = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} \frac{\bar{C}_{Bj}}{\bar{C}} C_{M_{X_{Bj}}} + \sum_{j=1}^{N_P} \frac{A_{Pj}}{A_R} \frac{\bar{C}_{Pj}}{\bar{C}} F_{Sj} C_{M_{X_{Pj}}} \quad (177)$$

$$C_{M_Y} = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} \frac{\bar{C}_{Bj}}{\bar{C}} C_{M_{Y_{Bj}}} + \sum_{j=1}^{N_P} \frac{A_{Pj}}{A_R} \frac{\bar{C}_{Pj}}{\bar{C}} F_{Sj} C_{M_{Y_{Pj}}} \quad (178)$$

$$C_{M_Z} = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} \frac{\bar{C}_{Bj}}{\bar{C}} C_{M_Z} + \sum_{j=1}^{N_P} \frac{A_{Pj}}{A_R} \frac{\bar{C}_{Pj}}{\bar{C}} F_{S_j} C_{M_{Z_{P_j}}} \quad (179)$$

The induced drag for the total configuration  $C_{D_i}$  is computed in the Trefftz plane with equations derived in Appendix F. The center of pressure for the total configuration is given by the following equations.

$$\left( \frac{X}{\bar{C}_R} \right)_{C.P.} = \left( \frac{X}{\bar{C}_R} \right)_{C.G.} + \frac{C_Y C_{M_Z} C_Z C_{M_Y}}{C_X^2 + C_Y^2 + C_Z^2} \quad (180)$$

$$\left( \frac{Y}{\bar{C}_R} \right)_{C.P.} = \left( \frac{Y}{\bar{C}_R} \right)_{C.G.} + \frac{C_Z C_{M_X} - C_X C_{M_Z}}{C_X^2 + C_Y^2 + C_Z^2} \quad (181)$$

$$\left( \frac{Z}{\bar{C}} \right)_{C.P.} = \left( \frac{Z}{\bar{C}_R} \right)_{C.G.} + \frac{C_X C_{M_Y} - C_Y C_{M_X}}{C_X^2 + C_Y^2 + C_Z^2} \quad (182)$$

$F_{S_j}$  is a symmetry indicator which is equal to 2.0 in equations (174), (176), and (178) and equal to 0.0 in equations (175), (177), and (179) when the panel has an image.

## COMPUTER PROGRAM RESULTS

### Airfoil Section Velocities

In order to establish the degree of accuracy that can be obtained by the use of a source-vortex lattice procedure with second order corrections to account for the interference between lift and thickness and to account for the fact that the boundary conditions and the perturbation velocities are satisfied and computed, respectively, on the chordal plane, the results from the program have been compared with two dimensional exact solutions for a Karman-Trefftz airfoil in figures (4) and (5). Also, comparisons are made with data for a forty-five degree swept wing with an aspect ratio of five and a taper ratio of one. The airfoil section on this wing is a twelve percent thick R.A.E. 101. These comparisons are in figure (6), (7), and (8) for zero angle of attack and in figures (9), (10), and (11) at 4.2 degrees angle of attack. For all of the above cases discrete solutions were obtained with twenty subpanels in the chordwise direction and ten subpanels in the spanwise direction. Both the upper and lower surface pressures were plotted for the zero angle of attack data to indicate the degree of accuracy involved with the test data. The wing section was symmetrical.

### Section Induced Drag Due to Thickness

The section induced drag  $C_{d_{ti}} C/C_{avg}$ , or potential form drag due to thickness, is computed in the present program by means of a source lattice with equation (24) of Appendix F. The source lattice as shown in figure (12) agrees quite well with the exact solution by R. T. Jones for a sixty degree swept, ten percent thick biconvex section, taper ratio one, and aspect ratio six wing given in reference (51). It should be noted that Woodward's equations, derived in Appendix D, for the constant and linearly varying distributed source density panels can be superimposed to obtain the same solution, for the biconvex section, as obtained by Jones. Even though Woodward, in reference (40), only computes induced velocities at the centroid of trapezoidal panels, it was shown that the correct solution for the longitudinal perturbation velocity is also obtained at any spanwise location, including the edges.

If two taper ratio one semi-infinite swept panels are joined at their side edges to form a wing center section or kink the Woodward distributed source equations will give the same center section solution as obtained by Kuchemann and Weber in reference (2). In addition, they will also give the correct spanwise variation of the kink effect due to thickness, which must be obtained by semi-empirically determined interpolation curves in the solution by Kuchemann and Weber. The Woodward equations also treat the spanwise variation of thickness and other planform induced effects such as taper ratio, tip, and crank effects equally well. These same effects are also correctly treated by the distributed constant density trapezoidal panel equations

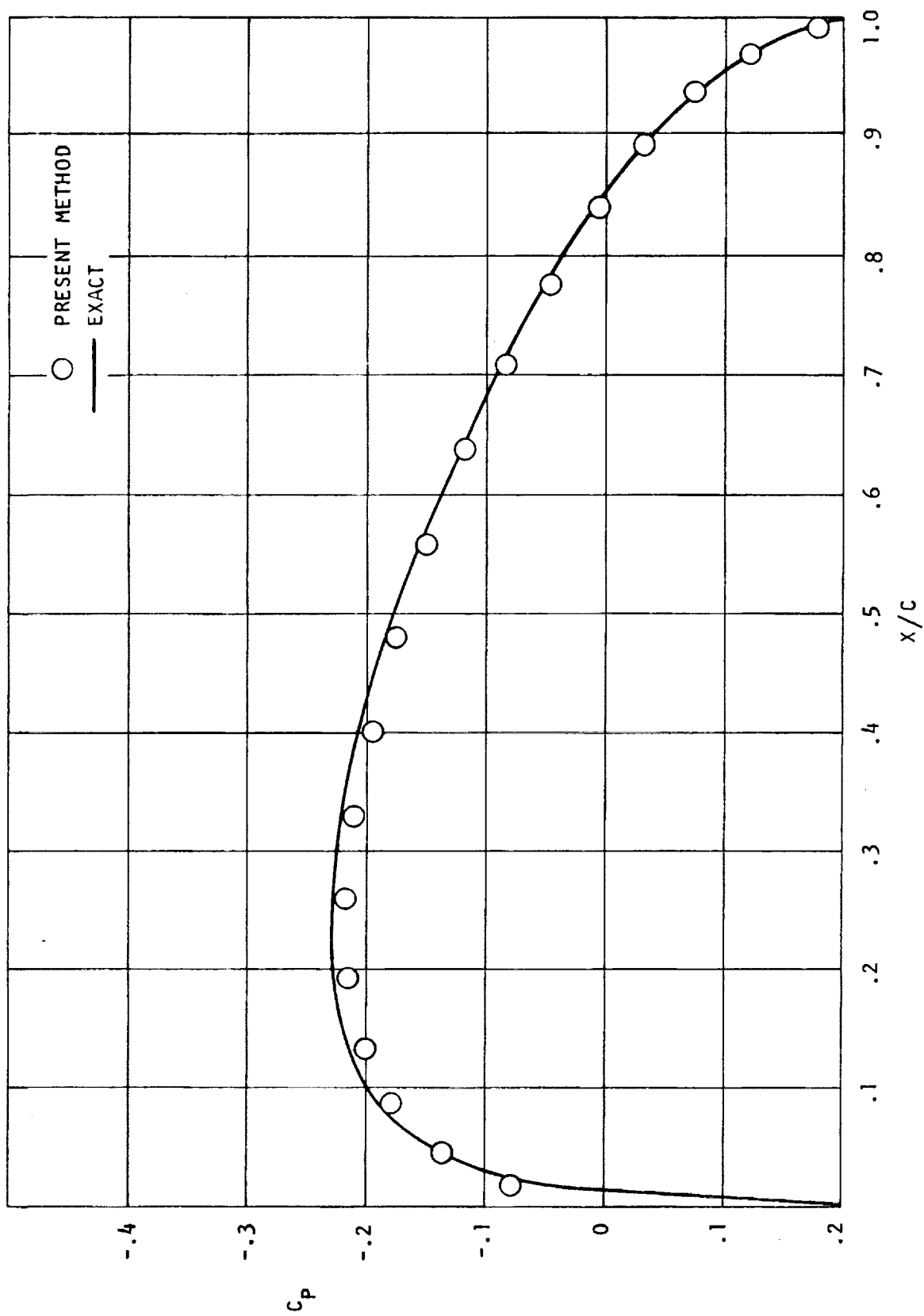


Figure 4.- Karman-trefftz airfoil pressure distribution at 0.0 degree angle of attack.

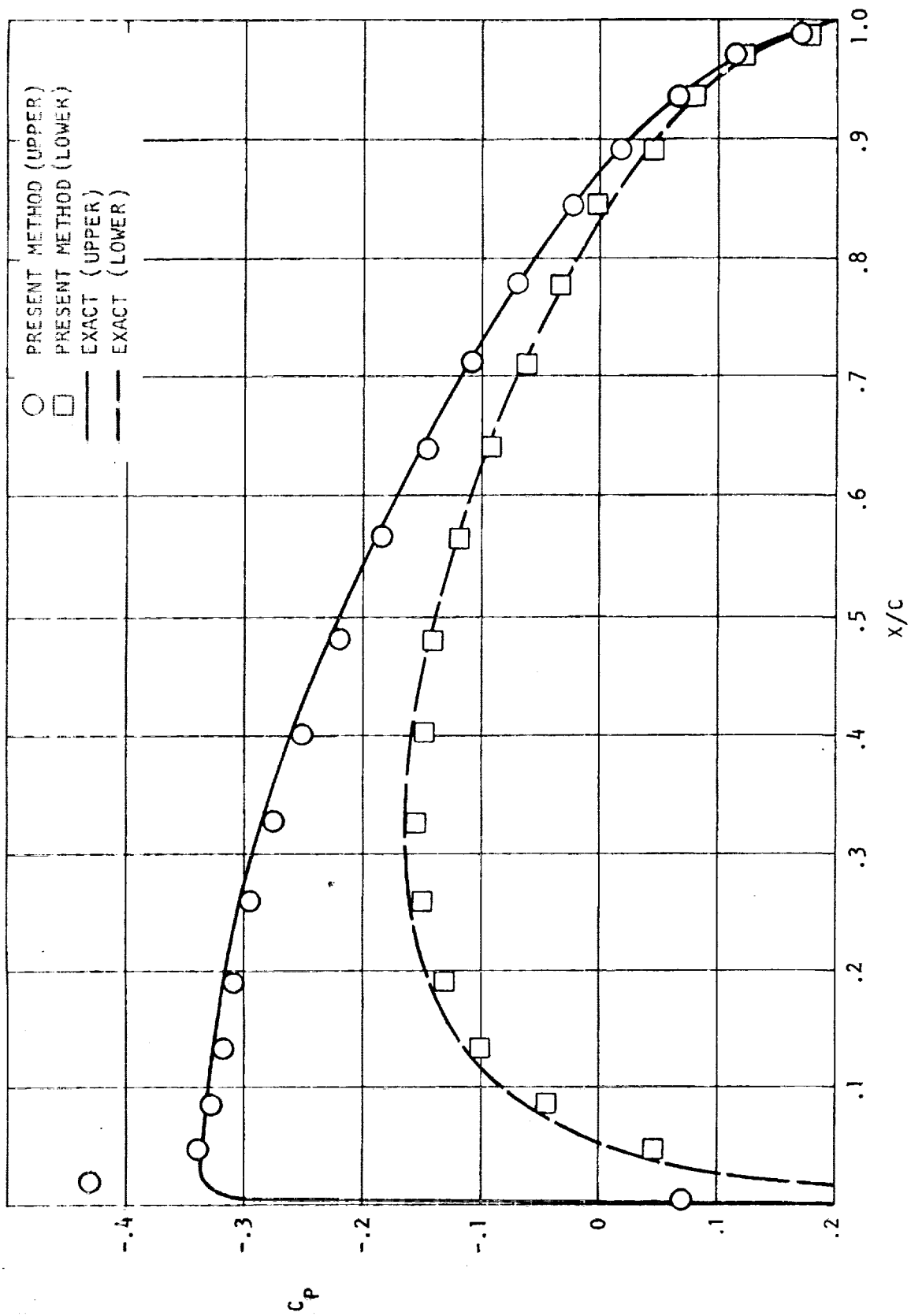


Figure 5.- Karman-trefftz airfoil pressure distribution at 1.0 degree angle of attack.

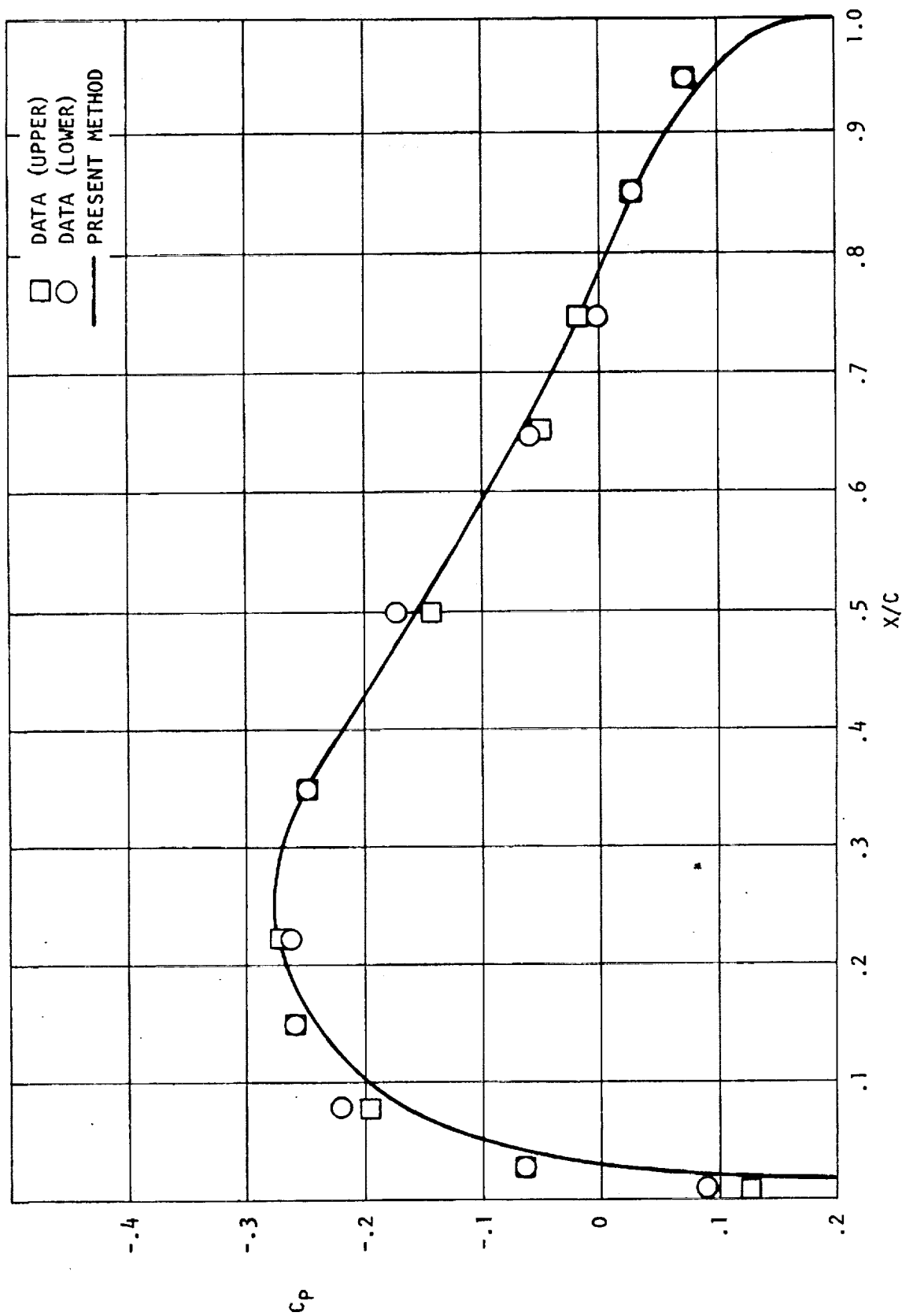


Figure 6.- Swept wing alone pressure distribution at 24.5 percent semi-span, 0.0 degree angle of attack.

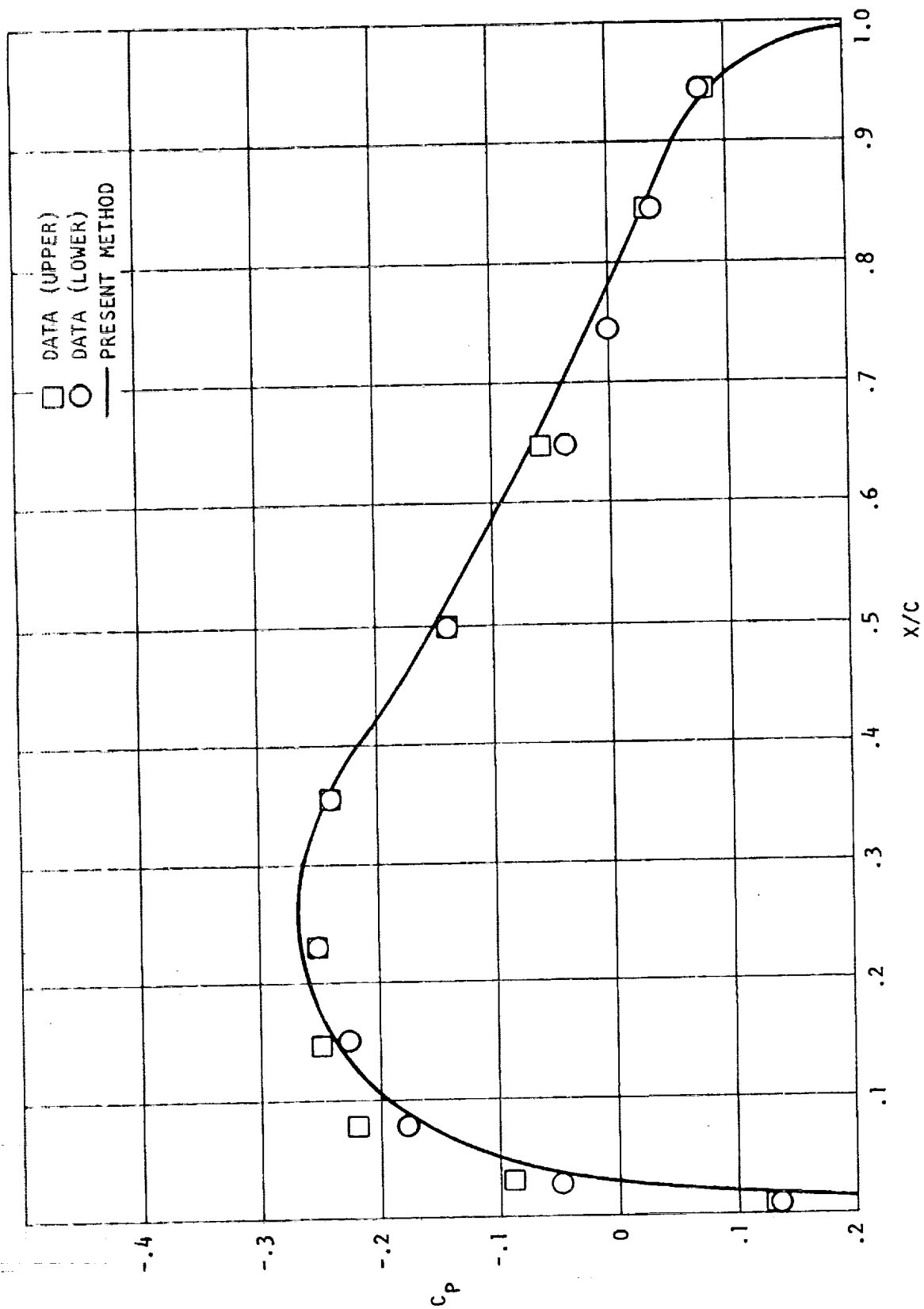


Figure 7.- Swept wing alone pressure distribution at 65.3 percent semi-span, 0.0 degree angle of attack.

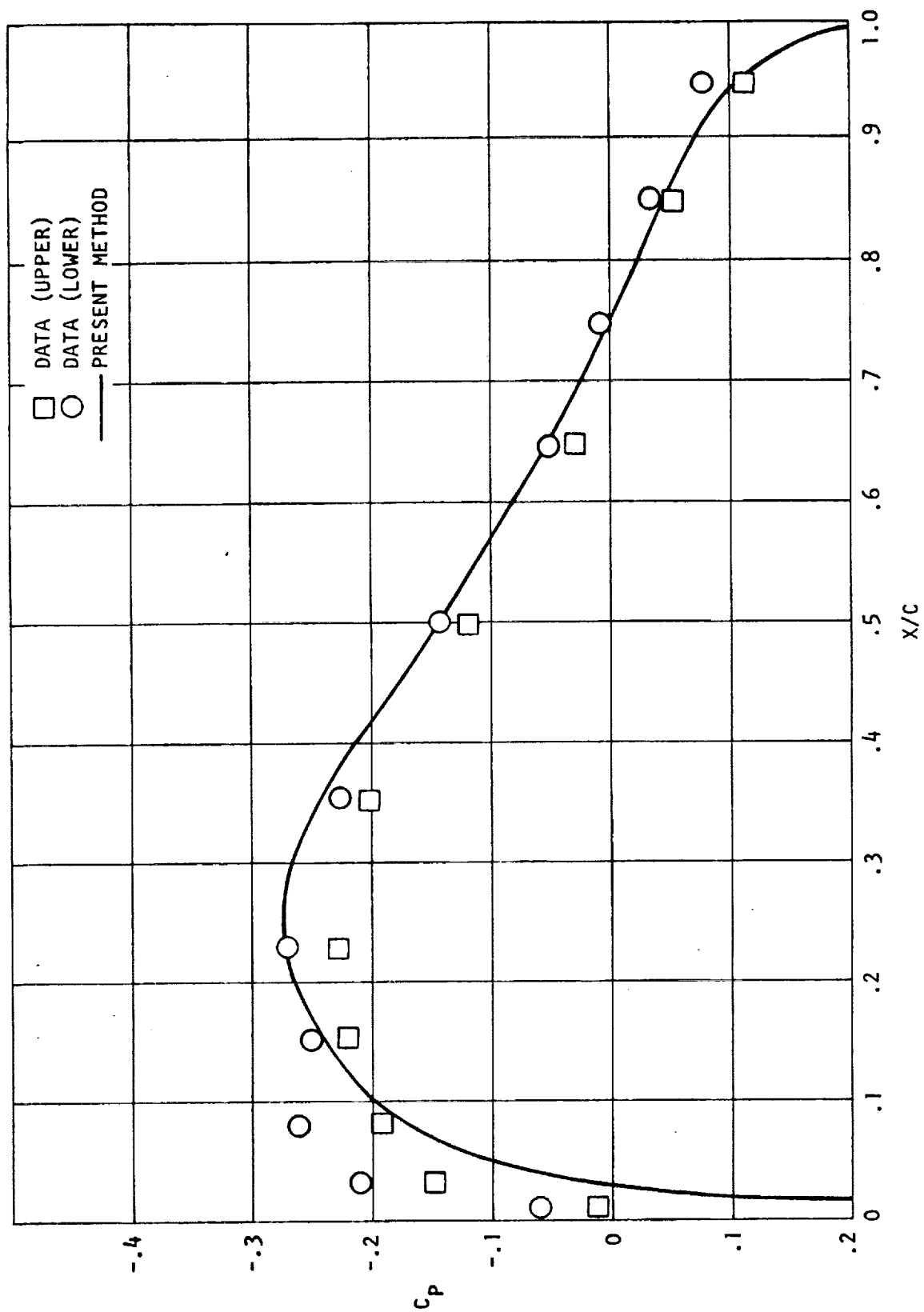


Figure 8.- Swept wing alone pressure distribution at 89.8 percent semi-span, 0.0 degree angle of attack.



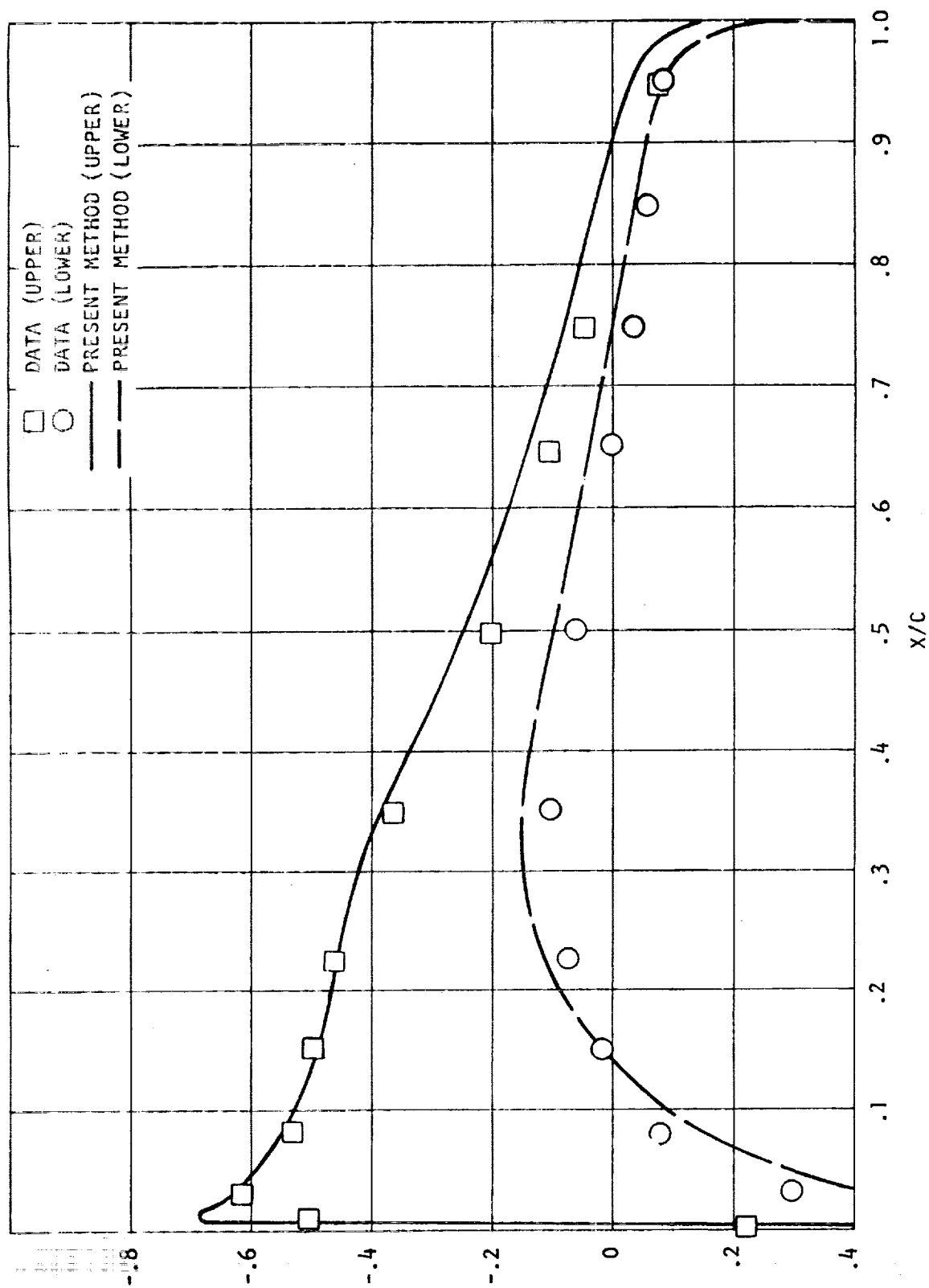


Figure 9.- Swept wing alone pressure distribution at 24.5 percent semi-span, 4.2 degrees angle of attack.

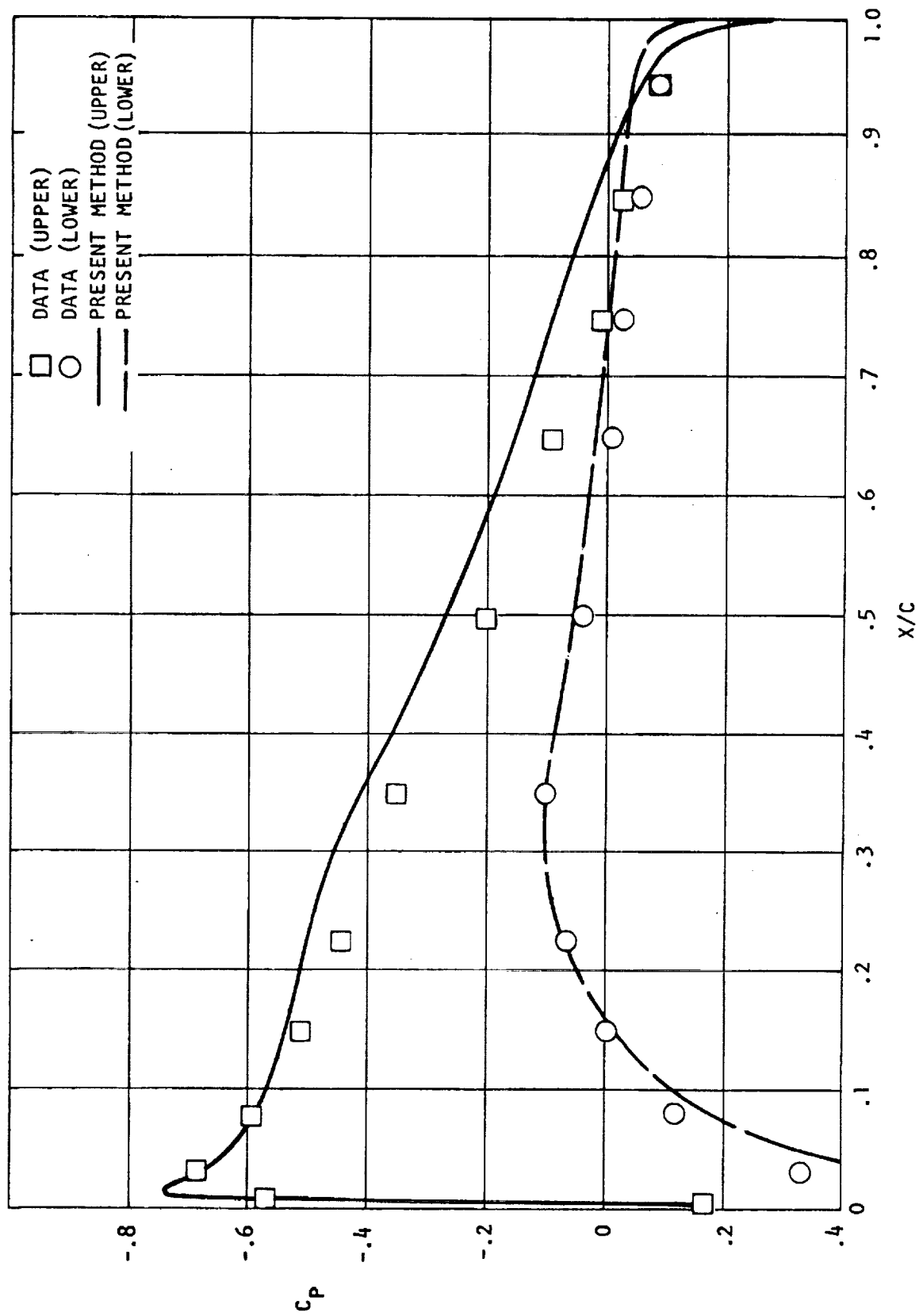


Figure 10.- Swept wing alone pressure distribution at 65.3 percent semi-span, 4.2 degrees angle of attack.

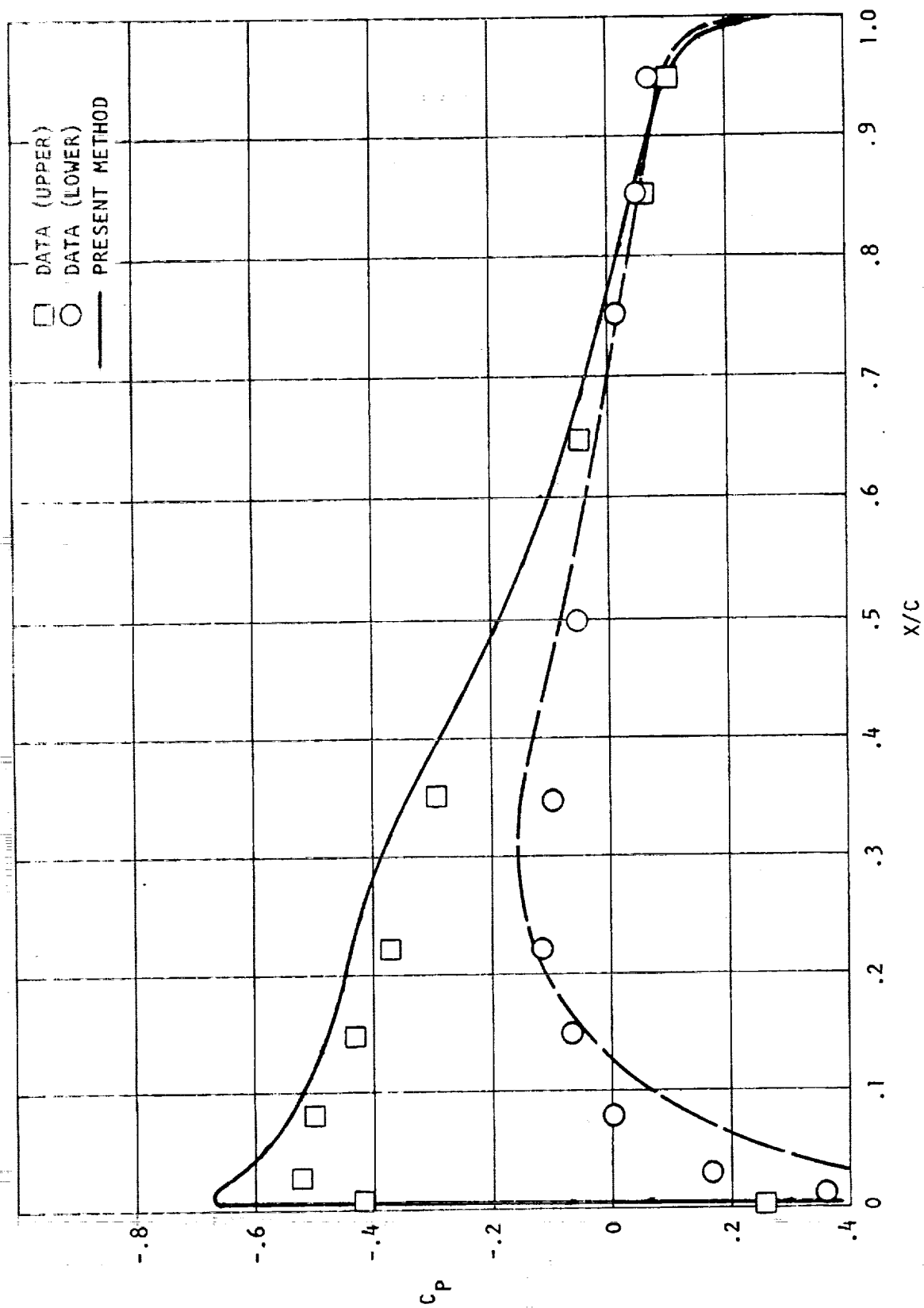


Figure 11.- Swept wing alone pressure distribution at 89.8 percent semi-span, 4.2 degrees angle of attack.

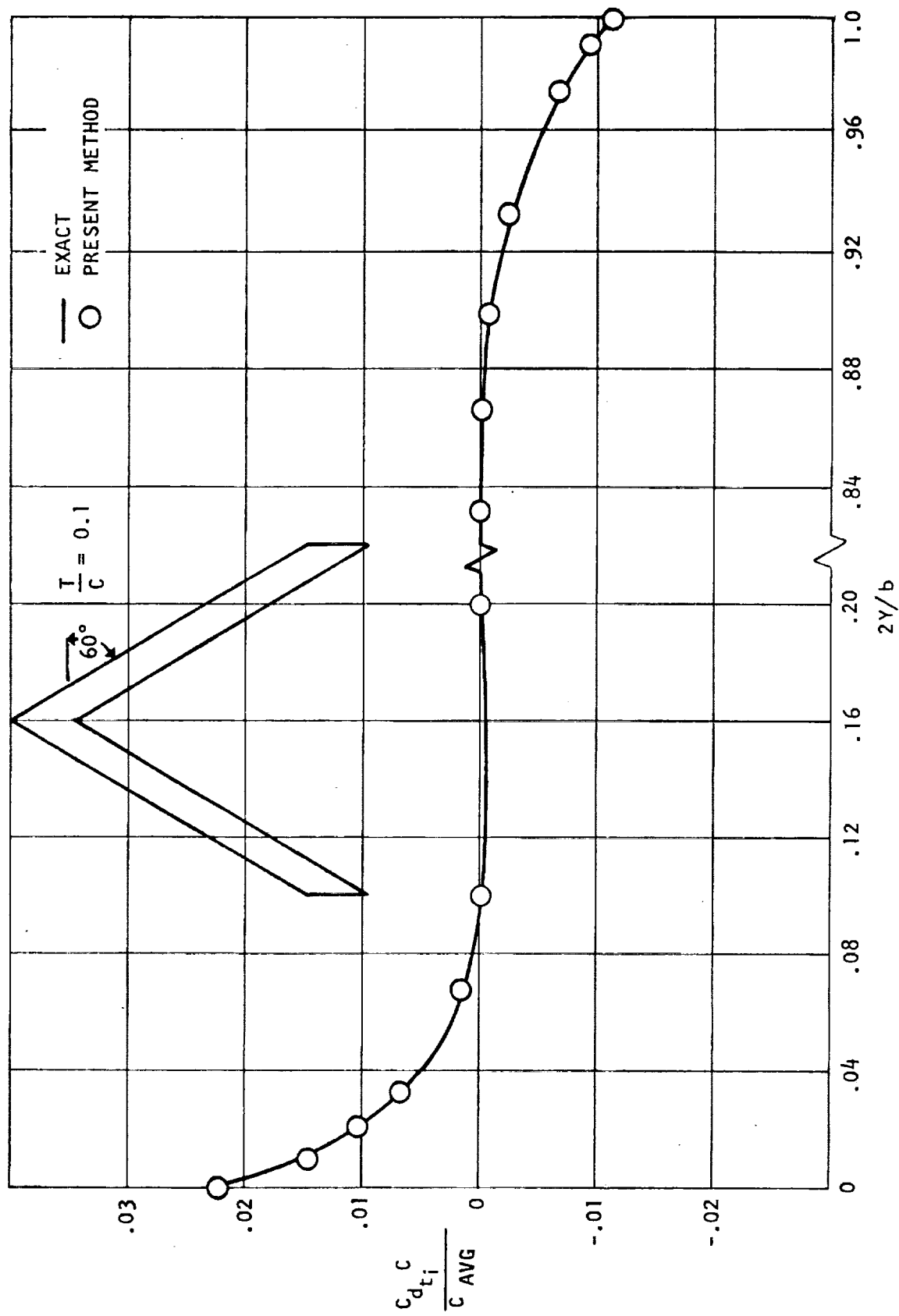


Figure 12.- Spanwise distribution of potential form drag due to thickness.

derived by Hess and Smith in reference (22). In fact Woodward's constant density source panel influence equations are identical to those derived by Hess and Smith.

As pointed out by Kuchmann and Weber the perturbation velocity due to thickness for a taper ratio one finite aspect ratio swept wing can be divided into two parts; 1) the two dimensional infinite sheared solution, and 2) the kink and tip effects. The two dimensional solution does not produce any section potential form drag provided the airfoil sections are closed. However, the kink and tip effects do produce a section drag and thrust, respectively, which when integrated across the span will give zero drag for the complete wing. Other planform effects such as taper ratio and cranks will also produce a section drag due to thickness which also integrates to zero for the complete wing.

All of the above methods will give the correct spanwise distribution of section potential form drag due to thickness. However, as also pointed out by Kuchmann and Weber the source lattice does not give the correct edge effect right at the tip, kink, or crank for a finite number of source lines in the chordwise direction. However, since this effect is only a function of the chordwise component of the thickness distribution gradient and the value of the inverse gudermannian function, with its argument being the sweep of the source line, at the point where the perturbation velocity is being computed, this effect is easily added. The region on either side of the tip, kink, or crank which is not properly handled by the source lattice is a function of the number of source lines in the chordwise direction and for practical solutions, which require about twenty source lines per chord to represent the distribution of thickness, this region is of no significance. Therefore, due to the superior numerical efficiency associated with the source lattice influence equations the source lattice appears to be the best aerodynamic finite element for predicting the perturbation velocity and potential form drag due to thickness.

#### Section Induced Drag Due to Lift

There are two basic approaches that have been tried in the past to solve the problem of predicting the spanwise distribution of induced drag or section potential form drag due to lift, 1) to accurately solve for the thin wing net pressure distribution, including the strength of the leading edge singularity, utilizing precise integration techniques to solve the aerodynamic influence integral equation, and 2) to utilize a vortex lattice procedure in conjunction with the Kutta-Joukowski theorem. Both of these approaches have failed to predict a spanwise distribution of induced drag due to lift which when integrated is equal to the induced drag computed in the Trefftz plane. The reason for this is that in these attempts the assumption

that the vorticity is constant in the spanwise direction along constant percent chord lines, even for only an infinitesimal distance, leads to a nonanalytic influence function for which no finite value exists for the induced velocity at span stations where the gradients of the constant percent chord lines are discontinuous.

Therefore, at span stations where the constant percent chord lines are kinked or cranked the constant vorticity distributed panel procedures, such as Woodward's, give a logarithmic singularity in the downwash. The lifting surface theories, such as Multhopp's or Wagner's, also produce a logarithmic singularity in the downwash which cannot be handled. Wagner's theory, as given in reference (50), has been investigated in great detail and is commented on in appendix E.

In the case of the skewed vortex-lattice the downwash at the control point is not singular, however, the Cauchy principal value does not exist for the downwash on the vortex line at a span station where the vortex lines have discontinuous sweeps. Therefore, the Kutta-Joukowski theorem will give an infinite section drag at these stations.

All of these problems can be eliminated by using an unswept horseshoe vortex lattice. It is proven in appendix F that if the bound vortex lines are all parallel and the horseshoe lattice is evenly spaced in the lateral direction, the integral of the spanwise distribution of induced drag and the induced drag computed in the Trefftz plane are identical for all planform shapes. This is also true for multiple lifting surfaces, provided they are all parallel, and for lifting surfaces with jet flaps.

It is not proposed that only unswept horseshoe vortex-lattice procedures be used to compute the net pressures or loads on wings of arbitrary shape. However, this appears to be the only numerical integration procedure known at this time which will always give the same value for the induced drag in the near and far fields. During studies of the error involved in using a skewed lattice it was determined that the error in the section induced drag was limited to a very small region on either side of the discontinuity in the sweep of the vortex lines. Also, since most of the wing is represented better by skewed vortex lines (because the lines of constant pressure do coincide with constant percent chord lines over most of the wing) a good overall answer can probably be obtained at less expense, (since fewer skewed vortices are needed in general to represent a wing than unswept vortices) if a skewed vortex lattice is used to compute the net pressures and the unswept vortex lattice is used to compute the drag once the net pressures are known.

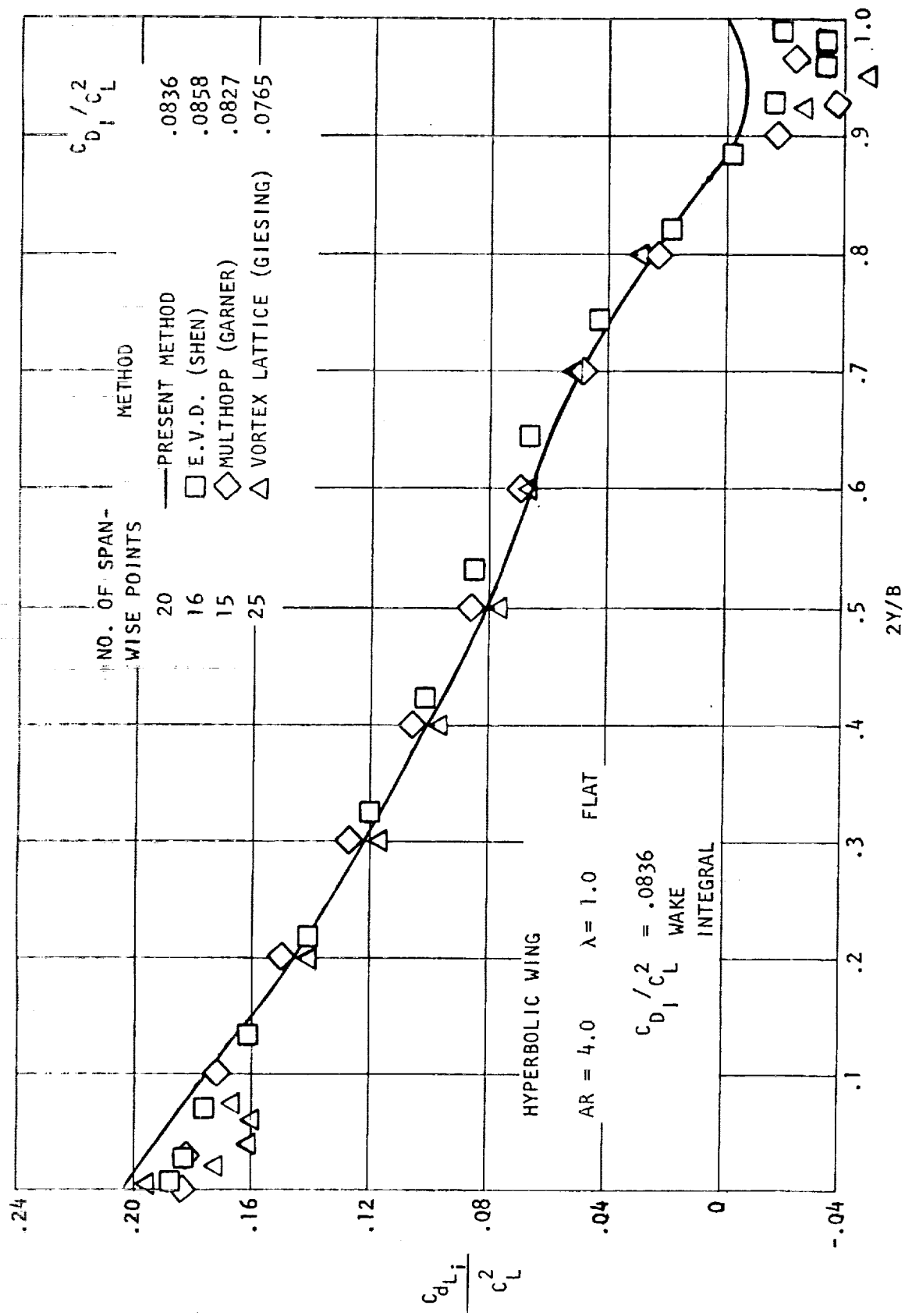


Figure 13.- Spanwise distribution of potential form drag due to lift.

△

A comparison of the section induced drag divided by the section lift four different procedures is shown in figure (13).

### Sphere Surface Velocity

The velocity over the surface of a sphere is compared to the exact solution in figure (14).

### X-15 Wing-Fuselage-Horizontal Tail-Vertical Tail

Surface velocities and pressure coefficients, section force and moment coefficients, and total configuration force and moment coefficients are computed for the X-15 wing-fuselage-horizontal tail-vertical tail shown in figure (15). The program input for this configuration plus the ventral is given in the sample input section. The program output for this configuration is given in the sample output section. In the program output the fuselage, wing, horizontal tail and vertical tail are designated as components 1, 2, 3, and 4, respectively.

Some results for this configuration are shown in figures (16), (17), (18), and (19). The data for these comparisons were obtained from References (59), (60), and (61). The force data is at .6 Mach number and the pressure coefficient data is at .2 Mach number. All of the theoretical results are for zero Mach number.

The total configuration  $C_{L\alpha}$  was determined experimentally to be .061. The program predicts .0617.



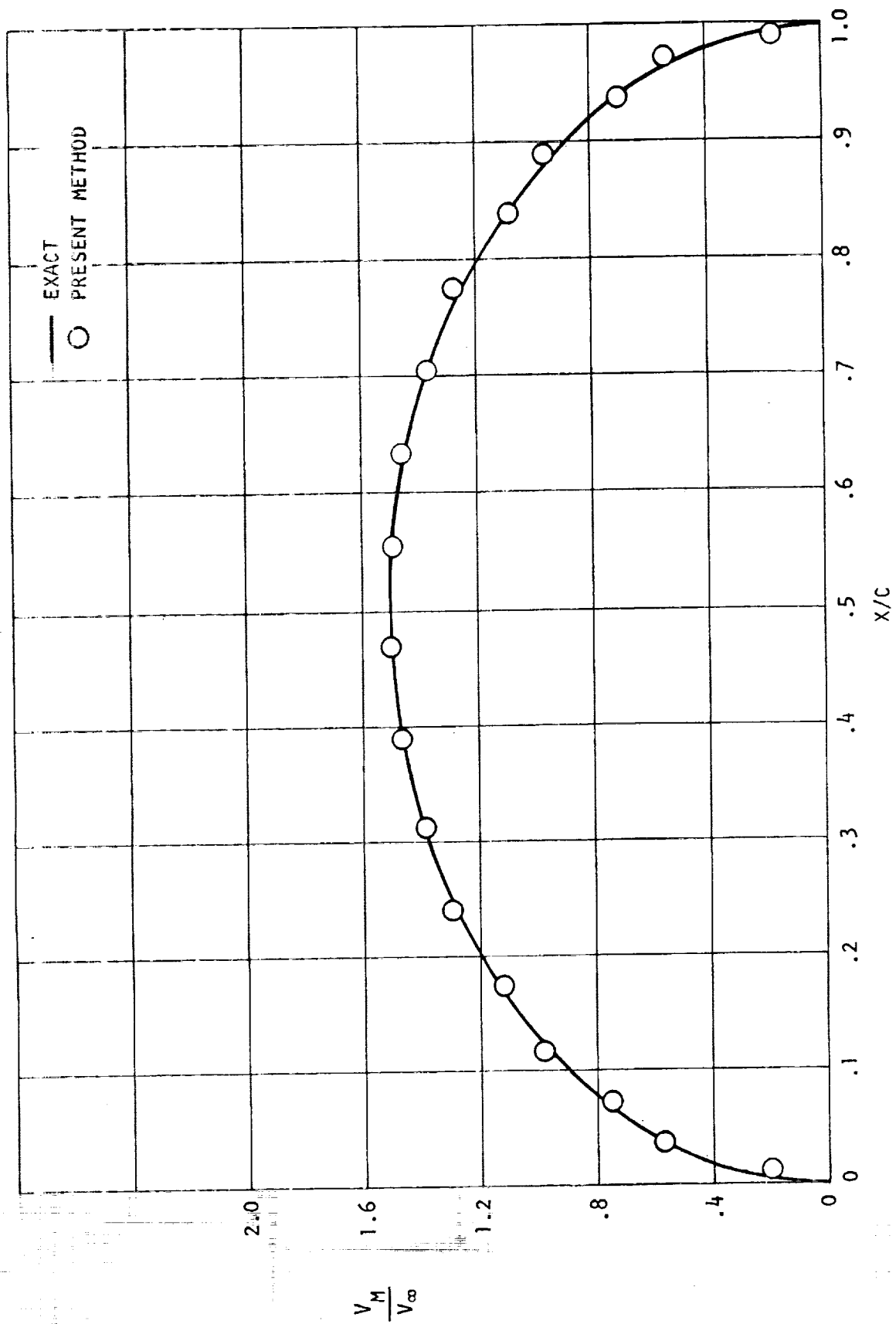


Figure 14.- Velocity ratio around a sphere.

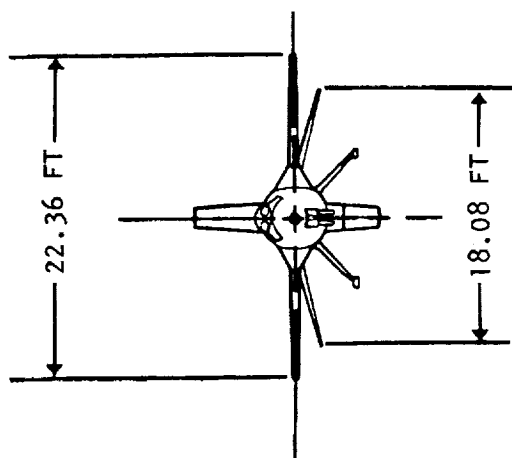
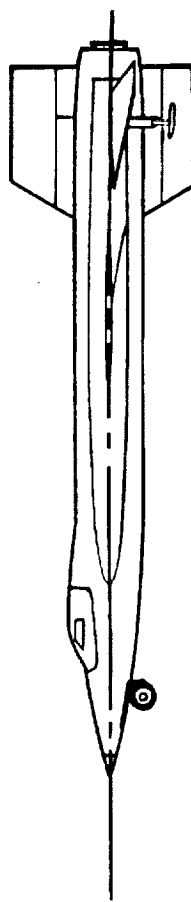
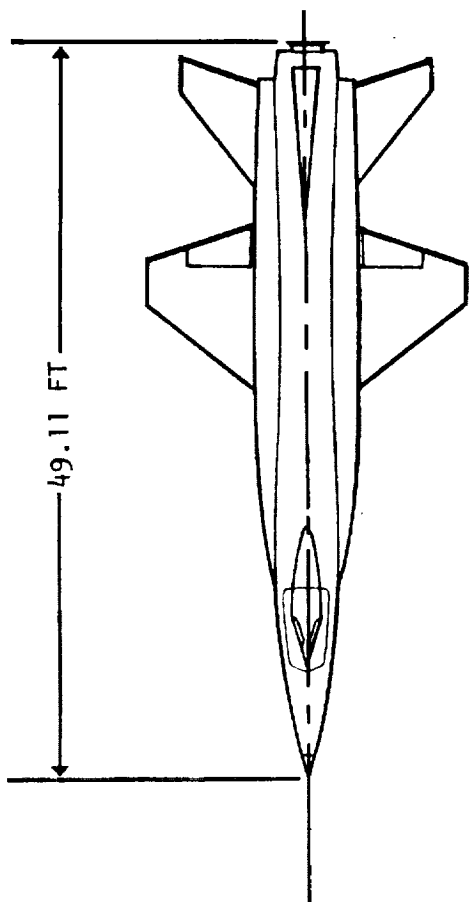


Figure 15. X-15 Configuration

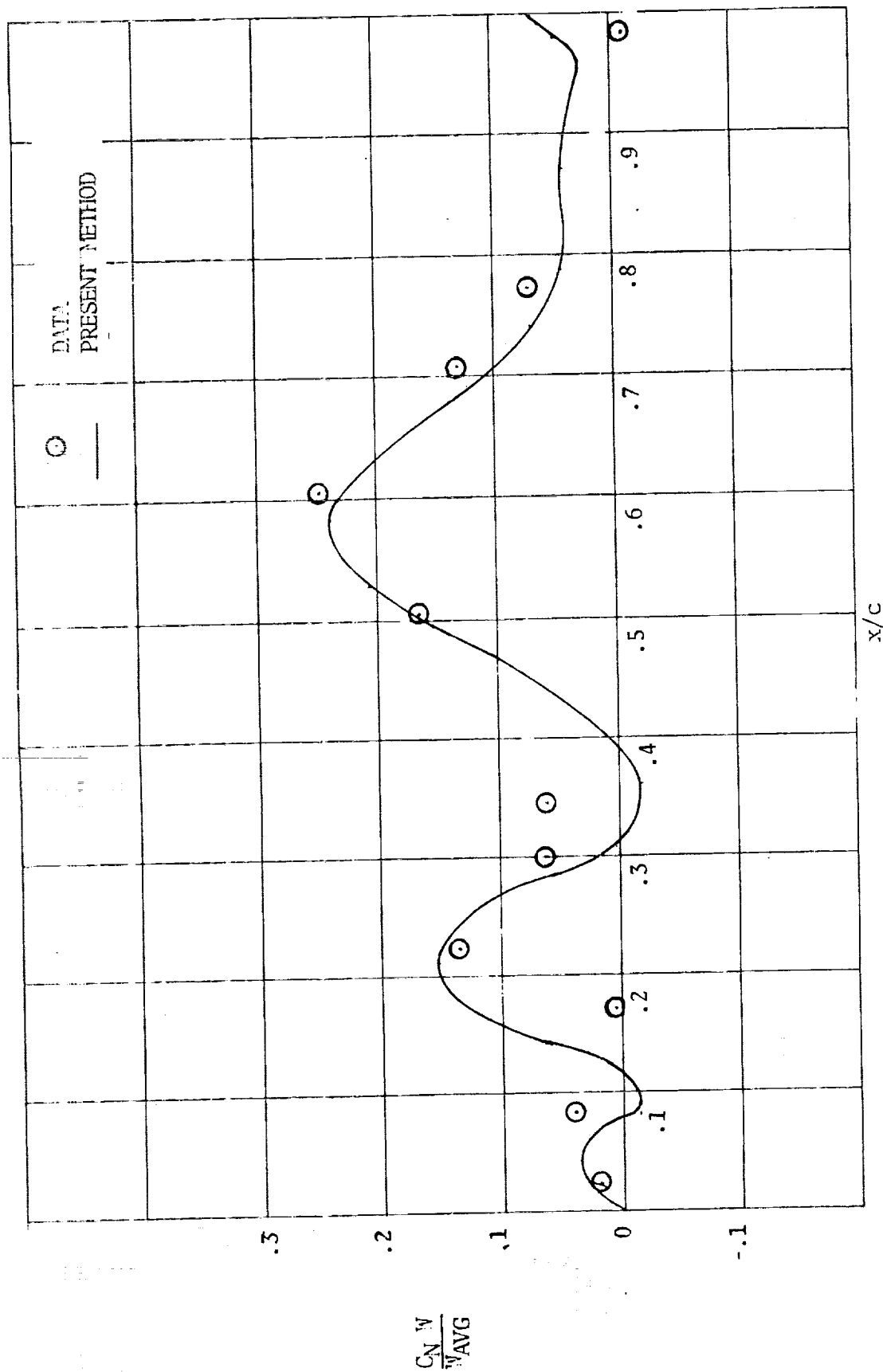


Figure 16. - Normal Load Distribution on X-15 Fuselage at Five Degrees Angle of Attack

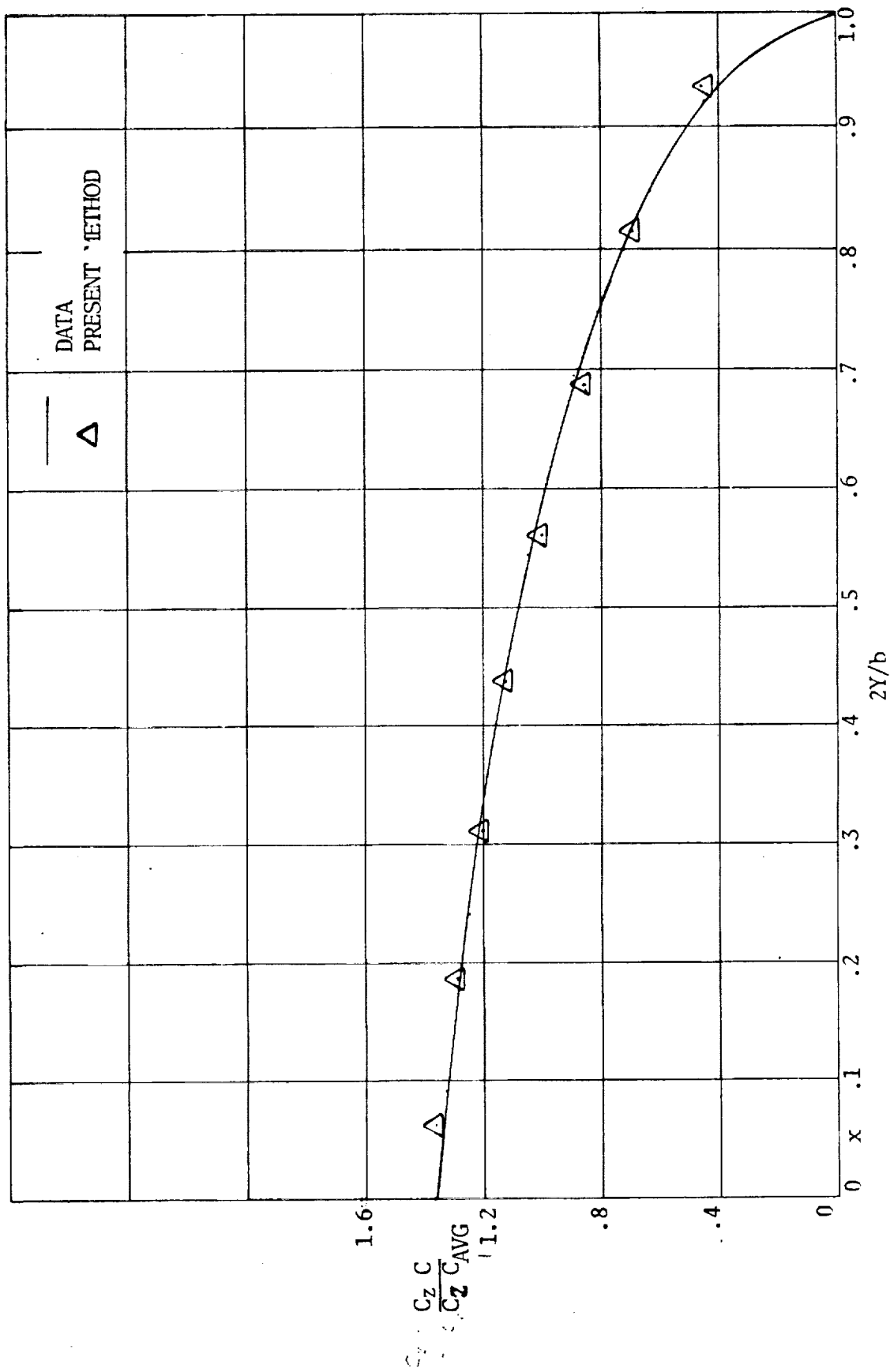


Figure 17. - Unit Span Load Distribution on X-15 Wing

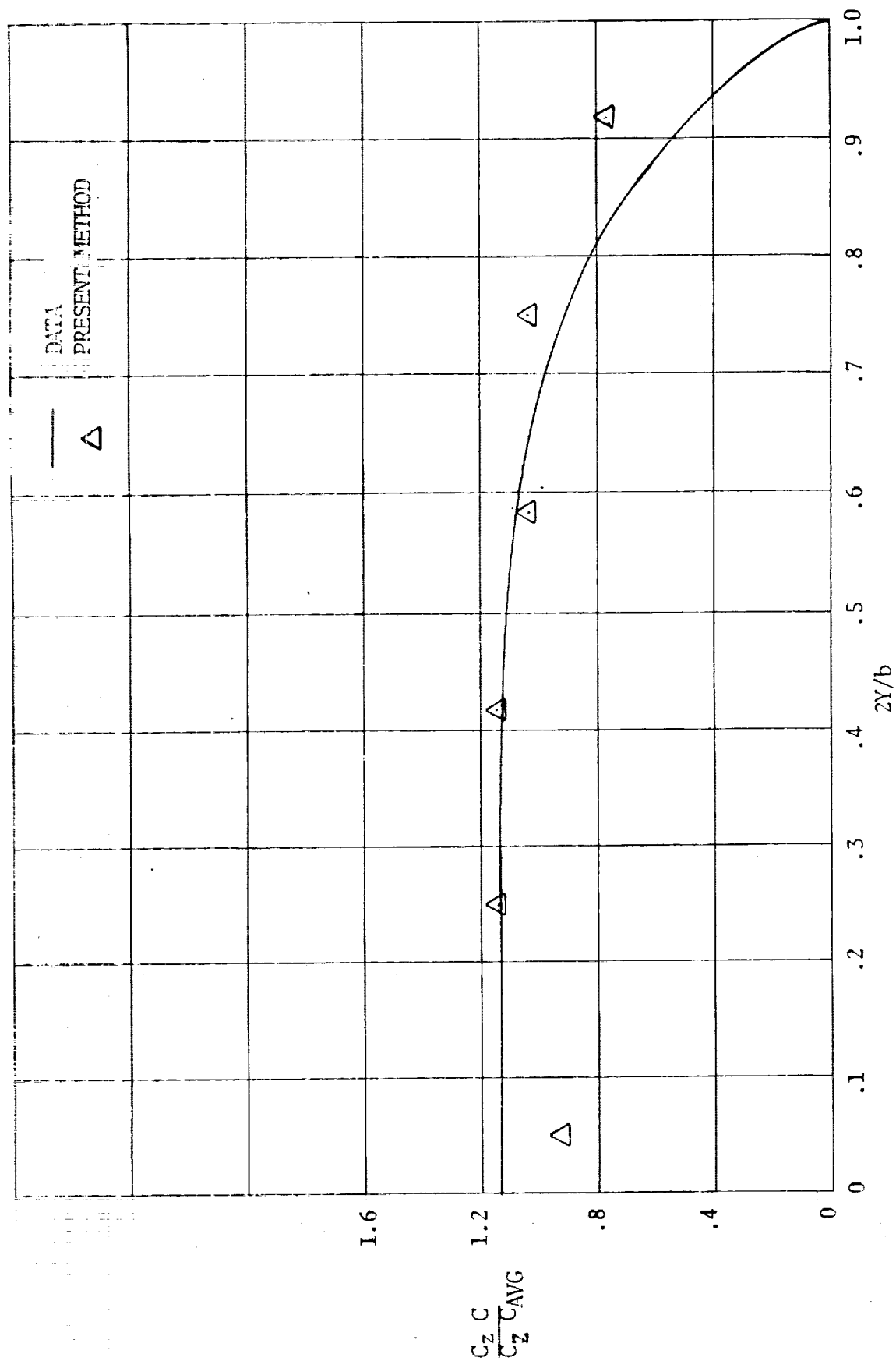


Figure 18.- Unit Span Load Distribution on X-15 Horizontal Tail

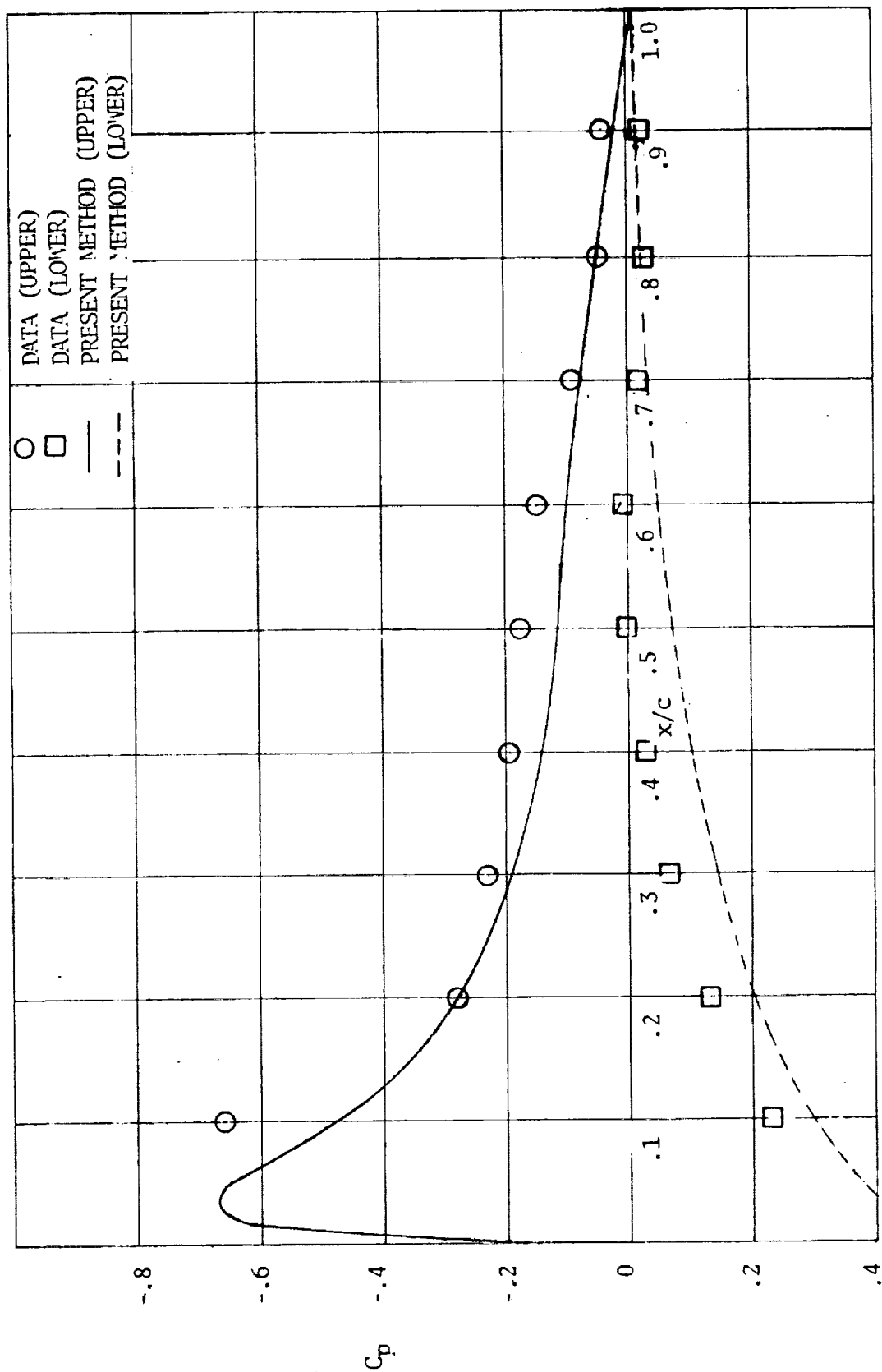


Figure 19.- Chordwise Pressure Distribution on X-15 Wing at 17.8 Percent Semispan at Five Degrees Angle of Attack

## COMPUTER PROGRAM USAGE

### Program Setup

There are two types of data decks which can be used in this program 1) the NASA Langley input format described in reference (52) and 2) the NR input format described in the next two sections of this report. Each of these input formats requires a different program setup.

Program setup for NASA format. - In this form of data setup, routine "OUTIN" preprocesses data from Langley's format and creates a card image file for use by the rest of the subroutines. Changes or additions to the data associated with bodies and panels in the Langley form of input created by "OUTIN" are made by body and panel "INFO" decks. There should be a body and panel "INFO" deck for each body and panel in the Langley input array, respectively. These body and panel "INFO" decks utilize the NR data array format.

Additional configuration bodies and panels not described in the Langley data array are added by means of additional body and panel input decks utilizing the NR data array format.

Most of the input from the Langley data array can be directly converted to the NR data array format. The only exception is a nacelle which is converted from a body of revolution to a ring wing. This occurs if the pod data has a nonzero value at the nose of the pod. If the radius at the nose of the pod is zero, the pod is considered a solid body of revolution.

The input data in the NR data array format ahead of the body and panel descriptions pertain to the total configuration and must always be input as an "INFO" deck. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed in figure 21. Figure 20 contains the entire deck setup.

**ORIGINAL PAGE IS  
OF POOR QUALITY**

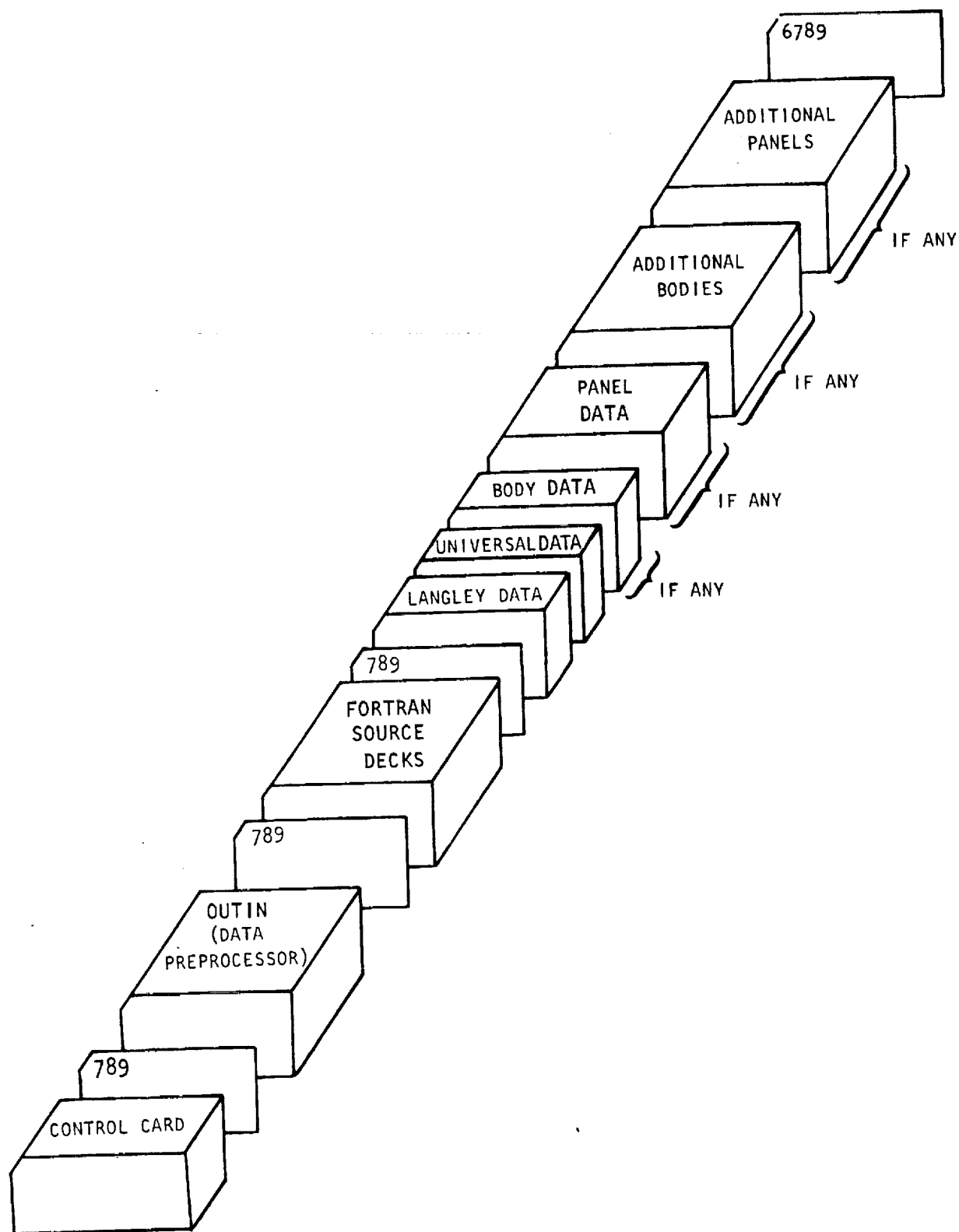


Figure 20.- Deck setup for use with NASA Langley data array format.



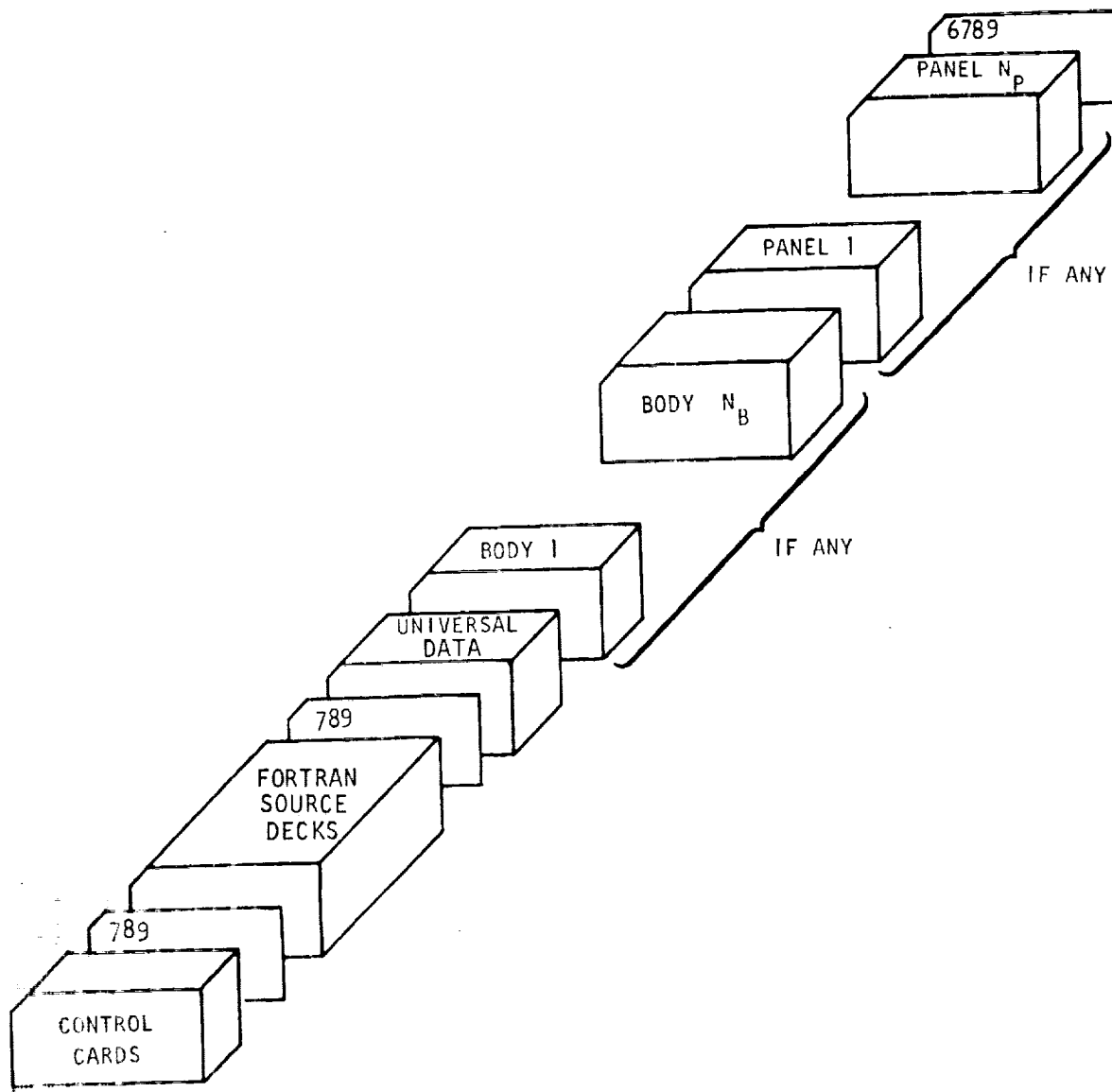


Figure 22.- Deck setup for use with NR data array format.

Program setup for NR format. - In this form of data setup all input decks utilize the NR data array format. The total configuration or universal "INFO" deck is followed by an input deck for each body and then a deck for each panel. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed for this form of data input in figure 21. Figure 22 contains the entire deck setup.

Card No.

- |    |                            |
|----|----------------------------|
| 1. | SEQUENCE CARD              |
| 2. | CHARGE CARD                |
| 3. | PFSTAD, CM31000, T7777, P6 |
| 4. | RUN, S.                    |
| 5. | SET, O.                    |
| 6. | LGO.                       |

Figure 21.- Control card deck sequence

Input Format

The NR data array format is given in this section. Data locations of all input data with limits on size of inputs are designated. A more detailed description of the input is given in the next section.

# FORTRAN FIXED IO DIGIT DECIMAL DATA

DECK NO. \_\_\_\_\_

PROGRAMMER \_\_\_\_\_

DATE \_\_\_\_\_

PAGE \_\_\_\_\_

of \_\_\_\_\_

JOB NO. \_\_\_\_\_

NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	1	No. of bodies	Universal data
13		No. of panels	
25		Mach No.	
37		$A_p$ (reference area)	(units) <sup>2</sup>
49	73	Not used	
61			
1	5	$\bar{c}$ (reference longitudinal length)	(units)
13		$b$ (reference lateral length)	(units)
25		$X_{C.G.}$	(units)
37		$Y_{C.G.}$	(units)
49	73	$Z_{C.G.}$	(units)
61			
1	10	$\alpha$ (angle of attack)	(deg)
13		$\beta$ (angle of sideslip)	(deg)
25		$P^* = P/(2V_\infty/b)$ (nondimensional roll rate)	
37		$Q^* = q/(2V_\infty/c)$ (nondimensional pitch rate)	
49	73	$R^* = r/(2V_\infty/b)$ (nondimensional yaw rate)	
61			
1	15	Body Data	
13		Component No.	
25		$A_{Bj}$ (Body reference area)	(units) <sup>2</sup>
37		$C$ (Clord)	(units)
49	73	Indicator for [A] matrix: (0.) compute new matrix and save on tape; (-1.) bypass matrix calculation and read from tape	
61		Symm indicator: (0.) body and vortex strengths are symm about x-z plane; (1.) if only body is symm; (-1.) no symm	

FORM 114C-7 ME 7-64

# FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.	PROGRAMMER	DATE	PAGE	of	JOB NO.
NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH			
1	2.0	Body coordinate table input indicator: (0.) for $(R_i \text{ vs } \theta_i)$ vs $X_i$ ;			
13		(1.) for $[(Z_{Bi} - \Delta Z_B)/Z_{BM}]$ vs $(Y_{Bi} - \Delta Y_B)/Y_{BM}$ vs $X_i$			
25		Body multiplication factor indicator (0.) for $Y_{BM} = Z_{BM}$ (1.) $Y_{MB} = Z_{MB}$			
37		Body camber indicator (0.) body section put 1 to camber line (1.)			
49		body section put 1 to body X axis.			
61		Body longitudinal subpanel indicator: (0.) even $\Delta\phi$ ; (-1.) even $\Delta X$ ;			
		(1.) given $x$			
		Body lateral vertex indicator: (0.) even $\Delta\theta$ or $\Delta(S/S_{max})$ (1.)			
		given $\theta$ or $(S/S_{max})$ ; (-1.) same as body definition stations			
1	2.5	Area of influence in longitudinal direction			
13		Area of influence in lateral direction			
25		$X_{Bo}$ (units)			
37		$Y_{Bo}$ (units)			
49		Body origin			
61		$Z_{Bo}$			
1	3.0	No. of subpanels in longitudinal direction (max 299)			
13		No. of subpanels in the lateral directions (max 40)			
25		No. of longitudinal subpanel divisions (odd integer)			
37		No. of lateral subpanel divisions (odd integer)			
49		No. of longitudinal constraint functions (max 50)			
61					
1	3.5	No. of lateral constraint functions (max 20)			
13		No. of Longitudinal constraint segments (max 50)			
25		No. of lateral constraint segments (max 20)			
37		No. of control points in longitudinal direction (max 299)			
49		No. of control points in lateral direction (max 40)			
61					

# FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.

PROGRAMMER

DATE

PAGE

of

JOB NO.

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1		Body coordinate table
13		No. of X stations (max 44)
25		$X_{B11}$ (units)
37		$X_{B12}$ (units)
49		:
61		:
1		List of X stations where $\theta$ 's or $[(Y_B - \Delta Y_B)/Y_{B1}]$ 's are defined
13		No. of $X_{B\theta}$ stations (max 44)
25		$X_{B\theta 1}$ (units)
37		$X_{B\theta 2}$ (If the list is the same as (units)
49		: in loc 41-89 it can be omitted)
61		:
1		Lateral coordinates for 1st $X_{\theta}$ station
13		No. of $\theta$ 's, $[(Y_B - \Delta Y_B)/Y_{B1}]$ 's, or $S$ 's if number in (max 39)
25		$\theta_1$ (deg)
37		$\theta_2$ (deg)
49		:
61		:
1		Lateral coordinates for 2nd $X_{\theta}$ station
13		No. of $\theta$ 's, $[(Y_B - \Delta Y_B)/Y_{B1}]$ 's, or $S$ 's if number in location (max 39)
25		$\theta_1$ (deg)
37		$\theta_2$ (deg)
49		:
61		:

FORTRAN	FIXED	IO	DIGIT	DECIMAL	DATA
---------	-------	----	-------	---------	------

DECK NO.	PROGRAMMER	DATE	PAGE	of	JOB NO.
NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH		
1		$R_i$ or $[(Z_{B_i} - Z_B)/Z_{B_M}]$ values at $X_{B_i}$ stations	(max 800)		
13		$R_1$ or $[(Z_{B_1} - Z_B)/Z_{B_M}]_1$			
25		$R_2$ or $[(Z_{B_1} - Z_B)/Z_{B_M}]_2$	(Input along longitudinal direction		
37		:	at first lateral station, then second		
49		:	lateral station, and etc)		
61					
1		Multiplication factors and camber longitudinal stations			
13		No. of X stations	(max 49)		
25		$X_{B_1}$	(units)		
37		$X_{B_2}$	(units)		
49		:			
61		:			
1		Y Multiplication factors			
13		$Y_{B_M1}$	(units)		
25		$Y_{B_M2}$	(units)		
37		:			
49		:			
61		:			
1		Z multiplication factors			
13		$Z_{B_M1}$	(units)		
25		$Z_{B_M2}$	(units)		
37		:			
49		:			
61		:			

# FORTTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. PROGRAMMER DATE PAGE of JOB NO.

NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	1 7 5 0	$\Delta Y$ displacements	
13		$\Delta Y_1$	(units)
25		$\Delta Y_2$	(units)
37		:	
49		:	
61			
1	1 8 0 0	$\Delta Z$ displacements	
13		$\Delta Z_1$	(units)
25		$\Delta Z_2$	(units)
37		:	
49		:	
61			
1	1 8 5 0	Longitudinal subpanel grid	(max 300)
13		$Y_{E1}$	(units)
25		$Y_{E2}$	(units)
37		:	
49		:	
61			
1	2 1 5 0	List of X stations where $\phi_E$ or $(S/S_{\max})$ are defined	
13		No. of X stations	(max 49)
25		$X_{E1}$	(units)
37		$X_{E2}$ (omit, if same as loc 1850 - 2149)	(units)
49		:	
61			

FORM 114-C-17 REV 7-58

# FORTTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. PROGRAMMER DATE PAGE of JOB NO.

NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
2 2 0 0		Lateral vortex grid at first $N_{BE}$ station	
		$\phi_{E1}$ or $(S/S)_{max 1}$	(deg)
		$\phi_{E2}$ or $(S/S)_{max 2}$	(deg)
		:	
		:	
		Lateral vortex grid at second $N_{BE}$ station	
		$\phi_{E1}$ or $(S/S)_{max 1}$	(deg)
		$\phi_{E2}$ or $(S/S)_{max 2}$	(deg)
		:	
		:	
2 8 0 0		List of subpanels in longitudinal direction where control points are to be placed	
		$J_{M1}$	
		$J_{M2}$	
		:	
		:	
3 1 0 0		List of subpanels in lateral direction where control points are to be placed	
		$J_{N1}$	
		$J_{N2}$	



# FORTRAN FIXED IO DIGIT DECIMAL DATA

DECK NO. \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_ PAGE \_\_\_\_\_ of \_\_\_\_\_ JOB NO. \_\_\_\_\_

NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
1	3 1 4 0	Longitudinal constraint segments	
13		$J_{SM1}$	
25		$J_{SN2}$	
37		:	
49	73 80		
61			
1	3 1 9 0	Lateral constraint segments	
13		$J_{SN1}$	
25		$J_{SN2}$	
37		:	
49	73 80		
61			
1	3 2 1 0	Longitudinal constraint function sequence No.	
13		1. 1.0	
25		2. $1.0/[1 + [Y_B(N_{XB}/N_{YB}) + Z_B(N_{XB}/N_{ZB})]^2/(Y_B^2 + Z_B^2)]^{1/2}$	
37		3. $\cot(\phi_B/2)$	
49	73 80	4. $\cot[(\pi/2) - (\phi_B/2)]$	
61		5. $\sin[\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$	
1	3 2 1 5		
13		6. $\cos [\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$	
25		7. $[(X_B/C_B) - (X_B/C_B)_0]/[(X_B/C_B)_f - (X_B/C_B)_0]$	
37		8. $\sin[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$	
49	73 80	9. $\cos[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$	
61		10. $[(X_B/C_B) - (X_B/C_B)_0]/[(X_B/C_B)_f - (X_B/C_B)_0]^{1/2}$	

# FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.	PROGRAMMER	DATE	PAGE	of	JOB NO.
1	3 2 2 0				
13					
25					
37					
49					
61					
1	3 2 5 0				
13					
25					
37					
49					
61					
1	3 2 5 5				
13					
25					
37					
49					
61					
1	3 2 7 0				
13					
25					
37					
49					
61					

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
3270	$L_{S1}$	$(X_B/C_B)$
3320	$L_{S2}$	$(X_B/C_B)$
3370	$L_{S3}$	$(X_B/C_B)$
Lateral constraint function sequence No.		
1. 1.0		
2. $N_{ZB}/[N_{-3}^2 + N_{YB}^2]^{1/2}$		
3. $N_{YB}/[N_{ZB}^2 + N_{YB}^2]^{1/2}$		
4. $\sin[\pi(\phi_E - \phi_0)/(\phi_f - \phi_0)]$		
5. $\cos[\pi(\phi_E - \phi_0)/(\phi_f - \phi_0)]$		
6. $[\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$ or $[(\eta_B - \eta_0)/(\eta_f - \eta_0)]$		
7. $\sin[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$		
8. $\cos[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$		
9. $[\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]^2$ or $[(\eta_B - \eta_0)/(\eta_f - \eta_0)]^2$		
First special longitudinal constraint function		
$(X_B/C_B)_1$		
$L_{S1}$		$(X_B/C_B)_1$
$(X_B/C_B)_2$		
$L_{S2}$		$(X_B/C_B)_2$

DIGIT DECIMAL DATA

**JOB NO.** \_\_\_\_\_

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1	3 3 2 0	Second special longitudinal constraint function
13		$(X_B/C_B)_1$
25		$LS_2 (X_B/C_B)_1$
37		$(X_B/C_B)_2$
49		$LS_2 (X_B/C_B)_2$
61		:
1	3 3 7 0	Third special longitudinal constraint function
13		$(X_B/C_B)_1$
25		$LS_3 (X_B/C_B)_1$
37		$(X_B/C_B)_2$
49		$LS_3 (X_B/C_B)_2$
61		:
1	3 4 2 0	Panel Data
13		Component No.
25		Reference area (units) <sup>2</sup>
37		Reference chord (units)
49		Reference span (units)
61		Indicator to calculate near field drag; (1.) calculate; (0.) omit
1	3 4 2 5	Indicator to save [A] matrix (0.) compute new matrix (-1.) bypass matrix calculation
13		Symm indicator (0.) if panel and unknowns have an image (1.) if just panel (-1.) no image (2.) antisymmetric
25		Panel inboard trailing vortex is attached to panel (0.) positive normal side (1.) negative normal side
37		Trailing vortex indicator (0.) straight (1.) force free
49		Panel contour indicator (0.) local angles of attack (1.) percent deflection relative to chord line
61		

# FORTRAN FIXED 10 DIGIT DECIMAL DATA

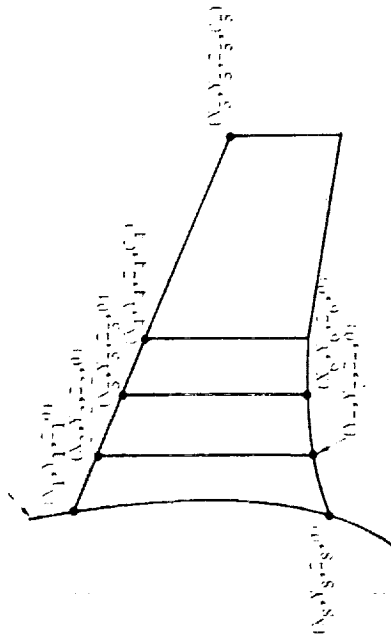
DECK NO. \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_ PAGE \_\_\_\_\_ of \_\_\_\_\_ JOB NO. \_\_\_\_\_

NUMBER	IDENTIFICATION	DESCRIPTION DO NOT KEY PUNCH
1		Panel longitudinal vortex indicator (0.) even $\Delta\phi$
13		(-1.) even $\Delta(X/C)$ (1.) given $(X/C)$
25		Panel lateral vortex indicator (0.) even $\Delta n$ (-1.) even $\Delta\theta$
37		(1.) given $n$
49		$x_{p0}$ (units)
61		$y_{p0}$ (units)
		$z_{p0}$ (units)
1		Area of influence in longitudinal direction
13		Area of influence in lateral direction
25		No. of longitudinal subpanels (max 40)
37		No. of lateral subpanels (max 40)
49		No. of longitudinal constraint functions (max 15)
61		
1		No. of lateral constraint functions (max 20)
13		No. of longitudinal control points (max 40)
25		No. of lateral control points (max 40)
37		No. of points defining panel perimeter
49		No. of body or panel to which inboard trailing vortex is attached
61		
1		No. of trailing vortex to which panel inboard trailing vortex is attached
13		No. of panel to which outboard trailing vortex is attached
25		No. of trailing vortex to which panel outboard trailing vortex is attached
37		No. of leading edge control surfaces
49		No. of trailing edge control surfaces
61		

FORM 114 C-17 REV 7-54

# FORTRAN FIXED IO DIGIT DECIMAL DATA

DECK NO.	PROGRAMMER	DATE	PAGE	OF	JOB NO.	DESCRIPTION	DO NOT KEY PUNCH
1						Panel perimeter description	
13	3 4 5 0					$X_1$ (Note: The chord is not given in the root section)	(units)
25						$Y_1$	(units)
37						$Z_1$	(units)
49		73				$C_1$	(units)
61						$X_2$	(units)
1	3 4 5 5					$Y_2$	(units)
13						$Z_2$	(units)
25						$C_2$	(units)
37							
49		73					
61							
1	3 6 0 0					Longitudinal stations where camber is defined	
13						No. of percent chord stations	(max 30)
25						$(X/C)_1$	
37						$(X/C)_2$	
49		73					
61							
1	3 6 3 0					Lateral stations where camber and twist are defined	
13						No. of percent semi-span stations	(max 30)
25						$\eta_1$	
37						$\eta_2$	
49		73					
61							



# FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_ PAGE \_\_\_\_\_ of \_\_\_\_\_ JOB NO. \_\_\_\_\_

NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH
3 6 6 0		Mean camber surface definition	
		$\alpha[(x/c)_1, \eta_1]$ on $Z_c/c [(x/c)_1, \eta_1]$	(Rad)
		$\alpha[(x/c)_2, \eta_1]$ or $Z_c/c [(x/c)_2, \eta_1]$	(Rad)
		:	
		$\alpha[(x/c)_1, \eta_2]$ or $Z_c/c [(x/c)_1, \eta_2]$	(Rad)
		:	
		Twist definition	
		$\epsilon(\eta_1)$	(Rad)
		$\epsilon(\eta_2)$	(Rad)
		:	
		Longitudinal stations where thickness is defined	
		No. of percent chord stations.	(30)
		$(x/c)_1$	
		$(x/c)_2$	
		:	
		Lateral stations where thickness is defined	
		No. of percent semi-span stations	(30)
		$\eta_1$	
		$\eta_2$	
		:	



# FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO.	PROGRAMMER	DATE	PAGE	of	JOB NO.	DESCRIPTION	DO NOT KEY PUNCH
1	4 7 2 0					Lateral Control Points	
13						$JN_1$	
25						$JN_2$	
37						:	
49						:	
61							
1	4 7 6 0					List of lateral constraint functions	
13						$W_1$ (Note: Standard functions are given by $\eta^w \sqrt{1 - \eta^2}$ and	
25						$W_2$ special functions are defined at given input data	
37						4780 array locations)	
49						4785	
61						:	
1	4 7 8 0					Special lateral constraint functions	
13						Type "p" Function L.E. Control Surface T.E. Control Surface	
25						Indicator 0. 1. -1.	
37						$\eta_b$	
49						$\Delta \eta_i$ $\eta_{k_i}$ $\eta_{f_i}$	
61						$\Delta \eta_o$ $\eta_{k_o}$ $\eta_{f_o}$	
1						(Note: Use same format for additional special functions)	
13	4 8 8 0					Special longitudinal constraint functions	
25						Type L.E. Control Surface T.E. Control Surface	
37						Indicator 1. -1. (Deg.)	
49						$\delta$ (Control Surface Angle)	
61						$\eta_{f_i}$ (Inboard Control Surface Edge)	
1						$\eta_{f_o}$ (Outboard Control Surface Edge)	
13						$x_1$ (Inboard L.E. or T.E. Corner of Control Surface) (Units)	
25							
37							
49							
61							

FORM 11A-C-17 REV. 7-58



# FORTRAN FIXED 10 DIGIT DECIMAL DATA

PROGRAMMER		DATE	PAGE	of	JOB NO.
DECK NO.	NUMBER	IDENTIFICATION	DESCRIPTION	DO NOT KEY PUNCH	
1	4 8 8 5		Special Longitudinal Constraint Functions Continued		
13			$x_0$ (Outboard L.E. or T.E. Corner of Control Surface) (Units)		
25			$x_{\eta_1}$ (Inboard Edge - Hinge Line Intersection) (Units)		
37			$x_{\eta_0}$ (Outboard Edge - Hinge Line Intersection) (Units)		
49			Note: Use same format for special longitudinal constraint functions due to additional control surfaces. Start data sets at 4890, 4900, and etc.)		
61					
1					
13					
25					
37					
49					
61					
1					
13					
25					
37					
49					
61					
1					
13					
25					
37					
49					
61					

## Input Description

Location	Description
1	Total number of bodies used to represent the configuration
2	Total number of panels used to represent the configuration. This number does not include those due to images when location 3426 has a zero in it.
3	Mach number
* 4	Total configuration reference area.
5	Total configuration longitudinal reference length.
6	Total configuration lateral reference length.
7	X component of center of gravity position vector.
8	Y component of center of gravity position vector.
9	Z component of center of gravity position vector.
10	Angle of attack.
11	Angle of sideslip.
12	Nondimensional roll rate $P/(2V_\infty/b)$ .
13	Nondimensional pitch rate $q/(2V_\infty/\bar{c})$ .
14	Nondimensional yaw rate $r/(2V_\infty/b)$ .
15	The number of the component. The bodies are numbered first and then the panels.
* 16	The body reference area.
17	The body chord.

\* Items not needed if data format from reference (52) is used.

Location	Description
18	This is an indicator to bypass the calculation of the aerodynamic influence matrices due to the vortices of this body. Input a zero to compute new matrices and a minus one if the calculation of the matrices is to be bypassed.
*19	This is a symmetry indicator used to take advantage of both symmetrical geometries and loadings. If both the body geometry and the vortex strength are symmetrical about the X-Z plane input a zero, if only the body geometry is symmetrical and not the vortex strengths input a one, and if neither symmetry exists input a minus one.
*20	This is an indicator used to signify whether cartesian or polar coordinates are used to input the body cross-sections. If polar coordinates are used input a zero and if cartesian coordinates are used input a one.
21	This is an indicator used to signify whether the lateral scaling factors are equal in the Y and Z directions. Input a zero if they are equal and a one if they are not equal. The $Z_{B_M}$ array is not input if $Y_{B_M} = Z_{B_M}$ .
*22	This indicator is used to signify whether the body cross-sections are placed perpendicular to the mean camber line or the X axis. Input a zero if the cross-section is to be placed perpendicular to the mean camber line or a one if it is to be placed perpendicular to the X axis.
23	This indicator signifies the type of subpanel spacing used on the body in the longitudinal direction. Input a zero if the subpanels are to be spaced at even increments of $\phi$ , where $\phi = \cos^{-1} [1 - 2(x/c)]$ , input a minus one if the subpanels are to be spaced at even increments of X, and input a one if the spacing is specified at a given set of X stations. The X stations are to be given in locations 1850 - 2149.
24	This indicator signifies the type of subpanel spacing used on the body in the lateral direction. Input a zero if the subpanels are to be spaced at even increments of $\theta$ or $\eta_B = (S/S_{max})$ , a one if the subpanel side edges are given at a set of $\theta$ or $\eta_B$ stations, and a minus one if the subpanel edges are at the same lateral stations as the body geometry is defined. The subpanel side edges are input in locations 2200 - 2799.

\* Items not needed if data format from reference (52) is used.

Location	Description
25	This is the longitudinal distance, in terms of the length of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a longitudinal distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.
26	This is the lateral distance, in terms of the width of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a lateral distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.
*27	This is the X component of the origin of the body coordinate frame.
*28	This is the Y component of the origin of the body coordinate frame.
*29	This is the Z component of the origin of the body coordinate frame.
30	This is the number of subpanels in the longitudinal direction on the body.
31	This is the number of subpanels in the lateral direction on the body.
32	This is the number of equally spaced divisions the body subpanels are divided into in the longitudinal direction. These divisions are used to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.

\* Items not needed if data format from reference (52) is used.

## Location

## Description

- 33 This is the number of equally spaced divisions the body subpanels are divided into in the lateral direction. These divisions are used to map the vortex grid closer to the actual body surface and to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.
- 34 This is the number of functions used to constrain the body surface vorticity in the longitudinal direction. The list of functions used is given in locations 3210-3249. These same functions are used over each longitudinal constraint function segment. The list defining these segments is given in locations 3140-3189. The maximum number of longitudinal constraint functions is 50.
- 35 This is the number of functions used to constrain the body surface vorticity in the lateral direction. The list of functions used is given in locations 3250-3269. These same functions are used over each lateral constraint function segment. The list defining these segments is given in locations 3190-3209. The maximum number of lateral constraint functions is 20.
- 36 This is the number of segments in the longitudinal direction over which the body longitudinal constraint functions are defined. The list of subpanels defining the segments is given in locations 3140-3189. The maximum number of segments is 50.
- 37 This is the number of segments around the circumference of the body over which the lateral constraint functions are defined. The list of subpanels defining the segments is given in locations 3190-3209. The maximum number of segments is 20.
- 38 This is the number of control points in the longitudinal direction on the body. The maximum number of points in the longitudinal direction is 299. The list of body control points in the longitudinal direction is given in location 2800, 3099.

\* Items not needed if data format from reference (52) is used.

Location	Description
39	This is the number of control points in the lateral direction on the body. The maximum number of points in the lateral direction is 40. This list of control points is given in locations 3100 - 3139.
*40	This is the number of longitudinal stations where the body cross-sections are defined. The maximum number is 44.
*41-89	This is the list of longitudinal stations where the body cross-sections are defined. This list starts at the nose and ends at the aft end if the body is solid. For a flow through body this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.
*85	This is the number of longitudinal stations where the body cross-section lateral stations are defined. The maximum number of these longitudinal stations is 44.
*86-129	This is the list of longitudinal stations where the body cross-section lateral stations are defined. The body cross-section lateral stations are constant between each longitudinal station in this list. If this list is the same as that in locations 41-89, it can be omitted. If the body is solid, this list starts at the nose and ends at the aft end. If the body is a flow through type, this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.
*130	This is the number of lateral stations at the first longitudinal station in the list at locations 91-129. The maximum number of these lateral stations is 39.
*131	This is the list of lateral stations at the first longitudinal station in the list at locations 91-129. These lateral stations where the body is defined can be given by the angle $\theta$ , the lateral fraction $[(Y_{B_1} \Delta Y) / Y_{B_M}]$ , or fraction of lateral circumferential length $S_{B_1 B_M}$ .
	Note: Lists of lateral stations at the other longitudinal stations, given in the list in locations 91-129, follow this data as shown in the input format.

\* Items not needed if data format from reference (52) is used.

Location	Description
*800-1599	In these locations the body cross-section radii or fractional distances $[(Z_{B_i} - \Delta Z_B)/Z_{B_M}]$ , for each of the $X_{B_i}$ longitudinal stations and $\theta_i$ or $[(Y_{B_i} - \Delta Y_B)/Y_{B_M}]$ lateral stations, are input.
*1600	This is the number of longitudinal stations where the multiplication factors $Y_{B_M}$ and $Z_{B_M}$ and the translation increments $\Delta Y$ and $\Delta Z$ are given. The maximum number of stations that can be used here is 49.
1601-1649	This is the list of longitudinal stations where the multiplication factors $Y_{B_M}$ and $Z_{B_M}$ and the translation increments $\Delta Y_B$ and $\Delta Z_B$ are given.
1650-1699	This is the list of multiplication factors $Z_{B_M}$ at the longitudinal stations given in locations 1601-1649.
1750-1799	This is the list of translation increments $\Delta Y_B$ at the longitudinal stations given in locations 1601-1649.
*1800-1849	This is the list of translation increments $\Delta Z_B$ at the longitudinal stations given in locations 1601-1649.
*1850-2149	This is the list of longitudinal stations where the body subpanel edges are defined.
2150	This is the number of longitudinal stations where the body lateral subpanel edges are defined. The maximum number of these longitudinal stations is 49.
2151-2199	This is the list of longitudinal stations where the body lateral subpanel edges are defined. This list can be omitted if it is the same as that in locations 1850-2149.
2200	This is the list of lateral subpanel edges at the first longitudinal station given in the list at locations 2151-2199. These stations can be defined by $\theta$ 's or fractions of body circumference $\eta_B = (S/S_{max})$ .

Note; Lists of lateral subpanel edges at the other longitudinal stations, given in the list at locations 2151-2199, follow this input as shown in the input format.

\* - Items not needed if data format from reference (52) is used.

Location	Description
2800-3099	This is the list of the number of the subpanels where the control points are located in the longitudinal direction on the body. The integer in this list indicates the number of the subpanel, aft of the nose of a solid body or aft of the tail end on the inner surface and around to the outer surface of a flow through body, at which a control point is placed. If no longitudinal constraint functions are used, this input can be omitted.
3100-3139	This is the list of control point locations in the lateral direction on the body. The integer in this list indicates the number of the subpanel in the lateral direction from the top of the body at which a control point is placed. If no lateral constraint functions are used, this input can be omitted.
3140-3189	This is the list of longitudinal constraint segment boundaries. The integers in this list indicate the subpanels between which the longitudinal constraint functions are applied. For example, if this list included the longitudinal number of every tenth subpanel, the longitudinal constraint functions would be applied over a range of ten subpanels and repeated over the segments defined by every tenth subpanel.
3190-3209	This is the list of lateral constraint segment boundaries. This list is utilized in the same manner as the longitudinal constraint segment boundaries.
3210-3249	This is the list of longitudinal constraint functions used on the body. The functions to be used are signified by entering here the sequence number of the function as given in the input format. If special functions are to be used the data location where the function is described is input here.
3250-3269	This is the list of lateral constraint functions used on the body. The same procedure is used here to signify the desired functions as is used in the definition of the longitudinal constraint functions.
3270-3419	In these locations the special longitudinal constraint functions are described as shown in the input format. These are described in tabular form.

\* Items not needed if data format from reference (52) is used.



Location	Description
3420	This is the component number for the panels.
*3421	This is the panel reference area.
3422	This is the panel reference chord.
3423	This is the panel lateral reference length.
3425	This is an indicator to bypass the calculation of the aerodynamic influence matrices due to the vortices of this panel. Input a zero if a new matrix is computed and a minus one if the calculation of the matrix is to be bypassed.
*3426	This is a symmetry indicator used to take advantage of a configuration's symmetry to save input effort and computer time. Input a zero if both the panel geometry and the vortex strengths have images on the port side of the X-Z plane, a one if just the panel geometry has an image, a minus one if neither have an image, and a two if the vortex strengths on the panel and its image are antisymmetric.
3427	This indicator is used to signify whether the panel is attached to the positive normal or negative normal side of another panel. Input a zero if it is attached to the positive normal side and a one if it is attached to the negative normal side.
3428	This indicator is used to signify whether the wake is force free or fixed. Input a zero if the wake is fixed and a one if it is force free. If any panel has a force free wake, the wake or a body it is attached to is also assumed to be force free.
*3429	This indicator signifies whether the mean camber surface of the panel is described in terms of local angles of attack or as fractions of chord. Input a zero if local angles of attack are input and a one if fractions of chord are input.

\* Items not needed if data format from reference (52) is used.

Location	Description
3430	This input indicates the type of longitudinal subpanel spacing that is used. If the subpanels are spaced at even increments of $\phi$ input a zero, if they are at equal increments of percent chord input a minus one, and if they are input as a given set of percent chord use a one.
3431	This input indicates the type of lateral subpanel spacing that is used on a panel. Input a zero if the spacing is at equal increments of $\eta$ , a minus one if it is at equal increments of $\theta$ , and a one if the spacing is specified at a given set of $\eta$ stations.
*3432	This is the X component of the panel origin point.
*3433	This is the Y component of the panel origin point.
*3434	This is the Z component of the panel origin point.
3435	This is the area of influence of a quadrilateral vortex on a panel in the longitudinal direction. This input is in terms of fraction of quadrilateral vortex length.
3436	This is the area of influence of a vortex on a panel in the lateral direction. This input is in terms of fraction of vortex width.
3437	This is the number of subpanels in the longitudinal direction on the panel. The maximum number of longitudinal subpanels is 40.
3438	This is the number of subpanels in the lateral direction on the panel. The maximum number of lateral subpanels is 40.
3439	This is the number of longitudinal constraint functions on the panel. This number does not include special functions due to control surfaces. The maximum number is 15.
3440	This is the number of lateral constraint functions on the panel. This number does include special functions due to control surfaces. The maximum number is 20.

\* Items not needed if data format from reference (52) is used.

Location	Description
3441	This is the number of control points in the longitudinal direction on the panel. The maximum number is 40.
3442	This is the number of control points in the lateral direction on the panel. The maximum number is 40.
*3443	This is the number of points used to define the panel perimeter. In the root section the perimeter is defined by X, Y, and Z components of points along the leading and trailing edges. The outboard section is defined by X, Y, and Z components of points along the leading edge and the local chord. The root section leading edge is input first, then the outboard section, and then the root section trailing edge. The maximum number of points is 50.
3444	This input is the number of the body or panel to which this panel's inboard trailing vortex is attached.
3445	This is the number of the trailing vortex leg or subpanel side edge to which this panel's inboard trailing vortex is attached. The subpanel side edges are numbered consecutively starting at the top of a body or at the inboard edge of a panel and going in the clockwise direction when an observer is looking in the negative X direction.
3446	This is the number of the panel to which the outboard trailing vortex of this panel is attached.
3447	This is the number of the trailing vortex leg or subpanel side edge to which this panel's outboard trailing vortex is attached.
3448	This is the number of leading edge control surfaces on the panel.
3449	This is the number of trailing edge control surfaces on the panel.
*3450-3599	In these locations the panel perimeter is described.

\* Items not needed if data format from reference (52) is used.

Location	Description
* 3600	This is the number of longitudinal stations where the panel camber is described. The maximum number is 30.
*3601-3629	This is the list of chord fractions which define the chordwise locations where the panel camber is defined.
* 3630	This is the number of lateral stations where the panel camber and twist is defined. The maximum number is 30.
*3631-3659	This is the list of lateral surface length fraction $\eta$ where the panel camber and twist is defined.
*3660-4099	This is the table of local angle of attack or deflection, in terms of fraction of chord, of the panel mean camber surface. The table is input as shown in the input format.
4100-4129	This is the table of panel twist.
* 4130	This is the number of longitudinal stations where the panel thickness is described. The maximum number is 30.
*4131-4159	This is the list of chord fractions which define the chordwise locations where the panel thickness is defined.
* 4160	This is the number of lateral stations where the panel thickness is defined. The maximum number is 30.
*4161-4189	This is the list of lateral surface length fraction $\eta$ where the panel thickness is defined.
*4190-4599	This is the table of panel thickness in terms of fraction of chord. The table is input as shown in the input format.
4600-4639	This is the list of longitudinal subpanel edge locations in terms of fraction of chord.
4640-4679	This is the list of lateral subpanel side edge locations in terms of $\eta$ .
4680-4719	This is the list of panel control points in the longitudinal direction.
* Items not needed if data format from reference (52) is used.	

Location	Description
4720-4759	This is the list of panel control points in the lateral direction.
4760-4779	This is the list of exponents W for the standard lateral constraint functions or data locations of the special lateral constraint functions.
4780-4879	In these locations the special panel lateral constraint functions are described. The input is as shown in the input format.
4880 - ...	In these locations the special panel longitudinal constraint functions are described. The input is as shown in the input format.

\* Items not needed if data format from reference (52) is used.

SAMPLE INPUT

111.0	4.0	20.0	28800.0	0.0
5123.23	268J32	339.1675	0.0	0.0
105.0	28800.0	580.0	0.0	-1.0
151.0	1.0	1.0	0.0	0.0
200.0	4.0	0.0	7.0	0.0
25		1.0	28.0	4.0
3028.0	0.0	580.0		
350.0	0.0	360.0		
402.0	0.0	360.0		
1302.0	1.0	1.0		
1332.0	0.0	3.0		
8001.0	0.0	30.0		
160038.0	20.0	80.0		
160515.0	70.0	130.0		
161060.0	120.0	180.0		
1615110.0	170.0	230.0		
1620160.0	220.0	280.0		
1625210.0	270.0	400.0		
1630260.0	320.0	4.0		
1635310.0	2.5	12.75		
16500.0	10.75	21.0		
16558.25	19.50	26.75		
166018.0	25.75	34.0		
166524.50	32.5	39.75		
167031.0	38.75	43.5		
167537.5	43.0	44.0		
168042.0	44.0	4.0		
168544.0	2.5	12.5		
17000.0	10.25	24.5		
17058.0	21.0	29.75		
171018.5	29.5	28.25		
171529.0	28.75	27.50		
172029.25	27.50	27.50		
172527.50	27.50	27.50		
173027.50	27.50	27.50		
173527.50	27.50	27.50		
1810 0.5	1.75	3.4		
1815 4.675	4.55	4.25		
1820 2.3	1.55	0.65		
			4.5	4.875
			3.75	3.15
			0.2	

1850	0.0	2.6	11.54	27.25	49.0
1855	77.0	110.0	148.0	189.0	233.0
1860	270.0	295.0	308.42	317.761	341.41
1865	374.4	407.39	431.53868	440.38	450.0
1870	460.	471.61	477.28644	492.79075	513.9715
1875	535.1572	550.6566	565.0	580.0	
5000	-1.				
3420	1.0	7566.58366	123.23	90.218	
3435			6.0	8.0	
3440		6.0	8.0	8.0	1.0
3445	2.0				
3450	308.42	43.5			316.9416
3455	54.9246			325.4632	66.3492
3460			333.9848	77.7738	
3465	95.6841	376.04	134.16		35.78
3470	433.23925	66.4965			436.8096
3475	55.21925				43.942
413028.0		.0	0.001	440.38	0.005
41350.0075		0.0125	0.025	0.0025	0.075
41400.10		0.15	0.20	0.050	0.30
41450.35		0.40	0.45	0.25	0.55
41500.60		0.65	0.67	0.50	0.75
4155 0.8		0.85	0.90	0.70	
41602.0		0.0	1.0	1.0	
4190.0		0.00402	0.00617	0.00824	0.00947
41950.01048		0.01124	0.01263	0.01395	0.01523
42000.01769		0.02001	0.02183	0.02317	0.02415
42050.02476		0.02499	0.02487	0.02437	0.02345
42100.02174		0.02085	0.019409	0.017008	0.014606
4215 0.012205		0.009803	0.005	0.0	0.00402
42200.00617		0.00824	0.00947	0.01048	0.01124
42250.01263		0.01395	0.01523	0.01769	0.02001
42300.02183		0.02317	0.02415	0.02476	0.02499
42350.02487		0.02437	0.02345	0.02174	0.02085
42400.019409		0.017008	0.014606	0.012205	0.009803
42450.005					
5000	-1.				
3420	2.0	3722.7580	123.23	67.662	
3435			6.0	6.0	



3440	16.0	6.0	8.0	1.0
3445	43.942			484.8699
3450	-2.918		498.1298	65.726
3455	553.9793	511.3898	76.618	-8.754
3460	65.726	109.294	-17.51	25.28
3465	-2.918	-5.836	556.3731	560.1875
3470	0.0	0.001	0.0025	43.942
3475	0.0125	0.025	0.050	0.0050
413023.0	0.15	0.20	0.250	0.075
41350.0075	0.40	0.45	0.50	0.30
41400.10	0.75	0.90	1.00	0.55
41450.35	0.0	1.0		
41500.60	0.00348	0.00538	0.00728	0.00847
41602.0	0.01052	0.01206	0.01353	0.01495
41900.0	0.02002	0.02184	0.02318	0.02417
41950.00969	0.02499	0.02492	0.02460	0.02346
42000.01768	0.00961	0.00498	0.0	0.00348
42050.02476	0.00728	0.00847	0.00969	0.01052
42100.01653	0.01353	0.01495	0.01768	0.02002
42150.00538	0.02318	0.02417	0.02476	0.02499
42200.01206	0.02460	0.02346	0.01653	0.00961
42250.02184	5864.375	123.23	55.0	
42300.02492	-1.0	6.0	5.0	
42350.00498	6.0	5.0	8.0	
5000 -1.		27.5		1.0
3420 3.0	38.5	470.5509	464.2006	457.8503
3425	483.25	49.5	82.5	60.5
3435	38.5		574.0	90.75
3440	0.0	0.25	0.5	574.0
3445 1.0				
3450 451.5				
3455				
3460 49.5				
3465 103.4491				
3470 574.0				
3475				
3480 27.5				
4130 5.0				
4135 1.0				

4160	2.0	0.0	1.0			
4190	0.00408	0.0259575	0.047835	0.0697125	0.09159	
4195	0.00551	0.0273825	0.049255	0.0711275	0.093	
5000	-1.					
3420	4.0	5024.12	123.23	46.0		
3425		-1.0				
3435			6.0	5.0		
3440		6.0	5.0	8.0	1.0	
3445	3.0					
3450	451.5					
3455		-36.7	-27.5	462.1242	456.8121	
3460	-45.9		467.4362		-55.1	
3465	106.5638	478.06		-73.5	95.94	
3470	462.1242		-45.9		456.8121	
3475		-36.7		451.5		
3480	-27.5					
4130	5.0	0.0	0.25	0.5	0.75	
4135	1.0					
4160	2.0	0.0	1.0			
4190	0.00408	0.0259575	0.047835	0.0697125	0.09159	
4195	0.00521	0.0270825	0.048955	0.0708275	0.0927	
5000	-1.					

## 113

# RODY VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 1

LATERAL STATION NO	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
	1.367594	5.909044	14.023102	25.607729	40.517242	58.564146	79.521490	103.125723	129.080009	157.057956	186.707727	217.656459	249.514952	281.882568	314.352265	346.515719	377.968455	408.314936	437.173539	464.181349	488.998727	511.313581	530.845289	547.348227	560.614862	570.478359	576.814677	579.544135
	.885863	2.742717	4.689037	6.823918	8.986607	11.263331	13.655558	15.920157	17.959463	20.478761	23.454137	26.221138	28.514243	30.266977	30.869521	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069	30.917069
	.887939	2.642261	4.410988	6.481009	8.847891	11.314363	16.967221	23.280504	25.290380	24.134684	20.871473	19.432546	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167	19.323167
	.590518	1.036237	1.059708	1.070975	1.064636	1.025605	1.084864	1.162270	1.134511	1.099374	1.047057	1.052650	1.067546	1.075833	1.082484	1.068542	1.046688	1.026233	1.010593	1.002017	1.000092	.999202	.997655	.996394	.997776	1.002328	1.002574	.998205
	-.068040	-.096102	-.100525	-.099997	-.091717	-.071717	-.049334	-.082142	-.139510	-.170262	-.177496	-.146017	-.135301	-.134304	-.113127	-.049121	-.034994	-.029783	-.026822	-.021138	-.011672	-.006057	-.004160	-.005571	-.010173	-.013911	-.013173	-.011736
	.646659	-.083023	-.133086	-.156986	-.141862	-.057008	-.179363	-.357618	-.306301	-.237612	-.127833	-.129392	-.157960	-.175454	-.184570	-.144195	-.096781	-.054041	-.022018	-.004485	-.000321	.001558	.004667	.007169	.004339	-.004855	-.005329	.003450

# LATERAL STATION 2

NO	XQ	YO	ZO	VM/V	VT/V	CP
1	1.367594	.885863	-.887939	.470534	-.063820	.774525
2	5.909044	2.742717	-2.642261	.912473	-.090469	.159209
3	14.023102	4.689037	-4.410988	.983299	-.101659	.022790
4	25.607729	6.823918	-6.481009	1.013680	-.104462	-.039529
5	40.517242	8.986606	-8.859000	1.034137	-.103806	-.080215
6	58.564146	11.263330	-11.284960	1.058615	-.091148	-.128974
7	79.521490	13.655557	-13.047635	1.065652	-.087291	-.143233
8	103.125723	15.920156	-14.403721	1.057658	-.093260	-.127339
9	129.080009	17.959462	-16.019709	1.049375	-.103364	-.111872
10	157.057956	20.478760	-17.601603	1.053666	-.104389	-.121109
11	186.707727	23.454135	-19.014641	1.071217	-.100577	-.157622
12	217.656459	26.221136	-19.318038	1.063202	-.092699	-.138991
13	249.514952	28.514240	-19.323169	1.050094	-.095140	-.111748
14	281.882568	30.266975	-19.323169	1.037424	-.103187	-.086895
15	314.352265	30.869518	-19.323169	1.004328	-.091440	-.017037
16	346.515719	30.917066	-19.323169	.988452	-.029404	.022097
17	377.968455	30.917066	-19.323169	.988548	-.016340	.022507
18	408.314936	30.917066	-19.323169	.990491	-.013453	.018747
19	437.173539	30.917066	-19.323169	.989040	-.016152	.021539
20	464.181349	30.917066	-19.323169	.985564	-.020544	.028242
21	488.998727	30.917066	-19.323169	.982855	-.018533	.033652
22	511.313581	30.917066	-19.323169	.982864	-.015374	.033742
23	530.845289	30.917066	-19.323169	.985097	-.013962	.029390
24	547.348227	30.917066	-19.323169	.987694	-.014689	.024245
25	560.614862	30.917066	-19.323169	.987333	-.017740	.024856
26	570.478359	30.917066	-19.323169	.982742	-.019734	.033826
27	576.814677	30.917066	-19.323169	.980573	-.017752	.038162
28	579.544135	30.917066	-19.323169	.985343	-.015788	.028851

# LATERAL STATION 3

NO	XO	YO	ZO	VM/V	VT/V	CP
1	1.367594	-0.885863	-0.887939	.470531	.054860	.775591
2	5.909044	-2.742718	-2.642261	.912469	.106063	.156151
3	14.023102	-4.689037	-4.410987	.983294	.108061	.021455
4	25.607729	-6.823919	-6.481008	1.013675	.110225	-.039686
5	40.517242	-8.986607	-8.858998	1.034131	.106109	-.080686
6	58.564146	-11.263332	-11.284958	1.058608	.092209	-.129153
7	79.521490	-13.655559	-13.047633	1.065645	.081929	-.142311
8	103.125723	-15.920158	-14.403718	1.057653	.092739	-.127230
9	129.080009	-17.959465	-16.019706	1.049371	.103859	-.111966
10	157.057956	-20.478763	-17.601599	1.053663	.104291	-.121082
11	186.707727	-23.454139	-19.014638	1.071213	.100677	-.157633
12	217.656459	-26.221140	-19.318035	1.063194	.092515	-.138941
13	249.514952	-28.514245	-19.323166	1.050077	.095113	-.111707
14	281.882568	-30.266980	-19.323166	1.037381	.103051	-.086779
15	314.352265	-30.869523	-19.323166	1.004228	.104047	-.019299
16	346.515719	-30.917072	-19.323166	.988233	.089126	.015453
17	377.968455	-30.917072	-19.323166	.988038	.088269	.015989
18	408.314936	-30.917072	-19.323166	.989610	.091745	.012256
19	437.173539	-30.917072	-19.323166	.989461	.096468	.011660
20	464.181349	-30.917072	-19.323166	.987045	.101795	.015381
21	488.998727	-30.917072	-19.323166	.983701	.103668	.021585
22	511.313581	-30.917072	-19.323166	.983235	.103617	.022512
23	530.845289	-30.917072	-19.323166	.985181	.104204	.018560
24	547.348227	-30.917072	-19.323166	.987455	.105814	.013735
25	560.614862	-30.917072	-19.323166	.986829	.108429	.014412
26	570.478359	-30.917072	-19.323166	.982309	.110354	.022891
27	576.814677	-30.917072	-19.323166	.980352	.108860	.027059
28	579.544135	-30.917072	-19.323166	.985285	.104955	.017773

# LATERAL STATION 4

NO	XO	YO	ZO	VM/V	VT/V	CP
1	1.367594	-0.885863	0.887940	.590515	.059083	.647801
2	5.909044	-2.742717	2.642261	1.036232	.111706	-.086256
3	14.023102	-4.689036	4.410989	1.059701	.106943	-.134403
4	25.607729	-6.823917	6.481010	1.070963	.100787	-.157120
5	40.517242	-8.986605	8.847892	1.064618	.094061	-.142259
6	58.564146	-11.263329	11.314365	1.025575	.072831	-.057108
7	79.521490	-13.655555	16.967224	1.084819	.044029	-.178770
8	103.125723	-15.920154	23.280508	1.162234	.081644	-.357453
9	129.080009	-17.959460	25.290384	1.134504	.139019	-.306426
10	157.057956	-20.478758	24.134688	1.099378	.170194	-.237598
11	186.707727	-23.454133	20.871477	1.047062	.177646	-.127898
12	217.656459	-26.221134	19.432549	1.052651	.145892	-.129359
13	249.514952	-28.514238	19.323171	1.067553	.135336	-.157985
14	281.882568	-30.266972	19.323171	1.075863	.134234	-.175501
15	314.352265	-30.869515	19.323171	1.082570	.125804	-.187786
16	346.515719	-30.917064	19.323171	1.068742	.108925	-.154075
17	377.968455	-30.917064	19.323171	1.047171	.107010	-.108019
18	408.314936	-30.917064	19.323171	1.027078	.108169	-.066589
19	437.173539	-30.917064	19.323171	1.010119	.107428	-.031881
20	464.181349	-30.917064	19.323171	1.000460	.102497	-.011426
21	488.998727	-30.917064	19.323171	.999125	.096834	-.007628
22	511.313581	-30.917064	19.323171	.998725	.094212	-.006328
23	530.845289	-30.917064	19.323171	.997547	.094273	-.003987
24	547.348227	-30.917064	19.323171	.996646	.096565	-.002628
25	560.614862	-30.917064	19.323171	.998281	.100756	-.006717
26	570.478359	-30.917064	19.323171	1.002773	.104440	-.016461
27	576.814677	-30.917064	19.323171	1.002819	.104177	-.016499
28	579.544135	-30.917064	19.323171	.998267	.102788	-.007102

# BODY SECTIONAL LOADS FOR COMPONENT 1

X/C	CXW/WAVG	CYH/HAVG	CZW/WAVG	CX	CY	CZ	CMX	CMY	CMZ	CDI
.001572	.050144	.000047	.003062	.010493	.004430	.114347	.000000	.000315	.000030	.019155
.007840	.029743	-.000190	.019492	.582145	-.000046	-.000455	2086.334115	29548.720723	41111.352224	
.020297	-.006168	-.000362	.032213							
.038787	-.017519	-.000101	.035236							
.063077	-.024219	-.000166	.029833							
.092861	-.022710	-.000183	-.020046							
.127765	-.050716	.000482	.000689							
.167351	-.090340	.000218	.120171							
.211120	-.043493	-.000174	.151413							
.258521	-.025014	-.000026	.118753							
.308959	-.016557	-.000035	.001799							
.361800	-.016400	.000035	-.020793							
.416379	-.015693	.000014	.040604							
.472009	-.011005	.000174	.103462							
.527991	-.002040	-.001718	.207591							
.583621	.000000	-.007340	.238464							
.638200	-0.000000	-.008775	.185028							
.691041	-.000000	-.009435	.120487							
.741479	.000000	-.009973	.069956							
.788880	.000000	-.010114	.043494							
.832649	-.000000	-.009853	.042609							
.872235	.000000	-.009701	.043201							
.907139	-.000000	-.009795	.035241							
.936923	-0.000000	-.010084	.025196							
.961213	-.000000	-.010683	.025866							
.979703	-0.000000	-.011288	.049624							
.992160	.000000	-.011363	.061495							
.998428	-.000000	-.010976	.039674							

## TOTAL BODY LOADS

CX	CY	CZ	CMX	CMY	CMZ	CDI
-.019493	-.004430	.114347	-.000000	.000315	-.000030	.019155
X/C CP	Y/C CP	Z/C CP	SX	SY	SZ	
.582145	-.000046	-.000455	2086.334115	29548.720723	41111.352224	



# PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 2

ETA= .062500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.181120	.656489	-.311229	.406818	-.491906	.403521	.895427
.204247	1.081112	.866413	-.296867	-.036583	-.256934	.247990	.504924
.437500	1.069295	.979515	-.220368	-.103482	-.191954	.029841	.221795
.687500	1.025080	.978685	-.168749	-.090304	-.079265	.034021	.113286
.887260	1.087280	1.067276	-.101465	-.031939	-.192473	-.140099	.052374
.983253	.952089	.946204	-.077220	.002996	.087564	.104688	.017125

ETA= .187500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.263527	.773080	-.222740	.195780	-.646114	.364017	1.010131
.204247	1.118429	.874160	-.144025	.082749	-.271627	.228997	.500624
.437500	1.054999	.946643	-.089491	.020618	-.121032	.103443	.224474
.687500	1.027543	.973618	-.066897	.006097	-.060319	.052031	.112351
.887260	1.011453	.988791	-.050859	.010839	-.025625	.022175	.047799
.983253	.995865	.989405	-.055099	.006077	.005217	.021041	.015824

ETA= .312500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.260104	.754962	-.217921	.179592	-.635353	.397779	1.033131
.204247	1.136498	.870238	-.133686	.096616	-.309499	.233351	.542850
.437500	1.061764	.939789	-.083033	.043308	-.134238	.114921	.249159
.687500	1.028805	.968763	-.064228	.026749	-.062564	.060783	.123348
.887260	1.009212	.984526	-.057023	.024083	-.021760	.030130	.051890
.983253	.999502	.992695	-.056792	.024304	-.002230	.013966	.016196

ETA= .437500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.269664	.739954	-.223053	.193428	-.661800	.415054	1.076854
.204247	1.142170	.864343	-.141068	.111018	-.324452	.240586	.565037
.437500	1.065755	.936223	-.098340	.067994	-.145504	.118863	.264367
.687500	1.031067	.967045	-.084377	.055413	-.070219	.061753	.131972
.887260	1.010733	.984547	-.080200	.053367	-.028013	.027820	.055833
.983253	1.000695	.993544	-.080154	.054449	-.007815	.009905	.017720

ETA= .562500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.272811	.734412	-.228597	.204819	-.672303	.418688	1.090992
.204247	1.143606	.861439	-.147200	.122977	-.329502	.242800	.572302
.437500	1.066636	.935267	-.106949	.082802	-.149150	.118420	.267570
.687500	1.031851	.967130	-.095781	.072593	-.073891	.059390	.133281
.887260	1.011707	.985262	-.093491	.071644	-.032291	.024125	.056416
.983253	1.001689	.994466	-.094099	.073051	-.012235	.005700	.017936

ETA= .687500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.279087	.726329	-.231136	.211646	-.689488	.427652	1.117140
.204247	1.145863	.858036	-.147972	.128280	-.334899	.247319	.582217
.437500	1.066623	.935111	-.109870	.090311	-.149757	.117412	.267168
.687500	1.031729	.967925	-.101516	.082613	-.074769	.056296	.131066
.887260	1.012103	.986104	-.100526	.082487	-.034457	.020795	.055252
.983253	1.002354	.995242	-.101382	.083853	-.014992	.002463	.017455

ETA= .812500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.283194	.720877	-.233841	.217733	-.701268	.432929	1.134197
.204247	1.144662	.858396	-.154770	.138599	-.334204	.243948	.578152
.437500	1.062995	.938607	-.125686	.109654	-.145754	.106993	.252747
.687500	1.029330	.970847	-.123072	.107472	-.074668	.045906	.120574
.887260	1.011507	.987656	-.124080	.109013	-.038542	.012653	.051195
.983253	1.002645	.996076	-.125143	.110385	-.020957	-.004353	.016604

ETA= .937500

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.268775	.734306	-.238602	.225210	-.666721	.410076	1.076796
.204247	1.126214	.876222	-.192559	.179171	-.305438	.200132	.505570
.437500	1.048174	.953330	-.196325	.183057	-.137213	.057653	.194866
.687500	1.021807	.978769	-.205868	.192867	-.086472	.004814	.091286
.887260	1.008961	.990936	-.212099	.199405	-.062989	-.021716	.041273
.983253	1.002336	.997257	-.214699	.202180	-.050773	-.035399	.015375

# PANFL SECTIONAL LOADS FOR COMPONENT 2

ETA	CXC/CAVG	CYC/CAVG	CZC/CAVG	CNC/CAVG	CMLEXC/CAVG	CMLEYC/CAVG	CMLEZC/CAVG
.062500	0.000000	-.000000	.443564	.443564	.001636	-.114514	-.000000
.187500	0.000000	-.000000	.416712	.416712	.001943	-.103769	-.000000
.312500	0.000000	-.000000	.384872	.384872	.001593	-.098811	-.000000
.437500	0.000000	-.000000	.365286	.365286	0.000000	-.093836	0.000000
.562500	0.000000	-.000000	.320442	.320442	0.000000	-.082251	0.000000
.687500	0.000000	-.000000	.274300	.274300	.000000	-.069529	0.000000
.812500	0.000000	-.000000	.220765	.220765	0.000000	-.054293	0.000000
.937500	0.000000	-.000000	.149743	.149743	0.000000	-.034260	0.000000
ETA	XLE/C CP	YLE/C CP	ZLE/C CP	CDT=0C/CAVG	CDLIC/CAVG	CTC/CAVG	CDTIC/CAVG
.062500	.258168	.003689	.000000	0.000000	0.000000	0.000000	0.000000
.187500	.249019	.004662	.000000	0.000000	0.000000	0.000000	0.000000
.312500	.256738	.004138	.000000	0.000000	0.000000	0.000000	0.000000
.437500	.256883	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.562500	.256680	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.687500	.253479	.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.812500	.245932	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.937500	.228791	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

## TOTAL PANEL LOADS

CX	0.000000	CY	-.000000	CZ	.212417	CMX	-.054188	CMY	-.000000	CMZ	-.000000
CDT=0	0.000000	CDLI	0.000000	CDI	.013985	CT	0.000000	CDTI	0.000000		
X/C CP	2.920645	Y/C CP	.659864	Z/C CP	.000000						

# PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 3

ETA= .083333  

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.025054	.956949	-.095312	.022608	-.059820	.083737	.143557
.204247	1.006177	.959123	-.023159	.057871	-.012929	.076734	.089663
.437500	.995129	.966151	-.016503	.026161	.009447	.065867	.056420
.687500	.996815	.977835	-.001801	.014707	.004358	.043623	.037265
.887260	.995186	.986316	.006937	.004910	.009558	.027157	.017599
.983253	.995323	.992755	.007548	.001089	.009275	.014437	.005162

ETA= .250000  

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.041796	.941670	-.010696	.095229	-.085454	.104190	.189644
.204247	1.025326	.959699	.022339	.067095	-.051793	.074477	.126270
.437500	1.013987	.972254	.050509	.045808	-.030721	.052625	.083345
.687500	1.006866	.980639	.065026	.032162	-.019008	.037313	.055321
.887260	1.000507	.987937	.073115	.023021	-.004361	.023451	.029812
.983253	.996244	.992552	.076488	.019051	.001648	.014477	.012829

ETA= .416667  

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.079304	.908199	.110317	.294157	-.177068	.088647	.265715
.204247	1.042394	.945292	.166965	.250391	-.114462	.043727	.158190
.437500	1.021030	.967689	.201900	.222396	-.083266	.014118	.097384
.687500	1.010246	.979548	.215199	.204890	-.066908	-.001493	.065414
.887260	1.002111	.988479	.219766	.192880	-.052524	-.014293	.038231
.983253	.997383	.993598	.220956	.187760	-.043595	-.022490	.021105

ETA= .583333  

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.106353	.883410	.076604	.347538	-.229886	.098803	.328689
.204247	1.055318	.934881	.141176	.284012	-.133627	.045334	.178961
.437500	1.025908	.964988	.175525	.251135	-.083297	.005729	.089026
.687500	1.012533	.979137	.189930	.238172	-.061296	-.015435	.045861
.887260	1.003453	.988858	.197049	.232128	-.045746	-.031723	.014023
.983253	.998361	.994273	.199961	.229706	-.036709	-.041344	-.004634

ETA= .750000

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.109203	.882289	-.089254	.205859	-.238297	.179188	.417485
.204247	1.059027	.932806	-.026707	.143932	-.122252	.109158	.231410
.437500	1.028351	.964010	.007470	.110636	-.057562	.058445	.116008
.687500	1.013245	.979697	.019760	.099216	-.027055	.030350	.057405
.887260	1.003666	.989754	.025191	.094422	-.007980	.011472	.019452
.983253	.998732	.994922	.027556	.092344	.001775	.001603	-.000172

ETA= .916667

X/C	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U
.050240	1.130268	.862901	-.095507	.220482	-.286628	.206789	.493417
.204247	1.065859	.927537	-.032051	.157370	-.137082	.114911	.251993
.437500	1.026608	.967134	-.014929	.140738	-.054147	.044844	.098991
.687500	1.010001	.984119	-.010977	.137271	-.020222	.012666	.032888
.887260	1.002144	.992282	-.009366	.136017	-.004381	-.003125	.001256
.983253	.998607	.995968	-.008639	.135450	.002710	-.010298	-.013008

# PANEL SECTIONAL LOADS FOR COMPONENT 3

ETA	CXC/CAVG	CYC/CAVG	CZC/CAVG	CNC/CAVG	CMLEXC/CAVG	CMLEYC/CAVG	CMLEZC/CAVG
.083333	0.000000	.023200	.086572	.089626	.000203	-.027748	.007436
.250000	0.000000	.029254	.109203	.113054	.000714	-.036636	.009814
.416667	0.000000	.029709	.110471	.114396	.000519	-.035766	.009619
.583333	0.000000	.026035	.097158	.100585	0.000000	-.025209	.006755
.750000	0.000000	.026722	.099721	.103240	0.000000	-.026123	.007000
.916667	0.000000	.019853	.074087	.076701	0.000000	-.015315	.004104
ETA	XLE/C CP	YLE/C CP	ZLE/C CP	CDT=0C/CAVG	CDLIC/CAVG	CTC/CAVG	CDTIC/CAVG
.083333	.320525	.002183	-.000585	0.000000	0.000000	0.000000	0.000000
.250000	.335482	.006098	-.001633	0.000000	0.000000	0.000000	0.000000
.416667	.323759	.004378	-.001177	0.000000	0.000000	0.000000	0.000000
.583333	.259464	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.750000	.261961	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.916667	.206721	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

## TOTAL PANEL LOADS

CX	CY	CZ	CMX	CMY	CMZ
0.000000	.025550	.095285	.060188	-.145875	.039115
CDT=0	CDLI	CDI	CT	CDTI	
0.000000	0.000000	.004900	0.000000	0.000000	
X/C CP	Y/C CP	Z/C CP			
4.283246	.589289	-.158016			

# PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 4

ETA= .100000  
 X/C  
 .050240  
 .204247  
 .437500  
 .687500  
 .887260  
 .983253  
 VM/V UP  
 .932949  
 1.024072  
 .985155  
 .998520  
 1.012247  
 1.057625  
 VM/V LO  
 .932564  
 1.024938  
 .985719  
 .998490  
 1.011969  
 1.057486  
 VT/V UP  
 -.077877  
 -.091636  
 -.086986  
 -.055047  
 -.042618  
 -.021044  
 VT/V LO  
 -.076001  
 -.092151  
 -.088775  
 -.056996  
 -.044008  
 -.021831  
 CP UP  
 .123541  
 -.057121  
 .021903  
 -.000073  
 -.026460  
 -.119014  
 CP LO  
 .124549  
 -.058990  
 .020477  
 -.000231  
 -.026018  
 -.118753  
 CP L-U  
 .001008  
 -.001869  
 -.001426  
 -.000158  
 .000442  
 .000261

ETA= .300000  
 X/C  
 .050240  
 .204247  
 .437500  
 .687500  
 .887260  
 .983253  
 VM/V UP  
 .953746  
 .984768  
 .996926  
 1.004757  
 1.023596  
 1.072462  
 VM/V LO  
 .953741  
 .985292  
 .997314  
 1.004793  
 1.023507  
 1.072425  
 VT/V UP  
 .002525  
 -.016717  
 -.017246  
 -.011580  
 -.001439  
 .006250  
 VT/V LO  
 .002958  
 -.016948  
 -.017980  
 -.012328  
 -.001902  
 .005918  
 CP UP  
 .090363  
 .029953  
 .005841  
 -.009672  
 -.047750  
 -.150213  
 CP LO  
 .090370  
 .028912  
 .005042  
 -.009761  
 -.047571  
 -.150131  
 CP L-U  
 .000007  
 -.001041  
 -.000798  
 -.000090  
 .000180  
 .000082

ETA= .500000  
 X/C  
 .050240  
 .204247  
 .437500  
 .687500  
 .887260  
 .983253  
 VM/V UP  
 .954350  
 .985299  
 .999663  
 1.008530  
 1.029149  
 1.078532  
 VM/V LO  
 .954499  
 .985656  
 .999929  
 1.008589  
 1.029124  
 1.078520  
 VT/V UP  
 .029335  
 .015500  
 .014749  
 .017911  
 .022322  
 .024845  
 VT/V LO  
 .029413  
 .015293  
 .014223  
 .017342  
 .021885  
 .024437  
 CP UP  
 .088356  
 .028946  
 .000456  
 -.017453  
 -.059645  
 -.163848  
 CP LO  
 .088067  
 .028249  
 -.000061  
 -.017553  
 -.059576  
 -.163803  
 CP L-U  
 -.000288  
 -.000697  
 -.000517  
 -.000099  
 .000070  
 .000045

ETA= .700000  
 X/C  
 .050240  
 .204247  
 .437500  
 .687500  
 .887260  
 .983253  
 VM/V UP  
 .956354  
 .988521  
 1.001854  
 1.010658  
 1.030562  
 1.079426  
 VM/V LO  
 .956544  
 .988770  
 1.002031  
 1.010716  
 1.030563  
 1.079422  
 VT/V UP  
 .051852  
 .043495  
 .044823  
 .046103  
 .045263  
 .043048  
 VT/V LO  
 .051796  
 .043272  
 .044373  
 .045593  
 .044805  
 .042599  
 CP UP  
 .082698  
 .020934  
 -.0005720  
 -.023555  
 -.064108  
 -.167013  
 CP LO  
 .082341  
 .020461  
 -.006036  
 -.023625  
 -.064067  
 -.166968  
 CP L-U  
 -.000357  
 -.000473  
 -.000316  
 -.000070  
 .000041  
 .000045

ETA=	.900000								
X/C									
.050240	.968136	VM/V UP	VM/V LO	VT/V UP	VT/V LO	CP UP	CP LO	CP L-U	
.204247	.994229		.968297	.081258	.081133	.056111	.055819	-.000292	
.437500	1.005005		.994378	.086369	.086080	.004049	.003802	-.000246	
.687500	1.012164		1.005106	.089418	.088908	-.019030	-.018143	-.000112	
.887260	1.024652		1.012205	.089343	.088735	-.032458	-.032432	.000025	
.983253	1.069703		1.024664	.082011	.081632	-.056638	-.056601	.000038	
			1.069703	.069102	.068775	-.149039	-.148995	.000044	



# PANEL SECTIONAL LOADS FOR COMPONENT 4

ETA	CXC/CAVG	CYC/CAVG	CZC/CAVG	CNC/CAVG	CMLEXC/CAVG	CMLEYC/CAVG	CMLEZC/CAVG
.100000	0.000000	.000639	0.000000	-.000639	-.000000	0.000000	.000198
.300000	0.000000	.000420	0.000000	-.000420	-.000000	0.000000	.000120
.500000	0.000000	.000308	0.000000	-.000308	-.000000	0.000000	.000088
.700000	0.000000	.000207	0.000000	-.000207	0.000000	0.000000	.000055
.900000	-.001351	.000086	0.000000	-.000086	0.000000	0.000000	.000011

ETA	XLE/C CP	YLE/C CP	ZLE/C CP	CDT=0C/CAVG	CDLIC/CAVG	CTC/CAVG	CDTIC/CAVG
.100000	.310517	0.000000	.000306	0.000000	0.000000	0.000000	0.000000
.300000	.284980	0.000000	.000825	0.000000	0.000000	0.000000	0.000000
.500000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.700000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
.900000	.000527	.000250	0.000000	0.000000	0.000000	0.000000	0.000000

## TOTAL PANEL LOADS

CX	CY	CZ	CMX	CMY	CMZ
-.0000290	.000334	0.000000	-.000125	-.000181	.000424
CDT=0	CDLI	CDI	CT	CDTI	
0.000000	0.000000	-.000008	0.000000	0.000000	
X/C CP	Y/C CP	Z/C CP			
3.476786	.628013	.481290			

TOTAL CONFIGURATION LOADS

CX	CY	CZ	CMX	CMY	CMZ
-0195524	-0043623	.3081308	-0000265	-0647415	-0000556
(X/CR) CP	(Y/CR) CP	(Z/CR) CP			
339.3767286	-0000970	.0132752			

## CONCLUSIONS

The theory and program described herein are capable of predicting surface velocities and pressure coefficients and section, control surface, and total configuration loads and moments for a diverse class of airplanes. In particular the source - vortex lattice with second order corrections was shown to agree well with the exact solution for a Karman-Trefftz airfoil section. Also, the quadrilateral vortex element predicted surface velocities which agreed well with the exact solution for a sphere.

Because of the options built into the program simple configurations can be input with a minimum of effort while the capability exists to input complicated configurations in significant detail. The program is general enough to analyze complete configurations at angle of attack and in side slip, pitching motion, rolling motion, yawing motion, and with control surfaces deflected. The configuration can be run in each of these modes to obtain static and rotary stability derivatives or in a combination of the modes to predict the loads and moments on the configuration while in a quasi-steady maneuver. The program also has the capability to account for a free wake while operating in any of the above modes.

Within the scope of this study a significant effort was devoted toward understanding the limitations of and the relationships between the different types of aerodynamic finite elements and lifting surface theories available at present. It has been concluded that those used in this analysis are presently the most efficient and generally available. It was also shown that the spanwise variation of potential form drag due to both thickness and lift are correctly computed by the program described in this report.

## Appendix A

### NUMERICAL PROCEDURES

Discussions of the prime numerical procedures used within the program are given in this appendix. There are essentially three such procedures; (1) straight line interpolation and extrapolation, (2) controlled deviation interpolation, and (3) Householder's simultaneous equation solution.

For straight line interpolation and extrapolation about two given points  $(X_1, Y_1)$  and  $(X_2, Y_2)$ ;

$$Y = \alpha Y_2 + (1 - \alpha) Y_1 \quad (1)$$

where

$$\alpha = \frac{X - X_1}{X_2 - X_1} \quad (2)$$

The slope  $\frac{dY}{dX}$  for this case is given by;

$$\frac{dY}{dX} = \alpha' (Y_2 - Y_1) \quad (3)$$

where

$$\alpha' = 1/(X_2 - X_1) \quad (4)$$

In the case of the controlled deviation interpolation method (CODIM) parabolae are used to curve fit a set of four given points  $(X_{N-1}, Y_{N-1})$ ,  $(X_N, Y_N)$ ,  $(X_{N+1}, Y_{N+1})$ , and  $(X_{N+2}, Y_{N+2})$  to obtain interpolated  $Y$  and  $dY/dX$  values for  $X_N \leq x \leq X_{N+1}$ . Only that information, relative to this method, which is necessary to judiciously pick input points will be discussed here. A complete derivation is given in reference (20).

One parabola  $P_1$  is fit through  $(X_{N-1}, Y_{N-1})$ ,  $(X_N, Y_N)$ , and  $(X_{N+1}, Y_{N+1})$ . The other parabola  $P_2$  is fit through  $(X_N, Y_N)$ ,  $(X_{N+1}, Y_{N+1})$ , and  $(X_{N+2}, Y_{N+2})$ . This curve fitting process involves the solution of two sets of simultaneous equations. If,

$$P_1 = A_1X^2 + B_1X + C_1 \quad (5)$$

and

$$P_2 = A_2X^2 + B_2X + C_2 \quad (6)$$

Then

$$\begin{Bmatrix} A_1 \\ B_1 \\ C_1 \end{Bmatrix} = \begin{bmatrix} X_{N-1}^2 & X_{N-1} & 1 \\ X_N^2 & X_N & 1 \\ X_{N+1}^2 & X_{N+1} & 1 \end{bmatrix}^{-1} \begin{Bmatrix} Y_{N-1} \\ Y_N \\ Y_{N+1} \end{Bmatrix} \quad (7)$$

and

$$\begin{Bmatrix} A_2 \\ B_2 \\ C_2 \end{Bmatrix} = \begin{bmatrix} X_N^2 & X_N & 1 \\ X_{N+1}^2 & X_{N+1} & 1 \\ X_{N+2}^2 & X_{N+2} & 1 \end{bmatrix}^{-1} \begin{Bmatrix} Y_N \\ Y_{N+1} \\ Y_{N+2} \end{Bmatrix} \quad (8)$$

The interpolated values of  $Y$  and  $dY/dX$  between  $X_N$  and  $X_{N+1}$  are defined by either  $P_1$ ,  $P_2$ , or a linear combination of  $P_1$  and  $P_2$ . The amount of  $P_1$  and  $P_2$  used in the linear combination is determined by comparing both of the parabolae to the straight line.

$$S = \alpha Y_{N+1} + (1 - \alpha) Y_N \quad (9)$$

Therefore;

$$\frac{dD}{dX} = \frac{d\alpha}{dX} E_1 + \alpha \frac{dE_1}{dX} - \frac{d\alpha}{dX} E_2 + (1-\alpha) \frac{dE_2}{dX} \quad (18)$$

$$Y = \begin{cases} P_1 & \text{for } X = X_N \\ \frac{\alpha E_1 P_2 + (1-\alpha) E_2 P_1}{\alpha E_1 + (1-\alpha) E_2} & \text{for } X_N < X < X_{N+1} \\ P_2 & \text{for } X = X_{N+1} \end{cases} \quad (19)$$

and

$$\frac{dY}{dX} = \begin{cases} \frac{dP_1}{dX} & \text{for } X = X_N \\ \frac{dN}{dX} / D - N \frac{dD}{dX} / D^2 & \text{for } X_N < X < X_{N+1} \\ \frac{dP_2}{dX} & \text{for } X = X_{N+1} \end{cases} \quad (20)$$

In the case of an end interval  $X_{N-1} \leq x \leq X_N$ ;  $P_1$  is set equal to;

$$P_1 = S + K(P_2 - S) \quad (21)$$

where

$$K = 1 - \frac{||M_1| - |M_2||}{|M_1| + |M_2|} \quad (22)$$

where

$$\alpha = \frac{Y_{N+1} - Y_N}{X_{N+1} - X_N} \quad (10)$$

The parabola which has the least deviation from the straight line is given the greatest weight. The weighting factors  $E_1$  and  $E_2$  are determined as follows:

$$E_1 = |P_1 - S| \quad (11)$$

$$E_2 = |P_2 - S| \quad (12)$$

The weighted expression for  $Y$  in the range  $X_N \leq x \leq X_{N+1}$  is then;

$$Y = \frac{\alpha E_1 P_2 + (1 - \alpha) E_2 P_1}{E_1 + (1 - \alpha) E_2} \quad (13)$$

The derivative  $dY/dX$  in the range  $X_N \leq x \leq X_{N+1}$  is then;

$$\frac{dY}{dX} = \frac{dN}{dX} / D - N \frac{dD}{dX} / D^2 \text{ for } D \neq 0 \quad (14)$$

where

$$N = \alpha E_1 P_2 + (1 - \alpha) E_2 P_1 \quad (15)$$

$$D = \alpha E_1 + (1 - \alpha) E_2 \quad (16)$$

And then

$$\frac{dN}{dX} = \frac{d\alpha}{dX} E_1 P_2 + \alpha \frac{dE_1}{dX} P_2 + \alpha E_1 \frac{dP_2}{dX} - \frac{d\alpha}{dX} E_2 P_1 + (1 - \alpha) \frac{dE_2}{dX} P_1 + E_2 \frac{dP_1}{dX} \quad (17)$$

and

$$M_1 = \frac{Y_N - Y_{N-1}}{X_N - X_{N-1}} \quad (23)$$

$$M_2 = \frac{Y_N - Y_{N+1}}{X_N - X_{N+1}} \quad (24)$$

A similar procedure is followed for the other end interval  $X_{N+1} \leq x \leq X_{N+2}$ .

Householder's method for solving simultaneous equations is used in the solution of the aerodynamic influence equations. The method is applicable to both square and rectangular influence matrices. In the case of rectangular matrices it is not necessary to least square the equations first, since Householder's procedure least squares and triangularizes simultaneously. Also, the influence matrix is triangularized by means of orthogonal transformation matrices, which preserve the conditioning of the matrix. The combination of these two advantages, along with a reduction in the number of required computer operations, greatly improves the numerical accuracy and stability of the solution over that of the standard Gaussian reduction method.

A complete, but rather abstract, derivation of the method is given in reference (21). The method in the subroutine has been altered from the original to allow the operation on a single row of the matrix at a time. This reduces the required core allocation necessary to triangularize the matrix.

A derivation of the method, developed by the writer, will be given here in order to describe the basic philosophy of the method.

If  $[A]$  is the rectangular influence matrix, the upper triangle is given by;

$$[R] = [W][A] \quad (25)$$

where  $[W]$  is the combined orthogonal transformation matrix used by Householder to triangularize  $[A]$ .



The relationship between Householder's triangularized matrix  $[R]$  and that obtained by Gaussian elimination of the least squared influence matrix  $[A]^T[A]$ , is

$$[G] = [T][A]^T[A] = [T][R]^T[R] = [D][R] \quad (26)$$

where  $[G]$  is the triangular matrix obtained by Gaussian elimination of  $[A]^T[A]$ . The matrix  $[T]$  is the Gaussian transformation matrix used to triangularize  $[A]^T[A]$ . And,  $[D]$  is a diagonal matrix with the same diagonal as  $[R]$ . It can be seen from equation (26) that a nonsquare matrix must be least squared first, before applying the Gaussian transformation  $[T]$ . Whereas, the Householder transformation matrix  $[W]$  can be applied directly. The least squared matrix  $[A]^T[A]$  is usually more illconditioned than  $[A]$ , and therefore, less accurate results are obtained.

In the Householder method  $[W]$  is equal to the product of  $N+1$  individual orthogonal transformation matrices, where  $n$  equals the number of unknowns. There are  $N+1$  transformations because the augmented influence matrix, made up of the influence matrix itself plus the boundary condition matrix, added on as the last column, has  $N+1$  columns. Each transformation results in reducing all elements below the diagonal to zero for one column. The columns are reduced from left to right.

The individual transformation matrices  $[W]_m$  are defined by;

$$[W]_m = ([I] - 2 \{u\}_m \{u\}_m^T) \quad (27)$$

where  $[I]$  is a unit diagonal matrix and  $\{u\}_m$  is a column matrix defined by the unit vector  $\bar{u}_m = (\bar{a}_m - \alpha_m \bar{v}_m) / \mu_m$ . The vector  $\bar{a}_m$  is defined by the  $m$ th column of  $[A]$  where the elements on rows less than  $m$  are replaced by zeros. The unit vector  $\bar{v}_m$  is defined by a column matrix  $[v]_m$  with all zeros except for the  $m$ th row, which is equal to one. The constants  $\alpha_m$  and  $\mu_m$  are defined as,

$$\alpha_m = |a_m| \quad (28)$$

$$\mu_m = \sqrt{2 \alpha_m (\alpha_m - \bar{v}_m \cdot \bar{a}_m)} \quad (29)$$

It can be shown that  $\{a_m\}$  is reduced to  $\|a_m\| \{v_m\}$  if  $\{a_m\}$  is premultiplied by  $([I] - 2\{u_m\}\{u_m\}^T)$ . Also, that the first  $m-1$  rows of

$$[W_{m-1}][W_{m-2}] \dots [W_1][A] \quad (30)$$

remain unchanged by the  $m$ th transformation. The result after  $m$  transformations is then zeros below the diagonal for the first  $m$  columns and  $\|a_1\|, \|a_2\|, \dots, \|a_{m-2}\|, \|a_{m-1}\|, \|a_m\|$  on the diagonal. The elements above the diagonal have been defined by the  $m$  preceding transformations and will remain unchanged for the  $N+1-m$  remaining transformations.

$$([I] - 2\{u_m\}\{u_m\}^T) \{a_m\} = \|a_m\| \{v_m\} \quad (31)$$

$$\vec{u}_m = (\vec{a}_m - \alpha_m \vec{v}_m) / \mu_m \quad \text{or} \quad \{u_m\} = (\{a_m\} - \alpha_m \{v_m\}) / \mu_m \quad (32)$$

$$\alpha_m = \|a_m\| \quad (33)$$

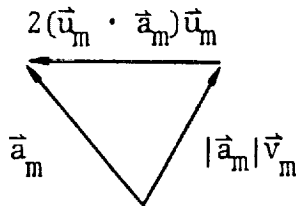
and

$$\mu_m = \sqrt{2\alpha_m(\alpha_m - \vec{v}_m \cdot \vec{a}_m)} \quad \text{or} \quad \mu_m = \sqrt{2\alpha_m(\alpha_m - \|a_m\| \{v_m\})} \quad (34)$$

remain to be proved. It is helpful in the derivation of equation (31) if the vector identity

$$\|\vec{a}_m\| \vec{v} + 2(\vec{u}_m \cdot \vec{a}_m) \vec{u}_m = \vec{a}_m \quad (35)$$

is observed from the following vector diagram.



Then from equation (35);

$$|\vec{a}_m| \vec{v}_m + 2 \vec{u}_m (\vec{u}_m \cdot \vec{a}_m) = \vec{a}_m \quad (36)$$

$$|\vec{a}_m| \vec{v}_m + 2 \vec{u}_m \vec{u}_m \cdot \vec{a}_m = \vec{a}_m \quad (37)$$

Therefore;

$$|\vec{a}_m| \vec{v}_m = (\hat{i}_m \hat{i}_m - 2 \vec{u}_m \vec{u}_m) \vec{a}_m \quad (38)$$

where  $\hat{i}_m \hat{i}_m$  and  $\vec{u}_m \vec{u}_m$  are dyadics. The unit vector  $\hat{i}_m$  is in the direction of  $\vec{a}_m$ .

Equation (38) can then be written in matrix notation as follows;

$$([I] - 2 \{u\}_m \{u\}_m^T) \{a\}_m = \{a\}_m \{v\}_m \quad (39)$$

which is equal to equation (31). In matrix or tensor notation it becomes evident that the dimensions of  $\{a\}_m$ ,  $\{v\}_m$ , and  $\{u\}_m$  are not limited to three.

$$\alpha_m = \|\{a\}_m\| \quad (40)$$

and

$$\mu_m = 2 \{a_m\}^T \{a_m\} \quad (41)$$

Then if equation (31) is premultiplied by  $\{a_m\}^T$

$$\{a_m\}^T \{a_m\} - 2 \{a_m\}^T \{u_m\} \{u_m\}^T \{a_m\} = \{a_m\}^T \{a_m\} \{v_m\} \quad (42)$$

And substituting  $\alpha_m$  and  $\mu_m$  into equation (42).

$$\alpha_m^2 - \frac{1}{2} \mu_m^2 = \alpha_m \{a_m\}^T \{v_m\} \quad (43)$$

or

$$\mu_m = \sqrt{2 \alpha_m (\alpha_m - \{a_m\}^T \{v_m\})} \quad (44)$$

In vector notation equation (44) is seen to be equal to;

$$\mu_m = \sqrt{2 \alpha_m (\alpha_m - \vec{v}_m \cdot \vec{a}_m)} \quad (45)$$

Also, if equation (44) is substituted back into equation (31)

$$\{a_m\} - \frac{\sqrt{2 \alpha_m (\alpha_m - \{a_m\}^T \{v_m\})}}{\alpha_m} \{a_m\} = \alpha_m \{v_m\} \quad (46)$$

Therefore;

$$\{u_m\} = \frac{\{a_m\} - \alpha_m \{v_m\}}{\sqrt{2 \alpha_m (\alpha_m - \{a_m\}^T \{v_m\})}} \quad (47)$$

or in vector notation

$$\vec{u}_m = \frac{\vec{a}_m - \alpha_m \vec{v}_m}{\sqrt{2 \alpha_m (\alpha_m - \vec{v}_m \cdot \vec{a}_m)}} \quad (48)$$

## Appendix B

### SKEWED SOURCE-VORTEX LATTICE INFLUENCE EQUATIONS

The general form of the skewed source and vortex lattice influence equations was derived in appendices B and C of reference (26), respectively. These equations will be specialized to the case where the inboard and outboard sweep angles for a given horseshoe are equal. The perturbation velocity due to a skewed vortex is then given by the following equations.

$$\frac{U_L}{V_\infty} = \frac{U_V}{V_\infty} \quad (1)$$

$$\frac{v_L}{V_\infty} = \frac{\Delta Y_V \left( \frac{V_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} - \frac{\Delta Z_V \left( \frac{W_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (2)$$

$$\frac{w_L}{V_\infty} = \frac{\Delta Z_V \left( \frac{V_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \frac{\Delta Y_V \left( \frac{W_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (3)$$

Where  $\Delta Y_V$  and  $\Delta Z_V$  are the changes in Y and Z across the width of the horseshoe vortex, respectively.

$$\frac{U_V}{V_\infty} = \frac{(\Gamma/V_\infty) E_{u_V}}{4\pi} \quad (4)$$

$$\frac{V_V}{V_\infty} = \frac{(\Gamma/V_\infty) E_{V_V}}{4\pi} \quad (5)$$

$$\frac{W_V}{V_\infty} = \frac{(\Gamma/V_\infty) E_{W_V}}{4\pi} \quad (6)$$

Where

$$E_{u_V} = \frac{\bar{Z}}{R_2^2} (I_2 + I_3) \quad (7)$$

$$E_{V_V} = \bar{Z} \left[ \frac{(I_1 + 1)}{R_1^2} - \frac{(I_4 + 1)}{R_3^2} - \frac{\bar{T}}{R_2^2} (I_2 + I_3) \right] \quad (8)$$

$$\Gamma_{w_V} = \frac{(\bar{Y} - \beta y_V)(I_4 + 1)}{R_3^2} - \frac{(\bar{Y} + \beta y_V)(I_1 + 1)}{R_1^2} - \frac{(\bar{X} - \bar{Y} \bar{T})(I_2 + I_3)}{R_2^2} \quad (9)$$

Let

$$\text{term 1} = \frac{(I_2 + I_3)}{R_2^2} \quad (10)$$

$$\text{term 2} = \frac{I_1 + 1}{R_1^2} \quad (11)$$

$$\text{term 3} = \frac{I_4 + 1}{R_3^2} \quad (12)$$

Then

$$E_{u_V} = \bar{Z} (\text{term 1}) \quad (13)$$

$$E_{v_V} = \bar{Z} [-\bar{T}(\text{term 1}) + (\text{term 2}) - (\text{term 3})] \quad (13)$$

$$E_{w_V} = -(\bar{X} - \bar{T} \bar{Y})(\text{term 1}) - (\bar{Y} + \beta y_V)(\text{term 2}) + (\bar{Y} - \beta y_V)(\text{term 3}) \quad (14)$$

Where

$$I_1 = \frac{\bar{X} + T y_V}{[(\bar{X} + T y_V)^2 + (\bar{Y} + \beta y_V)^2 + \bar{Z}^2]^{1/2}} = \frac{\bar{X} + T y_V}{R_5} \quad (15)$$

$$I_2 = \frac{\bar{Y} + \bar{T} \bar{X} + \beta y_V (1 + \bar{T}^2)}{[(\bar{X} + T y_V)^2 + (\bar{Y} + \beta y_V)^2 + \bar{Z}^2]^{1/2}} = \frac{\bar{Y} + \bar{T} \bar{X} + \beta y_V (1 + \bar{T}^2)}{R_5} \quad (16)$$

$$I_3 = \frac{\bar{Y} + \bar{T} \bar{X} - \beta y_V (1 + \bar{T}^2)}{[(\bar{X} - T y_V)^2 + (\bar{Y} - \beta y_V)^2 + \bar{Z}^2]^{1/2}} = - \frac{\bar{Y} + \bar{T} \bar{X} - \beta y_V (1 + \bar{T}^2)}{R_4} \quad (17)$$

$$I_4 = \frac{\bar{X} - T y_V}{[(\bar{X} - T y_V)^2 + (\bar{Y} - \beta y_V)^2 + \bar{Z}^2]^{1/2}} = \frac{\bar{X} - T y_V}{R_4} \quad (18)$$

$$R_1^2 = (\bar{Y} + \beta y_\nu)^2 + \bar{Z}^2 \quad (19)$$

$$R_2^2 = (\bar{X} - T Y)^2 + \bar{Z}^2 (1 + \bar{T}^2) \quad (20)$$

$$R_3^2 = (\bar{Y} - \beta y_\nu)^2 + \bar{Z}^2 \quad (21)$$

$$R_4^2 = (\bar{X} - T y_\nu)^2 + (\bar{Y} - \beta y_\nu)^2 + \bar{Z}^2 \quad (22)$$

$$R_5^2 = (\bar{X} + T y_\nu)^2 + (\bar{Y} + \beta y_\nu)^2 + \bar{Z}^2 \quad (23)$$

and

$$\bar{X} = X_q - X_v \quad (24)$$

$$\bar{Y} = \beta \frac{\Delta Y_V (Y_q - Y_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \beta \frac{\Delta Z_V (Z_q - Z_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (25)$$

$$\bar{Z} = -\beta \frac{\Delta Y_V (Y_q - Y_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \beta \frac{\Delta Z_V (Z_q - Z_V)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (26)$$

$$y_\nu = 1/2 \sqrt{\Delta Y_V^2 + \Delta Z_V^2} \quad (27)$$

$$\bar{T} = \frac{\tan \Lambda}{\beta} \quad (28)$$



where  $(X_V, Y_V, Z_V)$  and  $(X_q, Y_q, Z_q)$  are the locations of the influencing point and the point being influenced respectively.

The elements of  $[A_X]$ ,  $[A_Y]$ , and  $[A_Z]$  are computed for a unit strength of  $\Gamma/V_\infty$ .

Similarly, the perturbation velocity due to a skewed source line is given by;

$$\frac{U_t}{V_\infty} = \frac{U_S}{V_\infty} \quad (29)$$

$$\frac{V_t}{V_\infty} = \frac{\Delta Y_V \frac{V_S}{V_\infty}}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} - \frac{\Delta Z_V \frac{W_S}{V_\infty}}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (30)$$

$$\frac{W_t}{V_\infty} = \frac{\Delta Z_V \frac{V_S}{V_\infty}}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \frac{\Delta Y_V \frac{W_S}{V_\infty}}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (31)$$

where

$$\frac{U_S}{V_\infty} = \frac{(\Sigma / V_\infty) E_{u_S}}{4\pi} \quad (32)$$

$$\frac{V_S}{V_\infty} = \frac{(\Sigma / V_\infty) E_{v_S}}{4\pi} \quad (33)$$

$$\frac{W_S}{V_\infty} = \frac{(\Sigma / V_\infty) E_{w_S}}{4\pi} \quad (34)$$

and

$$E_{u_s} = \frac{\bar{T}}{\sqrt{1 + \bar{T}^2}} (\text{term 4}) + \frac{1}{\sqrt{1 + \bar{T}^2}} (\bar{X} - TY) (\text{term 1}) \quad (35)$$

$$E_{v_s} = \frac{1}{\sqrt{1 + \bar{T}^2}} (\text{term 4}) - \frac{\bar{T}}{\sqrt{1 + \bar{T}^2}} (\bar{X} - TY) (\text{term 1}) \quad (36)$$

$$E_{w_s} = \sqrt{1 + \bar{T}^2} \bar{Z} (\text{term 1}) \quad (37)$$

and where

$$\text{term 4} = \frac{1}{R_4} - \frac{1}{R_5} \quad (38)$$

The elements of  $[S_X]$ ,  $[S_Y]$ , and  $[S_Z]$  are computed for a unit strength of  $\Sigma/V_\infty$ . The influences of both the vortices and sources at the quarter chord of the subpanel are computed simultaneously due to the similarity in the vortex and source influence equations.

For the case where  $|\bar{Z}| \leq 2 y_\nu$ , set  $\bar{Z} = 0$  and if both

$$(\bar{X} - TY)^2 / \left[ (X + T y_\nu)^2 + (\bar{Y} + \beta y_\nu)^2 \right] \quad (39)$$

and

$$(\bar{X} - TY)^2 / \left[ (X - T y_\nu)^2 + (\bar{Y} - \beta y_\nu)^2 \right] \quad (40)$$

are less than  $(0.08716)^2$ , and  $|\bar{Y}| > \beta y_\nu$ , then use;

$$\frac{1}{2\sqrt{1 + \bar{T}^2}} \left| \frac{1}{(\bar{X} - T y_\nu)^2 + (\bar{Y} - \beta y_\nu)^2} - \frac{1}{(\bar{X} + T y_\nu)^2 + (\bar{Y} + \beta y_\nu)^2} \right| \quad (41)$$

in place of (term 1)

If  $|\bar{Y}| \leq \beta y_\nu$  and  $|\bar{X} - T y_\nu| < \bar{\ell}_M/4$  set (term 1) equal to zero.

## Appendix C

### QUADRILATERAL VORTEX INFLUENCE EQUATIONS

The Biot-Savart law can be used to calculate the influence of a finite vortex segment on a point in three-dimensional space. The incremental change in induced velocity at a point in space due to an incremental change in length of a finite vortex is given by the following expression.

$$dq = \frac{K \cos \phi}{4\pi h} d\phi \quad (1)$$

where;

$K$  = Vortex strength

$h$  = Perpendicular distance from the vortex segment to the point in space.

$\phi$  = Angle between the line formed by  $h$  and a line from the field point to a point on the vortex segment.

$q$  = Velocity induced by the finite vortex segment perpendicular to the plane formed by  $h$  and the vortex segment.

A vector expression for  $q$  can be determined from figure (D-1):

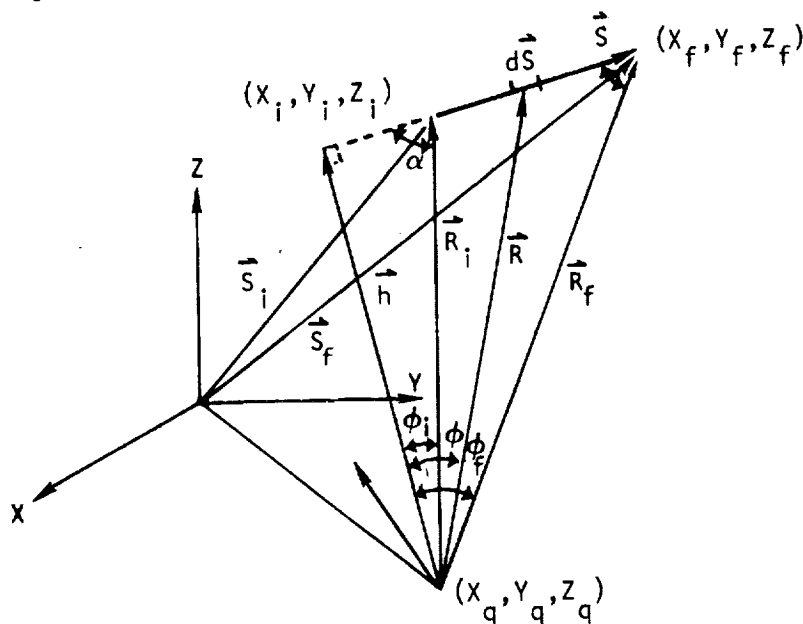


Figure C-1. Velocity induced by finite vortex segment.

The magnitude of the velocity  $\vec{q}$  induced at  $(X_q, Y_q, Z_q)$  by the vortex segment  $\vec{s}$  is given by the following equation after equation (1) has been integrated from  $\phi_i$  to  $\phi_f$ .

$$\left| \vec{q} \right| = \frac{k}{4\pi h} (\cos \beta - \cos \alpha) \quad (2)$$

where

$$\cos \beta = \frac{\vec{s} \cdot \vec{R}_f}{\left| \vec{s} \right| \left| \vec{R}_f \right|}$$

$$\cos \alpha = \frac{\vec{s} \cdot \vec{R}_i}{\left| \vec{s} \right| \left| \vec{R}_i \right|}$$

The vector  $\vec{h}$  is determined such as it satisfies the conditions of being perpendicular to  $\vec{s}$  and equal to the vector sum  $\vec{h} = \vec{R}_f - a\vec{s}$  where "a" defines the length of  $\vec{h}$ .

Since;

$$\vec{h} = \vec{R}_f - a\vec{s} \quad (3)$$

and

$$\vec{h} \cdot \vec{s} = 0 \quad (4)$$

then

$$\vec{h} \cdot \vec{s} = \vec{R}_f \cdot \vec{s} - a\vec{s} \cdot \vec{s} = 0 \quad (5)$$

therefore;

$$a = \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} \quad (6)$$

After substituting "a" into equation (3),  $\vec{h}$  is defined as;

$$\vec{h} = \vec{R}_f - \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} \vec{s} \quad (7)$$

Also, a unit vector  $\hat{q}$  in the direction of  $\vec{q}$  is seen to be equal to;

$$\hat{q} = \frac{\vec{R}_f \times \vec{s}}{|\vec{R}_f \times \vec{s}|} \quad (8)$$

The magnitude and direction of  $\vec{q}$  are then expressed in terms of the coordinates of the control point  $(X_q, Y_q, Z_q)$  and the end points of the vortex segment  $(X_i, Y_i, Z_i)$  and  $(X_f, Y_f, Z_f)$ . If  $\hat{i}$ ,  $\hat{j}$ , and  $\hat{k}$  are defined as unit vectors in the X, Y, and Z directions respectively, then;

$$\vec{s} = \vec{s}_f - \vec{s}_i = (X_f - X_i) \hat{i} + \beta (Y_f - Y_i) \hat{j} + \beta (Z_f - Z_i) \hat{k} \quad (9)$$

$$\vec{R}_i = \vec{s}_i - \vec{Q} = (X_i - X_q) \hat{i} + \beta (Y_i - Y_q) \hat{j} + \beta (Z_i - Z_q) \hat{k}$$

and

$$\vec{R}_f = \vec{s}_f - \vec{Q} = (X_f - X_q) \hat{i} + \beta (Y_f - Y_q) \hat{j} + \beta (Z_f - Z_q) \hat{k} \quad (10)$$

The value of "a" is then expressed as;

$$a = \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} = \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \quad (11)$$

and the components of  $\vec{h}$  by,

$$h_x = (X_f - X_q) - \left[ \frac{(X_f - X_q)(X_f - X_i) + (Y_f - Y_q)(Y_f - Y_i)\beta^2 + (Z_f - Z_q)(Z_f - Z_i)\beta^2}{(X_f - X_i)^2 + (Y_f - Y_i)^2\beta^2 + (Z_f - Z_i)^2\beta^2} \right] (X_f - X_i)$$

$$h_y = (Y_f - Y_q) - \left[ \frac{(X_f - X_q)(X_f - X_i) + (Y_f - Y_q)(Y_f - Y_i)\beta^2 + (Z_f - Z_q)(Z_f - Z_i)\beta^2}{(X_f - X_i)^2 + (Y_f - Y_i)^2\beta^2 + (Z_f - Z_i)^2\beta^2} \right] (Y_f - Y_i) \quad (12)$$

$$h_z = (Z_f - Z_q) - \left[ \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \right] (Z_f - Z_i)$$

The magnitude of  $\vec{q}$  is found by substituting

$$h = |\vec{h}| = \sqrt{h_x^2 + h_y^2 + h_z^2}$$

and the following expressions for  $\cos\alpha$  and  $\cos\beta$  into equation (2).

$$\cos \alpha = \frac{(X_f - X_i)(X_i - X_q) + \beta^2(Y_f - Y_i)(Y_i - Y_q) + \beta^2(Z_f - Z_i)(Z_i - Z_q)}{\sqrt{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \sqrt{(X_i - X_q)^2 + \beta^2(Y_i - Y_q)^2 + \beta^2(Z_i - Z_q)^2}} \quad (13)$$

$$\cos \beta = \frac{(X_f - X_i)(X_f - X_q) + \beta^2(Y_f - Y_i)(Y_f - Y_q) + \beta^2(Z_f - Z_i)(Z_f - Z_q)}{\sqrt{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \sqrt{(X_f - X_q)^2 + \beta^2(Y_f - Y_q)^2 + \beta^2(Z_f - Z_q)^2}}$$

The components of the vector  $\vec{q}$  are then given by the multiplication of the components of equation (8) by  $|\vec{q}|$ .

$$q_X = \frac{|\vec{q}| \left[ (Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i) \right] \beta^2}{|\vec{R} \times \vec{S}|}$$

$$q_Y = \frac{|\vec{q}| \left[ (X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q) \right] \beta^2}{|\vec{R} \times \vec{S}|} \quad (14)$$

$$q_Z = \frac{|\vec{q}| \left[ (X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i) \right] \beta^2}{|\vec{R} \times \vec{S}|}$$



where

$$|\vec{R} \times \vec{S}| = \left\{ \left[ (Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i) \right]^2 \beta^2 + \left[ (X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q) \right]^2 \beta^2 + \left[ (X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i) \right]^2 \beta^2 \right\}^{1/2} \quad (15)$$

The velocity induced at a control point by a vortex segment is then given by equation (14). Since a curved vortex can be represented by a number of straight segments, this equation can be used to compute the induced flow produced by a vortex of arbitrary shape.

The components of velocity induced by a quadrilateral vortex can be written as ratios computed by the product of influence matrices and the vortex strengths.

$$\left[ \frac{u}{V_\infty} \right] = \left[ A_X \right] \left[ \frac{K}{V_\infty} \right] \quad (16)$$

$$\left[ \frac{v}{V_\infty} \right] = \left[ A_Y \right] \left[ \frac{K}{V_\infty} \right] \quad (17)$$

and

$$\left[ \frac{w}{V_\infty} \right] = \left[ A_Z \right] \left[ \frac{K}{V_\infty} \right] \quad (18)$$

where the elements of  $A_X$ ,  $A_Y$ , and  $A_Z$ , are computed from the following equations.

$$A_X = \sum \frac{\beta^2 [\cos \beta - \cos \alpha] \left[ (Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i) \right]}{4\pi |\vec{h}| |\vec{RXS}|} \quad (19)$$

$$A_Y = \sum \frac{\beta [\cos \beta - \cos \alpha] \left[ (X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q) \right]}{4\pi |\vec{h}| |\vec{RXS}|} \quad (20)$$

and

$$A_Z = \sum \frac{\beta [\cos \beta - \cos \alpha] \left[ (X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i) \right]}{4\pi |\vec{h}| |\vec{RXS}|} \quad (21)$$

The  $\Sigma$  sign indicates that the contributions from all of the sides of the quadrilateral vortex are summed.

## Appendix D

### WOODWARD'S DISTRIBUTED PANEL INFLUENCE EQUATIONS

The equations will be derived for supersonic flow first, then for subsonic flow in subappendix A.

#### Preliminaries

Generalized potential function. If

$$\beta^2 \Omega_{xx} = \Omega_{yy} + \Omega_{zz}$$

Then

$$\begin{aligned} \Omega(x, y, z) = & -\frac{1}{2\pi} \frac{\partial}{\partial x} \iint \left( \frac{\partial \Omega}{\partial \nu} + \frac{\partial \Omega'}{\partial \nu'} \right) \sigma \, dS \\ & + \frac{1}{2\pi} \frac{\partial}{\partial x} \iint (\Omega - \Omega') \frac{\partial \sigma}{\partial \nu} \, dS \end{aligned} \quad (1)$$

where

$$\sigma = \cosh^{-1} \frac{x - \xi}{\beta \sqrt{(y - \eta)^2 + (z - \zeta)^2}}$$

and

$$\vec{\nu} = (-\beta^2 n_1, n_2, n_3) = -\vec{\nu}'$$

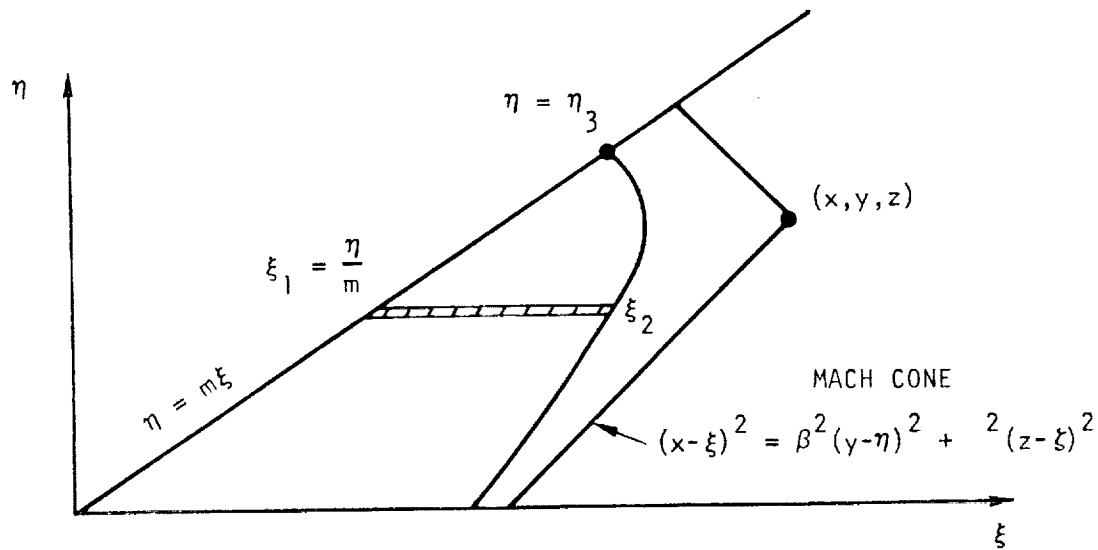
with  $n = (n_1, n_2, n_3)$  the unit normal to surface S

Region of integration. - In the  $(\xi, \eta, \zeta)$  coordinate system the plane of the semi-infinite triangular surface is determined by  $\zeta = a\xi$ . The lines  $\eta = 0$  and  $\eta = m\xi$  are the projections in the  $\xi, \eta$  plane of the triangle edges. The area of integration in equation (1) lies on the semi-infinite triangle and is within the Mach forecone from the point  $(x, y, z)$  given by  $\xi < x$  and  $(x - \xi)^2 > \beta^2(y - \eta)^2 + \beta^2(z - \xi)^2$ . The surface integral is carried out by integrating first over  $\xi$  and then over  $\eta$ .

The  $\xi$  integration goes from the leading edge  $\xi = \xi_1 = \eta/m$  to the intersection of the Mach forecone with the semi-infinite triangle,  $\xi = \xi_2(\eta)$ , where since  $\zeta = a\xi$ ,  $(x - \xi_2)^2 = \beta^2(y - \eta)^2 + \beta^2(z - a\xi_2)^2$ . The  $\eta$  integration goes from  $\eta = 0$  to the intersection of the Mach forecone the leading edge where  $\eta = \eta_3$ . Thus,

$$\left(x - \frac{\eta_3}{m}\right)^2 = \beta^2(y - \eta_3)^2 + \beta^2\left(z - \frac{a\eta_3}{m}\right)^2.$$

Looking down on  $\zeta$  axis



Thus

$$\xi_1 = \frac{\eta}{m} \quad (2)$$

$$(x - \xi_2)^2 = \beta^2(y - \eta)^2 + \beta^2(z - a\xi_2)^2 \quad (3)$$

$$(\eta_3 - mx)^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 (a\eta_3 - mz)^2 \quad (4)$$

These relations may be manipulated into other forms.

$$[(\eta_3 - y) - (mx - y)]^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 [a(\eta_3 - y) + (ay - mz)]^2$$

or

$$\begin{aligned} (\eta_3 - y)^2 [1 - \beta^2 (a^2 + m^2)] - 2[(mx - y) + \beta^2 a(ay - mz)](\eta_3 - y) \\ + (mx - y)^2 - \beta^2 (ay - mz)^2 = 0 \end{aligned}$$

Thus on the forecone

$$\begin{aligned} [1 - \beta^2 (a^2 + m^2)] (\eta_3 - y) &= (mx - y) + \beta^2 a(ay - mz) \\ - \sqrt{[(mx - y) + \beta^2 a(ay - mz)]^2 - [1 - \beta^2 (a^2 + m^2)] [(mx - y)^2 - \beta^2 (ay - mz)^2]} \end{aligned}$$

The following relation may be used to manipulate this further.

$$\begin{aligned} [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 &\equiv [(mx - y) + \beta^2 a(ay - mz)]^2 \\ &\equiv \beta^2 m^2 (z - ax)^2 + (1 - \beta^2 a^2) [(mx - y)^2 - \beta^2 (ay - mz)^2] \end{aligned} \quad (5)$$

Thus

$$\begin{aligned} \left[1 - \beta^2(a^2 + m^2)\right]\eta_3 &= m(x - \beta^2 az) - \beta^2 m^2 y \\ &\quad - \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} \end{aligned} \quad (6)$$

Or starting from (4) again after multiplying by  $(1 - \beta^2 a^2)$

$$\begin{aligned} (1 - \beta^2 a^2)(\eta_3 - mx)^2 &= \beta^2 m^2 (1 - \beta^2 a^2)(\eta_3 - y)^2 \\ &\quad + \beta^2 (1 - \beta^2 a^2) \left[ a(\eta_3 - mx) - m(z - ax) \right]^2 \end{aligned}$$

which becomes

$$\begin{aligned} (1 - \beta^2 a^2)^2 (\eta_3 - mx)^2 + 2\beta^2 am(z - ax)(1 - \beta^2 a^2)(\eta_3 - mx) \\ + \left[ \beta^2 am \right]^2 (z - ax)^2 &= \beta^2 m^2 \left[ (1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2 \right] \end{aligned}$$

or

$$\begin{aligned} \left[ (1 - \beta^2 a^2)(\eta_3 - mx) + \beta^2 am(z - ax) \right]^2 \\ = \beta^2 m^2 \left[ (1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2 \right] \end{aligned}$$

or

$$\left| \eta_3(1 - \beta^2 a^2) - m(x - \beta^2 az) \right|^2 = \beta^2 m^2 \left[ (1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2 \right] \quad (7)$$

### Surface Distribution of Sources

Boundary conditions. - For a distribution of sources on the plane  $\xi = a\xi$  we will assume

$$(a) \quad \phi = \phi' \quad (8)$$

$$(b) \quad \frac{\partial \phi}{\partial \nu} + \frac{\partial \phi'}{\partial \nu'} = \frac{\partial \phi}{\partial \xi} \nu_1 + \frac{\partial \phi}{\partial \xi} \nu_3 + \frac{\partial \phi'}{\partial \xi} \nu'_1 + \frac{\partial \phi'}{\partial \xi} \nu'_3$$

$$= (u - u') \nu_1 + (w - w') \nu_3$$

$$= \frac{1}{\sqrt{1 + a^2}} \left[ \beta^2 a(u - u') + (w - w') \right]$$

$$= \frac{2}{\sqrt{1 + a^2}} \left[ \bar{w} + \beta^2 a \bar{u} \right] = \text{const}$$

In equation (1), if we set  $\Omega = \phi$ , then (a) says the second integral vanishes and (b) means the quantity  $\partial \phi / \partial \nu + \partial \phi' / \partial \nu'$  may be removed from the integral. These assumptions may be checked after the integration is performed. The statement Woodward makes on the bottom of page 17, of reference (55),  $u' = -u$  and  $w' = -w$  is not true and not necessary.

Evaluation of the integral over  $\xi$ . - The integral which results for a surface distribution of sources is,

$$\phi(x, y, z) = -\frac{w + \beta^2 a \bar{u}}{\pi} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2}} d\eta \quad (9)$$

But

$$\begin{aligned} & \int \frac{d\xi}{\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2}} \\ &= \frac{-1}{2\sqrt{1-\beta^2 a^2}} \log \frac{(x-\xi) - \beta^2 a(z-a\xi) + \sqrt{(1-\beta^2 a^2)[(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2]}}{(x-\xi) - \beta^2 a(z-a\xi) - \sqrt{(1-\beta^2 a^2)[(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2]}} \end{aligned}$$

Which may be verified by differentiating

First we let

$$A = (x-\xi) - \beta^2 a(z-a\xi)$$

$$B^2 = \beta^2 [(1-\beta^2 a^2)(y-\eta)^2 + (z-a\xi)^2]$$

$$C^2 = (1-\beta^2 a^2) [(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-a\xi)^2]$$

and we note that

$$A^2 = B^2 + C^2$$



To verify the indefinite integral we must evaluate

$$\begin{aligned}\frac{\partial}{\partial \beta} \log \frac{A+C}{A-C} &= \frac{A' + C'}{A+C} - \frac{A' - C'}{A-C} \\ &= \frac{2(AC' - A'C)}{A^2 - C^2}\end{aligned}$$

where ' denotes  $\frac{\partial}{\partial \beta}$

But since  $B' = 0$

$$AA' = CC'$$

$$\text{or } C' = \frac{AA'}{C}$$

Therefore

$$\begin{aligned}\frac{-1}{2\sqrt{1-\beta^2 a^2}} \frac{\partial}{\partial \beta} \log \frac{A+C}{A-C} &= - \frac{A'(A^2 - C^2)}{C(A^2 - C^2)\sqrt{1-\beta^2 a^2}} \\ &= \frac{\sqrt{1-\beta^2 a^2}}{C} \\ &= \frac{1}{\sqrt{(x-\beta)^2 - \beta^2(y-\gamma)^2 - \beta^2(z-\alpha\beta)^2}}\end{aligned}$$

Q.E.D.

Now since from Eq (3) ,

$$(x - \xi_2)^2 = \beta^2 (y - \eta)^2 + \beta^2 (z - a\xi_2)^2$$

and from Eq (2)

$$\xi_1 = \frac{\eta}{m}$$

$$\begin{aligned} & \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} \\ &= \frac{1}{2\sqrt{1 - \beta^2 a^2}} \log \frac{(mx - \eta) - \beta^2 a(mz - a\eta) + \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 m^2 (\eta - \eta)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a(mz - a\eta) - \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 m^2 (\eta - \eta)^2 - \beta^2 (mz - a\eta)^2]}} \end{aligned}$$

(10)

Therefore from Eq (9)

$$\phi(x, y, z) =$$

$$- \frac{\bar{w} + \beta^2 a \bar{u}}{2\pi \sqrt{1 - \beta^2 a^2}} \int_0^{\eta_3} \log \frac{(mx - \eta) - \beta^2 a(mz - a\eta) + \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 m^2 (\eta - \eta)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a(mz - a\eta) - \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 m^2 (\eta - \eta)^2 - \beta^2 (mz - a\eta)^2]}} d\eta$$

(11)

Evaluation of the Integral over  $\eta$ . - To evaluate equation (11) we must first integrate by parts. If we let,

$$f = (mx - \eta) - \beta^2 a (mz - a\eta) \quad (12)$$

$$g^2 = \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]$$

$$h^2 = (1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]$$

then  $f^2 = g^2 + h^2$

Now let

$$u = \frac{1}{2} \log \frac{f+h}{f-h} \quad dv = d\eta$$

$$\begin{aligned} du &= \frac{fh' - hf'}{f^2 - h^2} d\eta = \frac{fh'h' - f'h^2}{h(f^2 - h^2)} d\eta \\ &= \frac{fh'h' - f'(f^2 - g^2)}{hg^2} d\eta = \frac{f[h'h' - f'f'] + f'g^2}{hg^2} d\eta \\ &= \frac{1}{hg^2} [f'g^2 - fgg'] d\eta \end{aligned} \quad (13)$$

and  $v = \eta$

but  $gg' = \beta^2 m^2 (1 - \beta^2 a^2)(\eta - y)$   
 $f' = -(1 - \beta^2 a^2)$

Now we can write

$$- \eta [g^2 f' - g' g f]$$

$$\begin{aligned}
 &= \eta (1 - \beta^2 a^2) \beta^2 m^2 \left\{ (z - ax)^2 + [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] (\eta - y) \right\} \\
 &= \beta^2 m^2 [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] [1 - \beta^2 a^2] (\eta - y)^2 + (z - ax)^2 \\
 &\quad + \beta^2 m^2 y (1 - \beta^2 a^2) \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] (\eta - y) \right\} \\
 &\quad + \beta^2 m^2 (z - ax)^2 \left\{ \eta (1 - \beta^2 a^2) - [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right\} \\
 &= \beta^2 m^2 [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2] \\
 &\quad + \beta^2 m^2 (1 - \beta^2 a^2) \left\{ y [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] + (z - ax)^2 \right\} (\eta - y) \\
 &\quad + \beta^2 m^2 (z - ax)^2 \left\{ y (1 - \beta^2 a^2) - [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right\}
 \end{aligned}$$

And since from the integration by parts

$$\frac{1}{2} \int \log \frac{f+h}{f-h} d\eta = \frac{1}{2} \eta \log \frac{f+h}{f-h} + \int \frac{-\eta (g^2 f' - g g' f)}{g^2 \sqrt{f^2 - g^2}} d\eta$$

We get for the integration by parts

$$\begin{aligned}
 & \frac{1}{2} \int \log \frac{(mx-\gamma) - \beta^2 a(mz-a\gamma) + \sqrt{(1-\beta^2 a^2)[(mx-\gamma)^2 - \beta^2 m^2(\gamma-y)^2 - \beta^2(mz-a\gamma)^2]}}{(mx-\gamma) - \beta^2 a(mz-a\gamma) - \sqrt{(1-\beta^2 a^2)[(mx-\gamma)^2 - \beta^2 m^2(\gamma-y)^2 - \beta^2(mz-a\gamma)^2]}} d\gamma \\
 &= \frac{\gamma}{2} \log \frac{(mx-\gamma) - \beta^2 a(mz-a\gamma) + \sqrt{(1-\beta^2 a^2)[(mx-\gamma)^2 - \beta^2 m^2(\gamma-y)^2 - \beta^2(mz-a\gamma)^2]}}{(mx-\gamma) - \beta^2 a(mz-a\gamma) - \sqrt{(1-\beta^2 a^2)[(mx-\gamma)^2 - \beta^2 m^2(\gamma-y)^2 - \beta^2(mz-a\gamma)^2]}} \\
 &\quad + [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \int \frac{d\gamma}{\sqrt{[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\gamma - y)^2 + (z - ax)^2]}} \\
 &\quad - \int \frac{(1 - \beta^2 a^2) [y[m(x - \beta^2 az) - y(1 - \beta^2 a^2)] + (z - ax)^2] (\gamma - y) + (z - ax)^2 [y(1 - \beta^2 a^2) - [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]]}{[(1 - \beta^2 a^2)(\gamma - y)^2 + (z - ax)^2] \sqrt{[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\gamma - y)^2 + (z - ax)^2]}} d\gamma
 \end{aligned}
 \tag{14}$$

The first integral in equation (14) can be evaluated using the relation

$$\int \frac{d\gamma}{\sqrt{A\gamma^2 + 2B\gamma + C}} = \frac{-1}{2\sqrt{A}} \log \frac{-(A\gamma+B) + \sqrt{A[A\gamma^2 + 2B\gamma + C]}}{-(A\gamma+B) - \sqrt{A[A\gamma^2 + 2B\gamma + C]}}
 \tag{15}$$

In our case expanding the denominator gives

$$A = (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)]$$

$$B = [-m(x - \beta^2 az) + \beta^2 m^2 y] (1 - \beta^2 a^2)$$

$$C = m^2 (x - \beta^2 az)^2 - \beta^2 m^2 [y^2 (1 - \beta^2 a^2) + (z - ax)^2]$$

and since we can write

$$-(A\eta + B) = [(mx - \eta) - \beta^2 a(mz - a\eta) + \beta^2 m^2 (\eta - y)] (1 - \beta^2 a^2)$$

$$\begin{aligned} A[A\eta^2 + 2B\eta + C] &= (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] \left\{ [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] \right\} \\ &= [1 - \beta^2 (a^2 + m^2)] (1 - \beta^2 a^2)^2 [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2] \end{aligned}$$

Therefore

$$\begin{aligned} &\int \frac{d\eta}{\sqrt{[m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}} \\ &= \frac{-1}{2\sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta^2 a^2)}} \log \frac{-(A\eta + B) + \sqrt{A[A\eta^2 + 2B\eta + C]}}{-(A\eta + B) - \sqrt{A[A\eta^2 + 2B\eta + C]}} \quad (16) \\ &= \frac{-1}{2\sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta^2 a^2)}} \log \frac{(mx - \eta) - \beta^2 a(mz - a\eta) + \beta^2 m^2 (\eta - y) + \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a(mz - a\eta) + \beta^2 m^2 (\eta - y) - \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}} \end{aligned}$$

The last integral in equation (14) may be evaluated using the method described in Appendix A of reference (55).

From Appendix A of reference (55);

If

$$y^4 + (c - ae^2) y^2 - b^2 e^2 = 0 \quad (17)$$

then

$$\begin{aligned}
 & \int \frac{(A + Bv) dv}{(v^2 + e^2) \sqrt{\hat{a}v^2 + 2bv + c}} \\
 &= \frac{Ab\gamma + B\gamma^3}{\gamma^4 + b^2e^2} \tan^{-1} \frac{\gamma \sqrt{\hat{a}v^2 + 2bv + c}}{\gamma^2 - bv} \\
 &+ \frac{A\frac{\gamma^3}{e} + Bbe\gamma}{2(\gamma^4 + b^2e^2)} \log \frac{-(v\gamma^2 + be^2) + \gamma e \sqrt{\hat{a}v^2 + 2bv + c}}{-(v\gamma^2 + be^2) - \gamma e \sqrt{\hat{a}v^2 + 2bv + c}}
 \end{aligned}
 \tag{18}$$

From equation (14) we can write

$$\begin{aligned}
 & [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] \\
 &= (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] (\eta - y)^2 \\
 &- 2[m(x - \beta^2 az) - y(1 - \beta^2 a^2)] (\eta - y) (1 - \beta^2 a^2) \\
 &+ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2
 \end{aligned}$$

Therefore if we factor out a  $(1 - \beta^2 a^2)$  we can write the denominator as

$$\begin{aligned} & [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2] \sqrt{[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2]} \\ & = (1 - \beta^2 a^2)^{3/2} (v^2 + e^2) \sqrt{\hat{a}v^2 + 2bv + c} \end{aligned} \quad (19)$$

where

$$\hat{a} = [1 - \beta^2 (a^2 + m^2)]$$

$$b = - [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]$$

$$c = \left[ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2 \right] (1 - \beta^2 a^2)^{-1}$$

$$e = (z - ax) (1 - \beta^2 a^2)^{-1/2}$$

$$v = \eta - y$$

Referring to equation (17)

$$\begin{aligned} c - \hat{a}e^2 &= \frac{[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2 - [1 - \beta^2 (a^2 + m^2)] (z - ax)^2}{(1 - \beta^2 a^2)} \\ &= \frac{b^2}{(1 - \beta^2 a^2)} - e^2 (1 - \beta^2 a^2) \end{aligned}$$

$$\gamma^4 + (c - \hat{a}e^2) - b^2 e^2 = \left[ \gamma^2 + \frac{B^2}{(1 - \beta^2 a^2)} \right] [\gamma^2 - e^2 (1 - \beta^2 a^2)] = 0$$

Therefore we can choose

$$\gamma^2 = e^2 (1 - \beta^2 a^2)$$

or

$$\gamma = (z - ax)$$

(20)



Using this the numerator of the second integral in (12) becomes

$$(z - ax)^2 \left[ y(1 - \beta^2 a^2) - [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right] + (1 - \beta^2 a^2) \left[ y[m(x - \beta^2 az) - y(1 - \beta^2 a^2)] + (z - ax)^2 \right] (\eta - \gamma) \\ = \gamma^2 \left[ y(1 - \beta^2 a^2) + b \right] + (1 - \beta^2 a^2) \left[ -yb + \gamma^2 \right] \gamma$$

Therefore from (18) the coefficient of  $\tan^{-1}$  is

$$\frac{b\gamma^3 \left[ y(1 - \beta^2 a^2) + b \right] + (1 - \beta^2 a^2) \gamma^3 \left[ -yb + \gamma^2 \right]}{(1 - \beta^2 a^2)^{3/2} (\gamma^4 + b^2 e^2)} = \frac{\gamma(1 - \beta^2 a^2)}{(1 - \beta^2 a^2)^{3/2}} \left[ \gamma^4 + \frac{b^2 \gamma^2}{1 - \beta^2 a^2} \right] \\ = \frac{\gamma}{\sqrt{1 - \beta^2 a^2}} \\ = \frac{z - ax}{\sqrt{1 - \beta^2 a^2}}$$

since  $\gamma = (z - ax)$  and  $\gamma^2 = e^2 (1 - \beta^2 a^2)$

Using (19) and (18) again the coefficient of  $\frac{1}{2} \log$  is

$$\frac{-\gamma^5 \left[ y(1 - \beta^2 a^2) + b \right] + b e^2 \gamma (1 - \beta^2 a^2) \left[ -yb + \gamma^2 \right]}{e (\gamma^4 + b^2 e^2) (1 - \beta^2 a^2)^{3/2}} \\ = \frac{[-\gamma^5 - b^2 e^2 \gamma] \gamma (1 - \beta^2 a^2)}{(z - ax) (\gamma^4 + b^2 e^2) (1 - \beta^2 a^2)} = -\gamma$$

And from (19)

$$(1 - \beta^2 a^2) [\hat{a}v^2 + 2bv + c]$$

$$= [m(x - \beta^2 az) - (v + y)(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [1 - \beta^2 a^2]v^2 + (z - ax)^2$$

$$\gamma^2 - bv = (z - ax)^2 + v[m(x - \beta^2 az) - y(1 - \beta^2 a^2)] - (v\gamma^2 + be^2)$$

$$-(v\gamma^2 + be^2) = \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] - v(1 - \beta^2 a^2) \right\} \frac{(z - ax)^2}{(1 - \beta^2 a^2)}$$

Therefore

$$\tan^{-1} \frac{\gamma \sqrt{\hat{a}v^2 + 2bv + c}}{\gamma^2 - bv} = \tan^{-1} \frac{(z - ax) \sqrt{[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}}{\sqrt{1 - \beta^2 a^2} [(z - ax)^2 + (\eta - y)[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]]}$$

$$\log \frac{-(v\gamma^2 + be^2) + \gamma e \sqrt{\hat{a}v^2 + 2bv + c}}{-(v\gamma^2 + be^2) - \gamma e \sqrt{\hat{a}v^2 + 2bv + c}}$$

$$= \log \frac{m(x - \beta^2 az) - \gamma(1 - \beta^2 a^2) + \sqrt{[m(x - \beta^2 az) - \gamma(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\gamma - y)^2 + (z - ax)^2]}}{m(x - \beta^2 az) - \gamma(1 - \beta^2 a^2) - \sqrt{[m(x - \beta^2 az) - \gamma(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\gamma - y)^2 + (z - ax)^2]}}$$

$$= \log \frac{(mx - \gamma) - \beta^2 a(mz - a\gamma) + \sqrt{(1 - \beta^2 a^2)[(mx - \gamma)^2 - \beta^2 m^2 (\gamma - y)^2 - \beta^2 (mz - a\gamma)^2]}}{(mx - \gamma) - \beta^2 a(mz - a\gamma) - \sqrt{(1 - \beta^2 a^2)[(mx - \gamma)^2 - \beta^2 m^2 (\gamma - y)^2 - \beta^2 (mz - a\gamma)^2]}}$$

because from (12)

$$\begin{aligned} & [m(x - \beta^2 az) - \gamma(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\gamma - y)^2 + (z - ax)^2] \\ &= (1 - \beta^2 a^2) [(mx - \gamma)^2 - \beta^2 m^2 (\gamma - y)^2 - \beta^2 (mz - a\gamma)^2] \end{aligned}$$

Therefore combining terms

$$\begin{aligned}
 & \frac{1}{2} \int \log \frac{(mx-\eta) - \beta^2 a(mz-ay) + \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-ay)^2]}}{(mx-\eta) - \beta^2 a(mz-ay) - \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-ay)^2]}} d\eta \\
 &= \frac{1}{2}(\eta-y) \log \frac{(mx-\eta) - \beta^2 a(mz-ay) + \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-ay)^2]}}{(mx-\eta) - \beta^2 a(mz-ay) - \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-ay)^2]}} \\
 &+ \frac{y(1-\beta^2 a^2) - m(x-\beta^2 az)}{2\sqrt{(1-\beta^2 a^2)[1-\beta^2(a^2+m^2)]}} \log \frac{(mx-\eta) - \beta^2 a(mz-ay) + \beta^2 m^2(\eta-y) + \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-ay)^2]}}{(mx-\eta) - \beta^2 a(mz-ay) + \beta^2 m^2(\eta-y) - \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-ay)^2]}} \\
 &+ \frac{(z-ax)}{\sqrt{1-\beta^2 a^2}} \tan^{-1} \frac{(z-ax)}{\sqrt{1-\beta^2 a^2}} \frac{\sqrt{[m(x-\beta^2 az) - \eta(1-\beta^2 a^2)]^2 - \beta^2 m^2[(1-\beta^2 a^2)(\eta-y)^2 + (z-ax)^2]}}{\sqrt{1-\beta^2 a^2} |(z-ax)^2 + (\eta-y)[m(x-\beta^2 az) - \eta(1-\beta^2 a^2)]|} \\
 & \hspace{15em} (21)
 \end{aligned}$$

Differentiation of the indefinite integral over  $\eta$  . - Equation (21) may be checked by differentiation

To differentiate the first log term we can refer back to the integration by parts of the integral over  $\eta$ . As before

let

$$f = (mx-\eta) - \beta^2 a(mz-ay)$$

$$g^2 = \beta^2 m^2 [(1-\beta^2 a^2)(\eta-y)^2 + (z-ax)^2]$$

$$h^2 = (1-\beta^2 a^2) [(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-ay)^2]$$

then  $f^2 = g^2 + h^2$

and  $\frac{d}{d\eta} \frac{1}{2} \log \frac{f+h}{f-h} = \frac{1}{hg^2} [f'g^2 - fg'g'] = \frac{f'}{h} - \frac{f g g'}{h g^2}$

Then since

$$f' = - (1 - \beta^2 a^2)$$

$$gg' = \beta^2 m^2 (1 - \beta^2 a^2) (\eta - y)$$

we can write

$$\frac{1}{2} \frac{\partial}{\partial \eta} (\eta - y) \log \frac{(mx - \eta) - \beta^2 a (mz - a\eta) + \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a (mz - a\eta) - \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}$$

$$= \frac{1}{2} \log \frac{(mx - \eta) - \beta^2 a (mz - a\eta) + \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a (mz - a\eta) - \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}$$

$$\frac{-(\eta - y)^2 (1 - \beta^2 a^2)}{\sqrt{[m(x - \beta^2 a z) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2}}$$

$$\frac{-(\eta - y)(1 - \beta^2 a^2)}{\sqrt{[m(x - \beta^2 a z) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}} \quad (22)$$

To differentiate the second log term in (21) we will first perform the following manipulation,

$$\begin{aligned}
& \left\{ m(x - \beta^2 a z) - \beta^2 m^2 y - [1 - \beta^2(a^2 + m^2)] \gamma \right\}^2 \\
&= \left\{ m(x - \beta^2 a z) - y(1 - \beta^2 a^2) - [1 - \beta^2(a^2 + m^2)](\gamma - y) \right\}^2 \\
&= \beta^2 m^2 (z - ax)^2 + (1 - \beta^2 a^2) [(mx - y)^2 - \beta^2 (ay - mz)^2] \\
&\quad - 2 [(mx - y) - \beta^2 a (mz - ay)] [1 - \beta^2(a^2 + m^2)] (\gamma - y) \\
&\quad + (1 - \beta^2 a^2) [1 - \beta^2(a^2 + m^2)] (\gamma - y)^2 \\
&\quad - \beta^2 m^2 [1 - \beta^2(a^2 + m^2)] (\gamma - y)^2 \\
&= \beta^2 m^2 [(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2] \\
&\quad + [1 - \beta^2(a^2 + m^2)] \left\{ (mx - y)^2 - 2(\gamma - y)(mx - y) + (\gamma - y)^2 \right. \\
&\quad \left. - \beta^2 [(ay - mz)^2 - 2a(mz - ay)(\gamma - y) + a^2(\gamma - y)^2] \right. \\
&\quad \left. - \beta^2 m^2 (\gamma - y)^2 \right\} \\
&= \beta^2 m^2 [(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2] \\
&\quad + [1 - \beta^2(a^2 + m^2)] [(mx - \gamma)^2 - \beta^2 m^2 (\gamma - y)^2 - \beta^2 (mz - a\gamma)^2]
\end{aligned}$$

Therefore if we let

$$f = m(x - \beta^2 a z) - \beta^2 m^2 y - [1 - \beta^2(a^2 + m^2)] \gamma$$

$$g^2 = \beta^2 m^2 [(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2]$$

$$h^2 = [1 - \beta^2(a^2 + m^2)] [(mx - \gamma)^2 - \beta^2 m^2 (\gamma - y)^2 - \beta^2 (mz - a\gamma)^2]$$

then  $f^2 = g^2 + h^2$

Since in this case  $\frac{d}{d\eta} \frac{1}{2} \log \frac{f+h}{f-h} = \frac{f'}{h}$  [using (13)]

we can write

$$f' = -[1 - \beta^2(a^2 + m^2)]$$

and

$$\begin{aligned} & \frac{1}{2} \frac{d}{d\eta} \log \frac{(mx-\eta) - \beta^2 a(mz-a\eta) + \beta^2 m^2(\eta-y) + \sqrt{[1-\beta^2(a^2+m^2)]}[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}{(mx-\eta) - \beta^2 a(mz-a\eta) + \beta^2 m^2(\eta-y) - \sqrt{[1-\beta^2(a^2+m^2)]}[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]} \\ & = \frac{-[1 - \beta^2(a^2 + m^2)]}{\sqrt{[m(x - \beta^2 az) - \beta^2 m^2 y - \eta[1 - \beta^2(a^2 + m^2)]]^2 - \beta^2 m^2[(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mx)^2]}} \\ & = \frac{-\sqrt{[1 - \beta^2(a^2 + m^2)](1 - \beta^2 a^2)}}{\sqrt{[m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}} \end{aligned} \quad (24)$$

This is because, using (5)

$$\begin{aligned} & (1 - \beta^2 a^2) \left\{ \left[ m(x - \beta^2 az) - \beta^2 m^2 y - \eta[1 - \beta^2(a^2 + m^2)] \right]^2 - \beta^2 m^2[(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mx)^2] \right\} \\ & = (1 - \beta^2 a^2) \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2) - [1 - \beta^2(a^2 + m^2)](\eta - y)]^2 - \beta^2 m^2(1 - \beta^2 a^2)(z - ax)^2 \right. \\ & \quad \left. - \beta^2 m^2[m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 + \beta^4 m^4(z - ax)^2 \right\} \\ & = [1 - \beta^2(a^2 + m^2)] \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 - 2(1 - \beta^2 a^2)[m(x - \beta^2 az) - y(1 - \beta^2 a^2)](\eta - y) \right. \\ & \quad \left. + (1 - \beta^2 a^2)[1 - \beta^2(a^2 + m^2)](\eta - y)^2 - \beta^2 m^2(z - ax)^2 \right\} \\ & = [1 - \beta^2(a^2 + m^2)] \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2) - (1 - \beta^2 a^2)(\eta - y)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] \right\} \\ & = [1 - \beta^2(a^2 + m^2)] \left\{ [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2[(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] \right\} \end{aligned}$$

We can also write,

$$\frac{d}{d\eta} \tan^{-1} \frac{f}{g} = \frac{\frac{f'}{g} - \frac{g'f}{g^2}}{\left(\frac{f}{g}\right)^2 + 1} = \frac{gff' - g'f^2}{f(f^2 + g^2)}$$

$$f = (z - ax) \sqrt{[m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2]}$$

$$ff^1 = -(z - ax)^2 (1 - \beta^2 a^2) \left| [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)] + \beta^2 m^2 (\eta - y) \right|$$

$$g = \sqrt{1 - \beta^2 a^2} \left| (z - ax)^2 + (\eta - y) [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right|$$

$$g^1 = \sqrt{1 - \beta^2 a^2} [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]$$

$$\begin{aligned} -\frac{gff' - g'f^2}{(z - ax)^2 \sqrt{1 - \beta^2 a^2}} &= (1 - \beta^2 a^2) [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) + \beta^2 m^2 (\eta - y)] \left\{ (z - ax)^2 + (\eta - y) [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right\} \\ &\quad + [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \left\{ [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] \right\} \\ &= (z - ax)^2 \left\{ [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)] (1 - \beta^2 a^2) + \beta^2 m^2 (1 - \beta^2 a^2) (\eta - y) - \beta^2 m^2 [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right\} \\ &\quad + [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)] [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \left\{ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) + (\eta - y)(1 - \beta^2 a^2) \right\} \\ &= [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)] \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \right\} \end{aligned}$$

$$\begin{aligned} f^2 + g^2 &= (z - ax)^2 \left\{ [m(x - \beta^2 az) - \eta(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2] \right\} \\ &\quad + (1 - \beta^2 a^2) \left\{ (z - ax)^2 + (\eta - y) [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right\}^2 \\ &= (z - ax)^2 \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 + (\eta - y)^2 (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] - \beta^2 m^2 (z - ax)^2 \right\} \\ &\quad + (1 - \beta^2 a^2) \left\{ (z - ax)^4 + (\eta - y)^2 [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 \right\} \\ &= (z - ax)^2 \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \right\} \\ &\quad + (1 - \beta^2 a^2) (\eta - y)^2 \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \right\} \\ &= \left\{ (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \right\} \left\{ [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \right\} \end{aligned}$$

Therefore

$$\frac{d}{d\eta} \tan^{-1} \frac{(z - ax) \sqrt{\left[ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right]}}{\sqrt{1 - \beta^2 a^2} \left[ (z - ax)^2 + (\eta - y) \left[ m(x - \beta^2 az) - y(1 - \beta^2 a^2) \right] \right]} \\ = \frac{-(z - ax) \sqrt{1 - \beta^2 a^2} \left[ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) \right]}{\left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right] \sqrt{\left[ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right]}} \quad (25)$$

Combining (22), (24), and (25)

$$\frac{d}{d\eta} \left\{ \frac{1}{2} (\eta - y) \log \frac{(mx - \eta) - \beta^2 a (mz - a\eta) + \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a (mz - a\eta) - \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}} \right. \\ + \frac{y(1 - \beta^2 a^2) - m(x - \beta^2 az)}{2 \sqrt{(1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)]}} \log \frac{(mx - \eta) - \beta^2 a (mz - a\eta) + \beta^2 m^2 (\eta - y) + \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a (mz - a\eta) + \beta^2 m^2 (\eta - y) - \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}} \\ \left. + \frac{(z - ax)}{\sqrt{1 - \beta^2 a^2}} \tan^{-1} \frac{(z - ax) \sqrt{\left[ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right]}}{\sqrt{1 - \beta^2 a^2} \left[ (z - ax)^2 + (\eta - y) \left[ m(x - \beta^2 az) - y(1 - \beta^2 a^2) \right] \right]} \right\} \\ = \frac{1}{2} \log \frac{(mx - \eta) - \beta^2 a (mz - a\eta) + \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a (mz - a\eta) - \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}} \\ + \left\{ -(\eta - y)(1 - \beta^2 a^2) \left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right] \right. \\ \left. - (1 - \beta^2 a^2)(\eta - y)^2 \left[ m(x - \beta^2 az) - y(1 - \beta^2 a^2) - (\eta - y)(1 - \beta^2 a^2) \right] + \left[ m(x - \beta^2 az) - y(1 - \beta^2 a^2) \right] \left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right] \right. \\ \left. - (z - ax)^2 \left[ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) \right] \right\} x \left[ \left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right] \sqrt{\left[ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2)(\eta - y)^2 + (z - ax)^2 \right]} \right]^{-1} \\ = \frac{1}{2} \log \frac{(mx - \eta) - \beta^2 a (mz - a\eta) + \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a (mz - a\eta) - \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}$$



Evaluation of the definite integral. - From (6) and (7)

$$m(x - \beta^2 az) - \eta_3(1 - \beta^2 a^2) = \theta m \sqrt{(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2} \quad (6)$$

$$m(x - \beta^2 az) - \beta^2 m^2 y - [1 - \beta^2(a^2 + m^2)] \eta_3 = \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} \quad (7)$$

Therefore using (6), (7) and (23) both of the  $\log$  terms in equation (21) are zero when  $\eta = \eta_3$ . However the  $\tan^{-1}$  term can be either zero or  $\pm\pi$  when  $\eta = \eta_3$ , depending upon whether the denominator is greater or less than zero. If it is greater than zero the limit is zero as  $\eta \rightarrow \eta_3$ . In (21) when  $\eta = \eta_3$  using (6) and then (5) and (7),

$$\begin{aligned} & (z - ax)^2 + (\eta_3 - y) [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \\ & \cdot \frac{[1 - \beta^2(a^2 + m^2)](z - ax)^2 + [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \cdot \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]}{1 - \beta^2(a^2 + m^2)} \\ & = \frac{\sqrt{(mx - y)^2 + (z - ax)^2} \cdot \beta^2(ay - mz)^2 \left\{ (1 - \beta^2 a^2) \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} - \beta m [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right\}}{[1 - \beta^2(a^2 + m^2)]} \end{aligned} \quad (26)$$

Now if we examine the numerator of (26) and use (5)

$$\begin{aligned} & (1 - \beta^2 a^2)^2 [(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2] - \beta^2 m^2 [m(x - \beta^2 az) - y(1 - \beta^2 a^2)]^2 \\ & = (1 - \beta^2 a^2) [1 - \beta^2(a^2 + m^2)] [(mx - y)^2 - \beta^2(ay - mz)^2] + [(1 - \beta^2 a^2)^2 - \beta^4 m^4] (z - ax)^2 \\ & = [1 - \beta^2(a^2 + m^2)] \left\{ (1 - \beta^2 a^2) [(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2] + \beta^2 m^2 (z - ax)^2 \right\} \end{aligned} \quad (27)$$

because

$$(1 - \beta^2 a^2)^2 - \beta^4 m^4 = (1 - \beta^2 a^2 - \beta^2 m^2)(1 - \beta^2 a^2 + \beta^2 m^2)$$

But if  $1 - \beta^2(a^2 + m^2) > 0$  then from (27)

$$(1 - \beta^2 a^2) \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} > \beta m |m(x - \beta^2 az) - y(1 - \beta^2 a^2)|$$

and (26) shows that the denominator of the  $\tan^{-1}$  term is  $> 0$ . But if  $1 - \beta^2(a^2 + m^2) < 0$  then (27) shows

$$(1 - \beta^2 a^2) \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} < \beta m |m(x - \beta^2 az) - y(1 - \beta^2 a^2)|$$

and the denominator of the  $\tan^{-1}$  term is  $> 0$  if and only if

$$m(x - \beta^2 az) - y(1 - \beta^2 a^2) > 0$$

But

$$m(x - \beta^2 az) - y(1 - \beta^2 a^2) = m[x - \beta \sqrt{y^2 + z^2}] + \beta m \sqrt{y^2 + z^2} - \beta^2 amz - y(1 - \beta^2 a^2)$$

But we can also write

$$\beta^2 m^2 (y^2 + z^2) - [\beta^2 amz + y(1 - \beta^2 a^2)]^2$$

$$= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2 m^2 z^2 - \beta^2 m^2 \beta^2 a^2 z^2 - 2\beta^2(1 - \beta^2 a^2)amyz + \beta^2 a^2(1 - \beta^2 a^2)y^2$$

$$= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2(1 - \beta^2 a^2)(m^2 z^2 - 2amyz + a^2 y^2)$$

$$= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2(1 - \beta^2 a^2)(mz - ay)^2 > 0 \text{ if } 1 - \beta^2(a^2 + m^2) < 0$$

And therefore

$$\beta m \sqrt{y^2 + z^2} - \beta^2 amz - y(1 - \beta^2 a^2) > 0$$

and

$$m(x - \beta^2 az) - y(1 - \beta^2 a^2) > 0$$

if

$$1 - \beta^2(a^2 + m^2) < 0$$

and

$$x > \beta(y^2 + z^2)$$

Therefore inside the Mach cone from the origin  $x^2 > \beta^2(y^2 + z^2)$  we have

$$(z - ax)^2 + (\eta_3 - y) [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] > 0 \quad (28)$$

and therefore because of (6) and (28)

$$\tan^{-1} \frac{(z - ax) \sqrt{[m(x - \beta^2 az) - \eta_3(1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2]}}{\sqrt{1 - \beta^2 a^2} \left| (z - ax)^2 + (\eta_3 - y) [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right|} = 0$$

If we change the sign of the denominator we would get  $\pi$  for  $z > ax$  and  $-\pi$  for  $z < ax$ . However when we evaluate the term when  $\eta = 0$  and subtract, the result is the same.

Therefore if we evaluate (21) for  $\eta = 0$  we get for (11)

$$\begin{aligned} \phi(x, y, z) = & - \frac{\bar{w} + \beta^2 a \bar{u}}{2\pi \sqrt{1 - \beta^2 a^2}} \int_0^{\eta_3} \log \frac{(mx - \eta) - \beta^2 a(mz - a\eta) + \sqrt{(1 - \beta^2 a^2)} [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}{(mx - \eta) - \beta^2 a(mz - a\eta) - \sqrt{(1 - \beta^2 a^2)} [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]} d\eta \\ & = \frac{\bar{w} + \beta^2 a \bar{u}}{\pi} \left\{ \frac{(z - ax)}{1 - \beta^2 a^2} \tan^{-1} \frac{m(z - ax) \sqrt{x^2 - \beta^2 (y^2 + z^2)}}{y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2} \right. \\ & + \frac{y(1 - \beta^2 a^2) - m(x - \beta^2 az)}{2(1 - \beta^2 a^2) \sqrt{1 - \beta^2 (a^2 + m^2)}} \log \frac{x - \beta^2 (az + my) + \sqrt{[x^2 - \beta^2 (y^2 + z^2)] [1 - \beta^2 (a^2 + m^2)]}}{x - \beta^2 (az + my) - \sqrt{[x^2 - \beta^2 (y^2 + z^2)] [1 - \beta^2 (a^2 + m^2)]}} \\ & \left. - \frac{y}{2 \sqrt{1 - \beta^2 a^2}} \log \frac{x - \beta^2 az + \sqrt{[x^2 - \beta^2 (y^2 + z^2)] (1 - \beta^2 a^2)}}{x - \beta^2 az - \sqrt{[x^2 - \beta^2 (y^2 + z^2)] (1 - \beta^2 a^2)}} \right\} \quad (29) \end{aligned}$$

Evaluation of the velocity components. - The velocity components may now be obtained by differentiation of the velocity potential. However, whoever attempts to take partial derivatives of (29) is in for a long days work. But the partial derivatives of (29) may be obtained quite simply if it is noted that all terms in the expressions for the velocity components must have coefficients involving either  $\log$  or  $\tan^{-1}$ . This is easily shown by differentiating the expression for  $\phi$  given by (11).

$$\begin{aligned}
u = \frac{\partial \phi}{\partial x} &= -\frac{\bar{w} + \beta^2 a \bar{u}}{2\pi\sqrt{1-\beta^2 a^2}} \frac{\partial}{\partial x} \int_0^{\eta_3} \log \frac{(mx-\eta) - \beta^2 a(mz-a\eta) + \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}}{(mx-\eta) - \beta^2 a(mz-a\eta) - \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}} d\eta \\
&= -\frac{\bar{w} + \beta^2 a \bar{u}}{2\pi\sqrt{1-\beta^2 a^2}} \left\{ \frac{\partial \eta_3}{\partial x} \log \frac{(mx-\eta_3) - \beta^2 a(mz-a\eta_3) + \sqrt{(1-\beta^2 a^2)[(mx-\eta_3)^2 - \beta^2 m^2(\eta_3-y)^2 - \beta^2(mz-a\eta_3)^2]}}{(mx-\eta_3) - \beta^2 a(mz-a\eta_3) - \sqrt{(1-\beta^2 a^2)[(mx-\eta_3)^2 - \beta^2 m^2(\eta_3-y)^2 - \beta^2(mz-a\eta_3)^2]}} \right. \\
&\quad \left. + \int_0^{\eta_3} \frac{\partial}{\partial x} \log \frac{(mx-\eta) - \beta^2 a(mz-a\eta) + \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}}{(mx-\eta) - \beta^2 a(mz-a\eta) - \sqrt{(1-\beta^2 a^2)[(mx-\eta)^2 - \beta^2 m^2(\eta-y)^2 - \beta^2(mz-a\eta)^2]}} d\eta \right\} \\
&= -\frac{\bar{w} + \beta^2 a \bar{u}}{\pi\sqrt{1-\beta^2 a^2}} \int_0^{\eta_3} \frac{m - \frac{(\eta-y)[m(x-\beta^2 az) - \eta(1-\beta^2 a^2)](1-\beta^2 a^2)}{(1-\beta^2 a^2)(\eta-y)^2 + (z-ax)^2}}{\sqrt{[m(x-\beta^2 az) - \eta(1-\beta^2 a^2)]^2 - \beta^2 m^2[(1-\beta^2 a^2)(\eta-y)^2 + (z-ax)^2]}} d\eta
\end{aligned}$$

because from (6) and (23a) the log term is zero when  $\eta = \eta_3$

However this integral expression for  $u$  involves integrals of the same form as in (13), (16), or (18). A closer examination shows that the resulting  $\log$  or  $\tan^{-1}$  terms will have arguments which are identical with those for  $\phi$ , although the coefficients may be different. Therefore all expressions must involve either  $\log$  or  $\tan^{-1}$  terms. The same arguments can be made for the  $v$  and  $w$  velocity components.

With this in mind the partial derivatives of  $\phi$  can be obtained from (29) by differentiating only the coefficients of the  $\log$  and  $\tan^{-1}$  terms. Differentiation of the  $\tan^{-1}$  or  $\log$  terms will not yield other  $\tan^{-1}$  or  $\log$  terms and therefore the sum of these terms must give zero. Therefore

$$\begin{aligned}
u = \frac{\partial \phi}{\partial x} &= -\frac{\bar{w} + \beta^2 a \bar{u}}{\pi(1-\beta^2 a^2)} \left\{ a \tan^{-1} \frac{m(z-ax) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{y[(y-mx) - \beta^2 a(ay-mz)] + (z-ax)^2} \right. \\
&\quad \left. + \frac{m}{2\sqrt{1-\beta^2(a^2 + m^2)}} \log \frac{x - \beta^2(a^2 + m^2) + \sqrt{[1-\beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(a^2 + m^2) - \sqrt{[1-\beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}} \right\}
\end{aligned}$$

# Constant Pressure Surface

Boundary Conditions. - In (1) we can set  $\Omega = u$

Assume that on S

$$(a) \quad u - u' = \text{const}$$

$$(b) \quad \left[ \frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} \right] = 0 \quad (30)$$

Now since

$$\frac{\partial u}{\partial \nu} = \frac{\beta^2 a}{\sqrt{1+a^2}} \frac{\partial u}{\partial \xi} + \frac{1}{\sqrt{1+a^2}} \frac{\partial u}{\partial \xi}$$

$$\frac{\partial u'}{\partial \nu'} = \frac{-\beta^2 a}{\sqrt{1+a^2}} \frac{\partial u}{\partial \xi} + \frac{-1}{1+a^2} \frac{\partial u}{\partial \xi}$$

and from (b)

$$\frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} = \frac{\beta^2 a}{\sqrt{1+a^2}} \frac{\partial}{\partial \xi} (u - u') + \frac{1}{\sqrt{1+a^2}} \frac{\partial}{\partial \xi} (u - u') = 0$$

Now from (a),  $u - u' = \text{const}$  on S implies that on S

$$\frac{\partial}{\partial \eta} (u - u') = 0$$

and

$$\frac{\partial}{\partial \xi} (u - u') + a \frac{\partial}{\partial \xi} (u - u') = 0$$

This means that since

$$\beta^2 a \frac{\partial}{\partial \xi} (u - u') + \frac{\partial}{\partial \xi} (u - u') = 0$$

and

$$\frac{\partial}{\partial \xi} (u - u') + a \frac{\partial}{\partial \xi} (u - u') = 0$$

then if

$$1 - \beta^2 a^2 \neq 0$$

$$\frac{\partial}{\partial \xi} (u - u') = \frac{\partial}{\partial \eta} (u - u') = \frac{\partial}{\partial \xi} (u - u') = 0 \text{ on } S$$

Note that assumptions (a) and (b) in (30) are independent assumptions.

(a) Says that  $(u - u')$  is constant on  $S$

(b) Effectively says that the derivative of  $u - u'$  in a direction normal to  $S$  is zero.

Woodward assumes, in reference (55), that (a) implies (b), which is not true.

If we examine the normal velocity on  $S$

$$u_n = \frac{-au + w}{\sqrt{1 + a^2}} \quad u'_n = \frac{au' - w}{\sqrt{1 + a^2}}$$

then since due to irrotationality  $\frac{\partial w}{\partial \xi} = \frac{\partial u}{\partial \xi}$

$$\sqrt{1 + a^2} \frac{\partial}{\partial \xi} u_n = -a \frac{\partial u}{\partial \xi} + \frac{\partial w}{\partial \xi} = -a \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \xi}$$

and

$$\sqrt{1 + a^2} \frac{\partial}{\partial \xi} (u_n + u'_n) = -a \frac{\partial}{\partial \xi} (u - u') + \frac{\partial}{\partial \xi} (u - u') = 0$$

and therefore the source strength, if any, is constant.

Evaluation of the integral over  $\xi$  - The integral to be evaluated is,

$$\begin{aligned} \phi(x, y, z) &= \frac{\Delta P}{4\pi q_\infty} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{\beta^2 a - \frac{(x - \xi)(z - a\xi)}{(y - \eta)^2 + (z - a\xi)^2}}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} d\xi d\eta \\ &= \frac{\Delta P}{4\pi a q_\infty} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{\beta^2 a^2 [(z - a\xi)^2 + (y - \eta)^2] - (z - a\xi)^2 + (z - a\xi)(z - ax)}{[(z - a\xi)^2 + (y - \eta)^2] \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} d\xi d\eta \\ &= \frac{\Delta P}{4\pi a q_\infty} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{(z - a\xi)(z - ax) + (y - \eta)^2}{[(z - a\xi)^2 + (y - \eta)^2] \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} d\xi d\eta \\ &= \frac{(1 - \beta^2 a^2) \Delta P}{4\pi a q_\infty} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} d\eta \end{aligned}$$



Except for the coefficient, the second integral is identical to the integral obtained for a surface distribution of sources. It has already been evaluated [see (9)]. The first integral over  $\xi$  is easy to evaluate,

$$\int \frac{[(z - ax)(z - a\xi) + (y - \eta)^2] d\xi}{[(y - \eta)^2 + (z - a\xi)^2] \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}}$$

$$= -\frac{1}{2} \log \frac{(x - z) + \sqrt{(x - z)^2 - \beta^2 [(z - az)^2 + (y - \eta)^2]}}{(x - z) - \sqrt{(x - z)^2 - \beta^2 [(z - az)^2 + (y - \eta)^2]}}$$

This can be verified by differentiating. Let,

$$f = x - z$$

$$g^2 = \beta^2 [(z - az)^2 + (y - \eta)^2]$$

$$h^2 = (x - z)^2 - \beta^2 [(z - az)^2 + (y - \eta)^2]$$

$$\text{and } \frac{\partial}{\partial z} \frac{1}{2} \log \frac{f+h}{f-h} = \frac{1}{hg^2} [f'g^2 - fgg'] \quad \text{from (13)}$$

$$f' = -1$$

$$gg' = -\beta^2 a (z - az)$$

Therefore

$$-\frac{1}{2} \frac{\partial}{\partial z} \log \frac{(x - z) + \sqrt{(x - z)^2 - \beta^2 [(z - az)^2 + (y - \eta)^2]}}{(x - z) - \sqrt{(x - z)^2 - \beta^2 [(z - az)^2 + (y - \eta)^2]}}$$

$$= \frac{(z - az)^2 + (y - \eta)^2 - a(x - z)(z - az)}{[(z - az)^2 + (y - \eta)^2] \sqrt{(x - z)^2 - \beta^2 (z - az)^2 - \beta^2 (y - \eta)^2}}$$

$$= \frac{(z - ax)(z - a\xi) + (y - \eta)^2}{[(z - a\xi)^2 + (y - \eta)^2] \sqrt{(x - \xi)^2 - \beta^2 (z - a\xi)^2 - \beta^2 (y - \eta)^2}}$$

Since from (3)

$$(x - \xi_2)^2 = \beta^2 [(z - a \xi_2)^2 + (y - \eta)^2]$$

$$\log \frac{(x - \xi_2) + \sqrt{(x - \xi_2)^2 - \beta^2 [(z - a \xi_2)^2 + (y - \eta)^2]}}{(x - \xi_2) - \sqrt{(x - \xi_2)^2 - \beta^2 [(z - a \xi_2)^2 + (y - \eta)^2]}} = 0$$

and from (2)

$$\xi_1 = \frac{\eta}{m}$$

$$\int_{\xi_1}^{\xi_2} \frac{(z - ax)(z - a\xi) + (y - \eta)^2}{[(y - \eta)^2 + (z - a\xi)^2] \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} d\xi$$

$$= \frac{1}{2} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}}$$

therefore

(32)

$$\begin{aligned} \phi(x, y, z) &= \frac{1}{8\pi a} \frac{\Delta P}{\rho_\infty} \int_0^{\eta_3} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2}} d\eta \\ &- \frac{1}{8\pi a} \frac{\Delta P}{\rho_\infty} \sqrt{1 - \beta^2 a^2} \int_0^{\eta_3} \log \frac{(mx - \eta) - \beta^2 a (mz - a\eta) + \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}}{(mx - \eta) - \beta^2 a (mz - a\eta) - \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2]}} d\eta \end{aligned}$$

The second integral was evaluated previously for a surface distribution of sources.

Evaluation of the indefinite integral over  $\eta$ . - The first integral in (32) must first be integrated by parts.

$$\text{let } f = mx - \eta$$

$$g^2 = \beta^2 m^2 (y - \eta)^2 + \beta^2 (mz - a\eta)^2$$

$$h^2 = (mx - \eta)^2 - \beta^2 m^2 (y - \eta)^2 - \beta^2 (mz - a\eta)^2$$

then let

$$u = \frac{1}{2} \log \frac{f+h}{f-h}$$

$$dv = d\eta$$

$$du = \frac{1}{hg^2} [f'g^2 - gg'f] d\eta$$

$$v = \eta$$

$$f' = -1$$

$$gg' = \beta^2 [a(a\eta - mz) + m^2(\eta - y)]$$

$$-[f'g^2 - gg'f] = \beta^2 [(a\eta - mz)^2 + m^2(\eta - y)^2 + (mx - \eta) [a(a\eta - mz) + m^2(\eta - y)]] \quad (33)$$

$$= \beta^2 [(a\eta - mz) [(a\eta - mz) + a(mx - \eta)] + m^2(\eta - y) [(\eta - y) + (mx - \eta)]]$$

$$= \beta^2 [-m(a\eta - mz)(z - ax) + m^2(\eta - y)(mx - y)]$$

$$\begin{aligned}
- \eta \left[ f'g^2 - gg'f \right] &= \beta^2 \left\{ m \left[ m(mx - y) - a(z - ax) \right] \eta^2 + m^2 \left[ z(z - ax) - y(mx - y) \right] \eta \right\} \\
&= \beta^2 \left\{ \frac{m \left[ m(mx - y) - a(z - ax) \right]}{(a^2 + m^2)} \left[ (a\eta - mz)^2 + m^2(\eta - y)^2 \right] \right. \\
&\quad \left. + \frac{2m^2 \left[ m(mx - y) - a(z - ax) \right] (az + my)\eta + m^2(a^2 + m^2) \left[ z(z - ax) - y(mx - y) \right] \eta}{(a^2 + m^2)} \right. \\
&\quad \left. - \frac{m \left[ m(mx - y) - a(z - ax) \right] m^2(y^2 + z^2)}{(a^2 + m^2)} \right\}
\end{aligned}$$

But

$$\begin{aligned}
&\left[ m(mx - y) - a(z - ax) \right] (az + my) + (a^2 + m^2) \left[ z(z - ax) - y(mx - y) \right] \\
&= \left[ m(az + my) - y(z^2 + m^2) \right] (mx - y) + \left[ z(z^2 + m^2) - a(az + my) \right] (z - ax) \\
&= -a(ay - mz)(mx - y) - m(ay - mz)(z - ax) \\
&= -(ay - mz)^2
\end{aligned}$$

Therefore

$$\begin{aligned}
 & \frac{1}{2} \int \log \frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2}} d\eta \\
 &= \frac{1}{2} \eta \log \frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2}} \\
 &\quad - \frac{m \left[ (az+my) - (a^2+m^2)x \right]}{(a^2+m^2)} \int \frac{d\eta}{\sqrt{(mx-\eta)^2 - \beta^2 (a\eta-mz)^2 - \beta^2 m^2 (\eta-y)^2}} \\
 &\quad + \frac{m^2}{a^2+m^2} \int \frac{\left[ (ay-mz)^2 + (my+az) \left[ m(mx-y) - a(z-ax) \right] \right] \eta \cdot m (y^2+z^2) \left[ m(mx-y) - a(z-ax) \right]}{\left[ (a\eta-mz)^2 + m^2 (\eta-y)^2 \right] \sqrt{(mx-\eta)^2 - \beta^2 \left[ (a\eta-mz)^2 + m^2 (\eta-y)^2 \right]}} d\eta
 \end{aligned}
 \tag{34}$$

The first integral in (34) may be evaluated using (16) because,

$$\begin{aligned}
 & \left[ m(x - \beta^2 az) - \eta(1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \right] \\
 &= (1 - \beta^2 a^2) \left| (mx - \eta)^2 - \beta^2 (a\eta - mz)^2 - \beta^2 m^2 (\eta - y)^2 \right|
 \end{aligned}
 \tag{35}$$

Therefore using (16) and (35)

$$\begin{aligned}
 & \int \frac{d\eta}{\sqrt{(mx-\eta)^2 - \beta^2 (a\eta-mz)^2 - \beta^2 m^2 (\eta-y)^2}} = \\
 & \frac{-1}{2\sqrt{1-\beta^2(a^2+m^2)}} \log \frac{(mx-\eta) - \beta^2 a(mz-ay) + \beta^2 m^2 (\eta-y) + \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2]}}{(mx-\eta) - \beta^2 a(mz-ay) + \beta^2 m^2 (\eta-y) - \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2]}}
 \end{aligned}
 \tag{36}$$

The last term in equation (34) must be changed to the form of equation (18) in order to be evaluated. We can write

$$\begin{aligned}
 (a\eta - mz)^2 + m^2 (\eta - y)^2 &= (a^2 + m^2) \eta^2 - 2m (az + my) \eta + m^2 (y^2 + z^2) \\
 &= (a^2 + m^2) \left[ \eta - \frac{m (az + my)}{a^2 + m^2} \right]^2 + \frac{m^2 [(a^2 + m^2)(y^2 + z^2) - (az + my)^2]}{a^2 + m^2} \\
 &= (a^2 + m^2) \left[ \eta - \frac{m (az + my)}{a^2 + m^2} \right]^2 + \frac{m^2 (ay - mz)^2}{(a^2 + m^2)}
 \end{aligned}$$

Now we introduce a change of variables. Let

$$u = \eta - \frac{m (az + my)}{a^2 + m^2}$$

Therefore

$$(a\eta - mz)^2 + m^2 (\eta - y)^2 = (a^2 + m^2) \left\{ u^2 + \frac{m^2 (ay - mz)^2}{(a^2 + m^2)^2} \right\} \quad (37)$$

$$\eta - mx = u + \frac{m (az + my) - mx (a^2 + m^2)}{(a^2 + m^2)} \quad (38)$$

$$\begin{aligned}
& (\eta - mx)^2 - \beta^2 (a\eta - mz)^2 - \beta^2 m^2 (\eta - y)^2 \\
&= \left[ 1 - \beta^2 (a^2 + m^2) \right] u^2 + 2 \frac{m (az + my) - mx (a^2 + m^2)}{(a^2 + m^2)} u \\
&\quad - \frac{\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[ (az + my) - x (a^2 + m^2) \right]^2}{(a^2 + m^2)^2}
\end{aligned} \tag{39}$$

Now (18) can be used. Let

$$\begin{aligned}
\hat{a} &= 1 - \beta^2 (a^2 + m^2) \\
b &= \frac{m \left[ (az + my) - x (a^2 + m^2) \right]}{(a^2 + m^2)} = \frac{m \left[ a (z - ax) - m (mx - y) \right]}{(a^2 + m^2)} \\
c &= \frac{-\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[ (az + my) - x (a^2 + m^2) \right]^2}{(a^2 + m^2)^2} \\
e &= \frac{m (ay - mz)}{(a^2 + m^2)}
\end{aligned} \tag{40}$$

and referring to (17)

$$c - \hat{a}e^2 = \frac{-m^2 (ay - mz)^2 + m^2 \left[ m (mx - y) - a (z - ax) \right]^2}{(a^2 + m^2)^2}$$

$$b^2 e^2 = \frac{m^2 \left[ m (mx - y) - a (z - ax) \right]^2 m^2 (ay - mz)^2}{(a^2 + m^2)^4}$$

$$\gamma^4 + (c - \hat{a}e^2) \gamma^2 - b^2 e^2$$

$$= \left\{ \gamma^2 - \frac{m^2 (ay - mz)^2}{(a^2 + m^2)^2} \right\} \left\{ \gamma^2 + \frac{m^2 \left[ m (mx - y) - a (z - ax) \right]^2}{(a^2 + m^2)^2} \right\} = 0$$

Therefore we can choose

$$\gamma = \frac{m (ay - mz)}{(a^2 + m^2)} = e \quad (41)$$



The numerator of the second integral in equation (34) can be written,

$$\begin{aligned}
 & \left\{ (a^2 + m^2) \gamma^2 - m (az + my) b \right\} \left\{ u + \frac{m (az + my)}{(a^2 + m^2)} \right\} + m^2 (y^2 + z^2) b \\
 &= \left\{ (a^2 + m^2) \gamma^2 - m (az + my) b \right\} u + m (az + my) \gamma^2 + \frac{m^2 b (ay - mz)^2}{(a^2 + m^2)} \\
 &= \left\{ (a^2 + m^2) \gamma^2 - m (az + my) b \right\} u + \gamma^2 [m (az + my) + b (a^2 + m^2)]
 \end{aligned}$$

Now referring to (18) we can write

$$\begin{aligned}
 A &= \frac{\gamma^2 [m (az + my) + b (a^2 + m^2)]}{a^2 + m^2} \\
 B &= \frac{(a^2 + m^2) \gamma^2 - m (az + my) b}{a^2 + m^2}
 \end{aligned}$$

The  $(a^2 + m^2)$  in the denominator of A and B comes from (37) which is in the denominator of (34). From (18) and (41)

$$\begin{aligned}
 \frac{Ab\gamma + B\gamma^3}{\gamma^4 + b^2 e^2} &= \frac{b\gamma [m (az + my) + b (a^2 + m^2)] + [(a^2 + m^2) \gamma^2 - m (az + my) b] \gamma}{(a^2 + m^2) (\gamma^2 + b^2)} \\
 &= \frac{\gamma (a^2 + m^2)}{(a^2 + m^2)} = \frac{m (ay - mz)}{(a^2 + m^2)}
 \end{aligned}$$

and

$$\frac{-A \frac{\gamma^3}{e} + Bbe\gamma}{\gamma^4 + b^2 e^2} = \frac{-\gamma^2 \left[ m (az + my) + b (a^2 + m^2) \right] + \left[ (a^2 + m^2) \gamma^2 - bm (az + my) \right] b}{(a^2 + m^2)(\gamma^2 + b^2)}$$

$$= \frac{-m (az + my)}{a^2 + m^2}$$

From (39) and (40)

$$\hat{a}u^2 + 2bu + c = (mx - \eta)^2 - \beta^2 (a\eta - mz)^2 - \beta^2 m^2 (\eta - y)^2$$

$$\gamma^2 - bu = \gamma^2 - b \left[ \eta - \frac{m (az + my)}{a^2 + m^2} \right] = \frac{m \left[ m (mx - y) - a (z - ax) \right] \eta}{(a^2 + m^2)} + \left[ \gamma^2 + b \frac{m (az + my)}{a^2 + m^2} \right]$$

$$= \frac{m \left[ m (mx - y) - a (z - ax) \right] \eta}{a^2 + m^2} + \frac{m^2 (ay^2 - 2amy + m^2 z^2) + m^2 \left[ a (z - ax) - m (mx - y) \right] (az + my)}{(a^2 + m^2)^2}$$

$$= \frac{m}{a^2 + m^2} \left\{ \left[ m (mx - y) - a (z - ax) \right] \eta + m \left[ z (z - ax) - y (mx - y) \right] \right\}$$

$$u + b = \eta - mx$$

from (38) and (40).

Therefore using (18), and (41) we get from (34)

$$\frac{m^2}{a^2 + m^2} \int \frac{\left[ (ay - mz)^2 + (my + az) \left[ m(mx - y) - a(z - ax) \right] \right] \eta - m \left[ m(mx - y) - a(z - ax) \right] (y^2 + z^2)}{\left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right] \sqrt{(mx - \eta)^2 - \beta^2 \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right]}} d\eta$$

$$= \frac{m(ay - mz)}{a^2 + m^2} \tan^{-1} \frac{(ay - mz) \sqrt{(mx - \eta)^2 - \beta^2 \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right]}}{\left[ m(mx - y) - a(z - ax) \right] \eta + m \left[ z(z - ax) - y(mx - y) \right]}$$

$$- \frac{m(az + my)}{2(a^2 + m^2)} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2}} \quad (42)$$

Therefore substituting (36), and (42) in (34)

$$\frac{1}{2} \int \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2}} d\eta =$$

$$\frac{1}{2} \left\{ \eta - \frac{m(az + my)}{a^2 + m^2} \right\} \log \frac{mx - \eta + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2}}{mx - \eta - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2}}$$

$$+ \frac{m[(az + my) - (a^2 + m^2)x]}{2(a^2 + m^2)} \log \frac{(mx - \eta) - \beta^2 a(mz - ay) + \beta^2 m^2 (\eta - y) + \sqrt{[1 - \beta^2(a^2 + m^2)] \left[ (mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2 \right]}}{(mx - \eta) - \beta^2 a(mz - ay) + \beta^2 m^2 (\eta - y) - \sqrt{[1 - \beta^2(a^2 + m^2)] \left[ (mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - ay)^2 \right]}}$$

$$+ \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay) \sqrt{(mx - \eta)^2 - \beta^2 \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right]}}{\left[ m(mx - y) - a(z - ax) \right] \eta + m \left[ z(z - ax) - y(mx - y) \right]} \quad (43)$$

Differentiation of the indefinite integral over  $\eta$ . - From (33)

$$\frac{1}{2} \frac{d}{d\eta} \log \frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2}} =$$

$$\frac{m^2 [y (mx - y) - z (z - ax)] + m [a (z - ax) - m (mx - y)] \eta}{[(a\eta - mz)^2 + m^2 (\eta - y)^2] \sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}}$$

(44)

From (24) using (35)

$$\frac{1}{2} \frac{d}{d\eta} \log \frac{(mx-\eta) - \beta^2 a (mz-ay) + \beta^2 m^2 (\eta-y) + \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2]}}{(mx-\eta) - \beta^2 a (mz-ay) + \beta^2 m^2 (\eta-y) - \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2 (\eta-y)^2 - \beta^2 (mz-ay)^2]}}$$

$$= \frac{-\sqrt{1 - \beta^2 (a^2 + m^2)}}{\sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}}$$

(45)

and

$$\frac{d}{d\eta} \tan^{-1} \frac{f}{g} = \frac{\frac{f'}{g} - \frac{g'f}{g^2}}{\left(\frac{f}{g}\right)^2 + 1} = \frac{gff' - g'f^2}{f(f^2 + g^2)}$$

where

$$f = (mz - ay) \sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]} \quad (46)$$

$$g = [m(mx - y) - a(z - ax)]\eta + m[z(z - ax) - y(mx - y)]$$

$$ff' = (mz - ay)^2 [( \eta - mx) - \beta^2 a(a\eta - mz) - \beta^2 m^2 (\eta - y)]$$

$$g' = [m(mx - y) - a(z - ax)]$$

$$\begin{aligned} f^2 + g^2 &= (mz - ay)^2 \left\{ (\eta - mx)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2] \right\} \\ &\quad + \left\{ [m(mx - y) - a(z - ax)]\eta + m[z(z - ax) - y(mx - y)] \right\}^2 \\ &= -\beta^2 (mz - ay)^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2] \\ &\quad + [m(z - ax) + a(mx - y)]^2 (\eta - mx)^2 \\ &\quad + [m(mx - y)(\eta - y) - (z - ax)(a\eta - mz)]^2 \\ &= -\beta^2 (mz - ay)^2 [(a\eta - mx)^2 + m^2 (\eta - y)^2] \\ &\quad + [a^2 (mx - y)^2 + 2am(mx - y)(z - ax) + m^2 (z - ax)^2] (\eta - mx)^2 \\ &\quad + m^2 (mx - y)^2 (\eta - y)^2 - 2m(mx - y)(z - ax)(\eta - y)(a\eta - mz) + (z - ax)^2 (a\eta - mz)^2 \end{aligned} \quad (47)$$

$$\begin{aligned}
&= -\beta^2 (mx - ay)^2 \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right] \\
&\quad + (mx - y)^2 \left[ (a\eta - mz) + m(z - ax) \right]^2 + m^2 (z - ax)^2 \left[ (\eta - y) - (mx - y) \right]^2 \\
&\quad - 2m (mx - y) (z - ax) \left[ (\eta - y) - (mx - y) \right] \left[ (a\eta - mz) + m(z - ax) \right] \\
&\quad + m^2 (mx - y)^2 (\eta - y)^2 - 2m (mx - y) (z - ax) (\eta - y) (a\eta - mz) + (z - ax)^2 (a\eta - mz)^2 \\
&= \left[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 \right] \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right]
\end{aligned}$$

$$\begin{aligned}
\frac{gff' - f^2 g'}{(ay - mz)^2} &= \left\{ \left[ m (mx - y) - a (z - ax) \right] (\eta - mx) + m \left[ (mx - y)^2 + (z - ax)^2 \right] \right\} \left\{ \left[ 1 - \beta^2 (a^2 + m^2) \right] (\eta - mx) + \beta^2 m \left[ a (z - ax) - m (mx - y) \right] \right\} \\
&\quad + \left[ a (z - ax) - m (mx - y) \right] \left\{ (\eta - mx)^2 - \beta^2 \left[ a (\eta - mx) - m (z - ax) \right]^2 - \beta^2 m^2 \left[ (\eta - mx) + (mx - y) \right]^2 \right\} \\
&= \left\{ \left[ m (mx - y) - a (z - ax) \right] (\eta - mx) + m \left[ (mx - y)^2 + (z - ax)^2 \right] \right\} \left\{ \left[ 1 - \beta^2 (a^2 + m^2) \right] (\eta - mx) + \beta^2 m \left[ a (z - ax) - m (mx - y) \right] \right\} \\
&\quad + \left[ a (z - ax) - m (mx - y) \right] \left\{ (\eta - mx)^2 \left[ 1 - \beta^2 (a^2 + m^2) \right] + 2\beta^2 m \left[ a (z - ax) - m (mx - y) \right] (\eta - mx) - \beta^2 m^2 \left[ (z - ax)^2 + (mx - y)^2 \right] \right\} \\
&= m (\eta - mx) \left\{ \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 + (z - ax)^2 \right] + \beta^2 \left[ a (z - ax) - m (mx - y) \right]^2 \right\} \\
&= m (\eta - mx) \left\{ (mx - y)^2 + (z - ax)^2 - \beta^2 \left[ m^2 (z - ax)^2 + 2am (z - ax) (mx - y) + a^2 (mx - y)^2 \right] \right\} \\
&= m (\eta - mx) \left\{ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mx)^2 \right\}
\end{aligned}$$

ORIGINAL PAGE IS  
OF POOR QUALITY

therefore

$$\begin{aligned} & \frac{d}{d\eta} \tan^{-1} \frac{(ay - mz) \sqrt{(\eta - mx)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}}{\left[ m(mx - y) - a(z - ax) \right] \eta + m \left[ z(z - ax) - y(mx - y) \right]} \\ &= \frac{m(ay - mz)(\eta - mx)}{\left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right] \sqrt{(\eta - mx)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}} \end{aligned} \quad (48)$$

Therefore combining terms in (43), (46), (47), and (48)

$$\begin{aligned} & \frac{1}{2} \frac{d}{d\eta} \int \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2}} d\eta = \\ & \frac{1}{2} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - a\eta)^2}} \\ & \cdot \left\{ \eta \cdot \frac{m(az + my)}{a^2 + m^2} \right\} \frac{m^2 [y(mx - y) - z(z - ax)] + m[a(z - ax) - m(mx - y)]}{\left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right] \sqrt{(\eta - mx)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}} \\ & - \frac{m[(az + my) \cdot (a^2 + m^2)x]}{(a^2 + m^2) \sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}} \\ & \cdot \frac{m^2 (ay - mz)^2 (\eta - mx)}{(a^2 + m^2) \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right] \sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}} \end{aligned}$$

Arranging all terms over a common denominator

$$= \frac{1}{2} \log \frac{(mx-y) + \sqrt{(mx-y)^2 - \beta^2 m^2 (y-z)^2 - \beta^2 (mz-ay)^2}}{(mx-y) - \sqrt{(mx-y)^2 - \beta^2 m^2 (y-z)^2 - \beta^2 (mz-ay)^2}} + \left\{ \frac{m}{(a^2+m^2)[(ay-mz)^2 + m^2(y-z)^2] \sqrt{(mx-y)^2 - \beta^2 m^2 (y-z)^2 - \beta^2 (mz-ay)^2}} \right\} \\ \times \left\{ \{ (a^2+m^2)\eta - m(az+my) \} \{ m[y(mx-y) - z(z-ax)] + [a(z-ax) - m(mx-y)]\eta \} \right. \\ \left. - [a(z-ax) - m(mx-y)] \{ (ay-mz)^2 + m^2(y-z)^2 \} + m(ay-mz)^2(\eta-mx) \right\}$$

The numerator can be written

$$\eta^2 \left\{ (a^2 + m^2) [a(z-ax) - m(mx-y)] - (a^2 + m^2) [a(z-ax) - m(mx-y)] \right\} \\ + \eta \left\{ -m(az+my) [a(z-ax) - m(mx-y)] + (a^2 + m^2) m [y(mx-y) - z(z-ax)] \right. \\ \left. + 2 [a(z-ax) - m(mx-y)] [az+my] m + m(ay-mz)^2 \right\} \\ - m^2 (az+my) [y(mx-y) - z(z-ax)] - m^2 [a(z-ax) - m(mx-y)] (y^2 + z^2) - m^2 x (ay-mz)^2 \\ = m \eta \left\{ (z-ax) [-a(az+my) - (a^2 + m^2)z + 2a[az+my] - m(ay-mz)] \right. \\ \left. + (mx-y) [m(az+my) + (a^2 + m^2)y - 2m[az+my] - a(ay-mz)] \right\} \\ + m^2 \left\{ (z-ax) [z(az+my) - a(y^2 + z^2) + mx(ay-mz)] \right. \\ \left. + (mx-y) [-y(az+my) + m(y^2 + z^2) + ax(ay-mx)] \right\} \\ = m^2 \left\{ (z-ax) [-zm(mx-y) + ay(mx-y)] \right. \\ \left. + (mx-y) [-ay(z-ax) + mz(z-ax)] \right\} = 0$$

Therefore (43) is verified.



Evaluation of the definite integral over  $\eta$ . - Because of (4) and (6) and analogous to the section in the evaluation of the definite integral for the surface distribution of sources, all of the terms in (43) will be zero when  $\eta = \eta_3$  provided the denominator of the  $\tan^{-1}$  term is greater than zero. At  $\eta = \eta_3$ , using (6), and then (5),

$$\begin{aligned}
 & \left[ m (mx - y) - a (z - ax) \right] \eta_3 = m \left[ z (z - ax) - y (mx - y) \right] \\
 & = \frac{m \left[ m (mx - y) - a (z - ax) \right] \left\{ (x - \beta^2 az) - \beta^2 my - \beta \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mx)^2} \right\}}{[1 - \beta^2 (a^2 + m^2)]} \\
 & \quad + m \left[ z (z - ax) - y (mx - y) \right] \\
 & = m \left\{ \left[ m (x - \beta^2 az) - y (1 - \beta^2 a^2) \right] (mx - y) + \left[ (1 - \beta^2 m^2) z - a (x - \beta^2 my) \right] (z - ax) \right. \\
 & \quad \left. - \beta \left[ m (my - x) - a (z - ax) \right] \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \right\} [1 - \beta^2 (a^2 + m^2)]^{-1} \\
 & = \frac{m \left\{ \left[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 \right] - \beta \left[ m (mx - y) - a (z - ax) \right] \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \right\}}{1 - \beta^2 (a^2 + m^2)} \\
 & = \frac{m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2}}{1 - \beta^2 (a^2 + m^2)} \left\{ \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} - \beta \left[ m (mx - y) - a (z - ax) \right] \right\}
 \end{aligned} \tag{49}$$

But

$$\begin{aligned}
 & \left[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 \right] - \beta^2 \left[ m (mx - y) - a (z - ax) \right]^2 \\
 & = \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 + (z - ax)^2 \right] + \beta^2 a^2 (mx - y)^2 + \beta^2 m^2 (z - ax)^2 \\
 & \quad + 2 \beta^2 am (mx - y) (z - ax) - \beta^2 (ay - mz)^2 \\
 & = \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 + (z - ax)^2 \right] + \beta^2 \left[ a (mx - y) + m (z - ax) \right]^2 - \beta^2 (ay - mz)^2 \\
 & = \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 + (z - ax)^2 \right]
 \end{aligned} \tag{50}$$

Therefore if  $1 - \beta^2 (a^2 + m^2) > 0$

$$\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} > \beta |m (mx - y) - a (z - ax)|$$

and from (49)

$$[m (mx - y) - a (z - ax)] \eta_3 + m [z (z - ax) - y (mx - y)] > 0$$

However also,

$$m (mx - y) - a (z - ax) = (a^2 + m^2)x - (my + az)$$

and

$$\begin{aligned} (a^2 + m^2) x^2 - (my + az)^2 &= (a^2 + m^2)^2 x^2 - (a^2 + m^2) (y^2 + z^2) + (ay - mz)^2 \\ &= \frac{a^2 + m^2}{\beta^2} \left\{ \left[ \beta^2 (a^2 + m^2) - 1 \right] x^2 + \left[ x^2 - \beta^2 (y^2 + z^2) \right] \right\} + (ay - mz)^2 > 0 \end{aligned}$$

if

$$1 - \beta^2 (a^2 + m^2) < 0$$

and

$$x^2 > \beta^2 (y^2 + z^2)$$

Therefore if  $x > 0$  also

$$(a^2 + m^2 x) > |my + az|$$

and

$$m(mx - y) - a(z - ax) > 0$$

if

$$1 - \beta^2 (a^2 + m^2) < 0 \quad x^2 > \beta^2 (y^2 + z^2)$$

Therefore from (50) if  $1 - \beta^2 (a^2 + m^2) < 0$

$$\left[ \beta^2 (a^2 + m^2) - 1 \right]^{-1} \left\{ \beta [m(mx - y) - a(z - ax)] \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \right\} > 0$$

and from (49), if  $x^2 > \beta^2 (y^2 + z^2)$

$$[m(mx - y) - a(z - ax)] \eta_3 + m[z(z - ax) - y(mx - y)] > 0$$

and therefore, using (4) and (54), the  $\tan^{-1}$  term in (45) is zero when  $\eta = \eta_3$ .

Therefore substituting  $\eta = 0$  into (43), and from (32),

$$\begin{aligned}
 \phi(x, y, z) &= \frac{\frac{\Delta P}{2_\infty}}{8\pi a} \left\{ \int_0^{\gamma_3} \log \frac{(mx-\eta) + \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\gamma-\eta)^2 - \beta^2 (a\gamma-mz)^2}}{(mx-\eta) - \sqrt{(mx-\eta)^2 - \beta^2 m^2 (\gamma-\eta)^2 - \beta^2 (a\gamma-mz)^2}} d\eta \right. \\
 &\quad \left. - \sqrt{1-\beta^2 a^2} \int_0^{\gamma_3} \log \frac{(mx-\eta) - \beta^2 a(mz-a\eta) + \beta^2 m^2 (\gamma-\eta) + \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2 (\gamma-\eta)^2 - \beta^2 (a\gamma-mz)^2]}}{(mx-\eta) - \beta^2 a(mz-a\eta) + \beta^2 m^2 (\gamma-\eta) - \sqrt{[1-\beta^2(a^2+m^2)][(mx-\eta)^2 - \beta^2 m^2 (\gamma-\eta)^2 - \beta^2 (a\gamma-mz)^2]}} d\eta \right\} \\
 &= \frac{1}{4\pi a} \frac{\Delta P}{2_\infty} \left\{ \frac{m(a\bar{z}+m\gamma)}{2(a^2+m^2)} \log \frac{x + \sqrt{x^2 - \beta^2(y^2+z^2)}}{x - \sqrt{x^2 - \beta^2(y^2+z^2)}} \right. \\
 &\quad - \frac{m[(a\bar{z}+m\gamma) - (a^2+m^2)x]}{2(a^2+m^2)\sqrt{1-\beta^2(a^2+m^2)}} \log \frac{x - \beta^2(a\bar{z}+m\gamma) + \sqrt{[1-\beta^2(a^2+m^2)][x^2 - \beta^2(y^2+z^2)]}}{x - \beta^2(a\bar{z}+m\gamma) - \sqrt{[1-\beta^2(a^2+m^2)][x^2 - \beta^2(y^2+z^2)]}} \\
 &\quad + \frac{m(ay-mz)}{a^2+m^2} \tan^{-1} \frac{(mz-ay)\sqrt{x^2 - \beta^2(y^2+z^2)}}{z(z-ax) - y(mx-y)} \\
 &\quad + (z-ax) \tan^{-1} \frac{m(z-ax)\sqrt{x^2 - \beta^2(y^2+z^2)}}{y[(y-mx) - \beta^2 a(ay-mz)] + (z-ax)^2} \\
 &\quad - \frac{m(x-\beta^2 a\bar{z}) - y(1-\beta^2 a^2)}{2\sqrt{1-\beta^2(a^2+m^2)}} \log \frac{x - \beta^2(a\bar{z}+m\gamma) + \sqrt{[1-\beta^2(a^2+m^2)][x^2 - \beta^2(y^2+z^2)]}}{x - \beta^2(a\bar{z}+m\gamma) - \sqrt{[1-\beta^2(a^2+m^2)][x^2 - \beta^2(y^2+z^2)]}} \\
 &\quad \left. - \frac{1}{2} y \sqrt{1-\beta^2 a^2} \log \frac{(x-\beta^2 a\bar{z}) + \sqrt{(1-\beta^2 a^2)[x^2 - \beta^2(y^2+z^2)]}}{(x-\beta^2 a\bar{z}) - \sqrt{(1-\beta^2 a^2)[x^2 - \beta^2(y^2+z^2)]}} \right\}
 \end{aligned}$$

Therefore combining terms

$$\begin{aligned}
 \phi(x,y,z) = & \frac{1}{4\pi a} \left( \frac{\Delta P}{Q_\infty} \right) \left\{ \frac{m(a\bar{z} + m\bar{y})}{2(a^2 + m^2)} \log \frac{x + \sqrt{x^2 - \beta^2(y^2 + z^2)}}{x - \sqrt{x^2 - \beta^2(y^2 + z^2)}} \right. \\
 & + \frac{a(a\bar{y} + m\bar{z}) \sqrt{1 - \beta^2(a^2 + m^2)}}{2(a^2 + m^2)} \log \frac{x - \beta^2(a\bar{z} + m\bar{y}) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(a\bar{z} + m\bar{y}) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}} \\
 & - \frac{m(m\bar{z} - a\bar{y})}{a^2 + m^2} \tan^{-1} \frac{(m\bar{z} - a\bar{y}) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{z(z - a\bar{x}) - y(m\bar{x} - \bar{y})} \\
 & + (z - a\bar{x}) \tan^{-1} \frac{m(z - a\bar{x}) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{y[(y - m\bar{x}) - \beta^2 a(a\bar{y} - m\bar{z})] + (z - a\bar{x})^2} \\
 & \left. - \frac{1}{2} y \sqrt{1 - \beta^2 a^2} \log \frac{x - \beta^2 a \bar{z} + \sqrt{(1 - \beta^2 a^2)[x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2 a \bar{z} - \sqrt{(1 - \beta^2 a^2)[x^2 - \beta^2(y^2 + z^2)]}} \right\} \quad (51)
 \end{aligned}$$

Evaluation of the velocity components. - Using the arguments presented in the section covering the evaluation of the velocity components for the source distribution, the partial derivatives of (51) may be obtained by differentiating only the coefficients of each term.

#### Modifications and Regions of Validity of the Velocity Potential Functions

Region of validity of  $\phi$ . - All of the integrations were performed without regard to the existence of the limits of integration, negative square roots, etc. Therefore the functions in (29) and (51) must be examined and possibly modified for some regions of  $(x, y, z)$  space.

Since all of the terms contain  $\sqrt{x^2 - \beta^2(y^2 + z^2)}$  the formula for  $\phi(x, y, z)$  given by (51) is only valid when  $x^2 > \beta^2(y^2 + z^2)$ , which means inside the Mach cone from the origin.

If  $1 - \beta^2(a^2 + m^2) < 0$  (supersonic leading edge)  
the log term which contains the square root of this  
quantity must be modified. We can write

$$\frac{i}{2} \log \frac{f+ig}{f-ig} = -\tan^{-1} \frac{g}{f} \quad (52)$$

and therefore if  $1 - \beta^2(a^2 + m^2) < 0$

$$\begin{aligned} & \frac{1}{2} \sqrt{1 - \beta^2(a^2 + m^2)} \log \frac{x - \beta^2(az + my) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(az + my) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}} \\ &= -\sqrt{\beta^2(a^2 + m^2) - 1} \tan^{-1} \frac{\sqrt{[\beta^2(a^2 + m^2) - 1][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(az + my)} \end{aligned} \quad (53)$$

If  $1 - \beta^2(a^2 + m^2) = 0$  there is a sonic leading edge.  
For the case where  $1 - \beta^2(a^2 + m^2) \rightarrow 0$  we can write

$$\begin{aligned} & \lim_{1 - \beta^2(a^2 + m^2) \rightarrow 0} \frac{1}{2} \sqrt{1 - \beta^2(a^2 + m^2)} \log \frac{x - \beta^2(az + my) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(az + my) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}} \\ &= [1 - \beta^2(a^2 + m^2)] \frac{\sqrt{x^2 - \beta^2(y^2 + z^2)}}{x - \beta^2(az + my)} \end{aligned} \quad (54)$$

all of the log terms appear to potentially involve logarithms of negative numbers. This difficulty may be avoided by taking the absolute value of the arguments. This is allowable since any log of -1 would have canceled in the definite integral over  $\gamma$ . The argument could not have passed through zero and therefore must have been always positive or always negative.

Since certain functions occur repeatedly we define

$$F1 = \tan^{-1} \frac{m(z - ax) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2}$$

$$F2 = \frac{1}{2\sqrt{1 - \beta^2(a^2 + m^2)}} \log \frac{x - \beta^2(mz + az) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(mz + az) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}$$

$$F3 = \tan^{-1} \frac{(mz - ay) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)}$$

(56)

$$F6 = \sqrt{1 - \beta^2 a^2} \frac{1}{2} \log \frac{x - \beta^2 a z + \sqrt{(1 - \beta^2 a^2)[x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2 a z - \sqrt{(1 - \beta^2 a^2)[x^2 - \beta^2(y^2 + z^2)]}}$$

Evaluation of the  $\tan^{-1}$  functions F1 and F3. - The terms F1 or F3 are always real inside the Mach cone from the origin,

$$x^2 - \beta^2(y^2 + z^2) \geq 0$$

However as the argument of these functions go to zero the functions may take on different values depending upon how zero is approached. Corresponding to each of the four quadrants, if

$$\theta = \tan^{-1} \frac{f}{g}$$

then

$$f \geq 0 \quad g > 0 \quad \text{means} \quad 0 \leq \theta < \frac{\pi}{2}$$

$$f \geq 0 \quad g < 0 \quad \text{means} \quad \frac{\pi}{2} < \theta < \pi$$

$$f \leq 0 \quad g > 0 \quad \text{means} \quad 0 \geq \theta > -\frac{\pi}{2}$$

$$f \leq 0 \quad g < 0 \quad \text{means} \quad -\frac{\pi}{2} > \theta \geq -\pi$$

(57)

Therefore from (52) and (57) as  $z \rightarrow ax$

$$\begin{array}{ll} 0 & y < 0 \text{ or } y > mx \\ \lim_{z \rightarrow ax} F1 = \pi & 0 < y < mx, \quad z > ax \\ -\pi & 0 < y < mx, \quad z > ax \end{array}$$

(58)

and as  $mz \rightarrow ay$

$$\begin{array}{ll} 0 & y < 0 \text{ or } y > mx \\ \lim_{mz \rightarrow ay} F3 = \pi & 0 < y < mx \quad mz > ay \\ -\pi & 0 < y < mx \quad mz < ay \end{array}$$

(59)



Velocity potential on  $x^2 = \beta^2(y^2 + z^2)$  if  $1 - \beta^2(a^2 + m^2) > 0$ . Referring

to (52)  $F_2$  and  $F_6$  are zero if  $x^2 - \beta^2(y^2 + z^2) = 0$ . The functions  $F_1$  and  $F_3$  will be zero if and only if the denominators of their arguments are greater than zero [see (57)]. First we note the following

$$\begin{aligned}
 x^2 - [\beta^2(my + az)]^2 &= [x^2 - \beta^2(y^2 + z^2)] + \beta^2(y^2 + z^2) - \beta^4[m^2y^2 + 2amyz + a^2z^2] \\
 &= [x^2 - \beta^2(y^2 + z^2)] + \beta^2[1 - \beta^2(a^2 + m^2)](y^2 + z^2) + \beta^4[a^2y^2 + 2amyz + m^2z^2] \\
 &= [x^2 - \beta^2(y^2 + z^2)] + \beta^2[1 - \beta^2(a^2 + m^2)](y^2 + z^2) + \beta^4(ay - mz)^2 \\
 &= [x - \beta^2(my + az)][x + \beta^2(my + az)]
 \end{aligned} \tag{60}$$

which means  $x > |\beta^2(my + az)|$

if  $x^2 - \beta^2(y^2 + z^2) \geq 0 \quad 1 - \beta^2(a^2 + m^2) > 0$

For  $F_1$  the denominator is

$$\begin{aligned}
 y [(y - mx) - \beta^2 a (ay - mz)] + (z - ax)^2 \\
 = y^2 [1 - \beta^2(a^2 + m^2)] - my [x - \beta^2(az + my)] + (z - ax)^2 \\
 > 0 \text{ if } y < 0 \text{ from (60)}
 \end{aligned}$$

$$y [(y - mx) - \beta^2 a (ay - mz)] + (z - ax)^2$$

$$= (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + mx (y - mx) - \beta^2 mz (ay - mz)$$

$$= (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + mxy - m^2 (x^2 - \beta^2 z^2) - \beta^2 amyz$$

$$= (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + my [x - \beta^2 (az + my)]$$

$$- m^2 [x^2 - \beta^2 (y^2 + z^2)] > 0$$

$$\text{if } x^2 - \beta^2 (y^2 + z^2) = 0, y > 0 \text{ due to (50) and (60)}$$

Therefore

$$y [(y - mx) - \beta^2 a (ay - mz)] + (z - ax)^2 > 0$$

on

$$x^2 = \beta^2 (y^2 + z^2) \quad (61)$$

if

$$1 - \beta^2 (a^2 + m^2) > 0$$

Therefore due to (57) and (61)

$$\lim F1 = 0$$

$$\text{as } x^2 - \beta^2 (y^2 + z^2) \rightarrow 0$$

if

$$1 - \beta^2 (a^2 + m^2) > 0$$

or F3

$$(z - ax) - y (mx - y) = y^2 + z^2 - x (az + my) \quad (62)$$

$$= - [x^2 - \beta^2 (y^2 + z^2)] \frac{1}{\beta^2} + \frac{x}{\beta^2} [x - \beta^2 (az + my)] > 0$$

if

$$x^2 - \beta^2 (y^2 + z^2) = 0, \text{ and } 1 - \beta^2 (a^2 + m^2) > 0 \text{ due to (54c) if } x > 0$$

Therefore examining (29) and (56) shows that for a surface distribution of sources or a constant pressure surface

$$\lim \phi = 0$$

$$\text{as } x^2 - \beta^2 (y^2 + z^2) \rightarrow 0$$

if

$$1 - \beta^2 (a^2 + m^2) \rightarrow 0 \quad (63)$$

Therefore  $\phi$  will be continuous, as it must be, if we define  $\phi$  to be zero outside the Mach cone from the origin.

$$\phi = 0$$

if

$$1 - \beta^2 (a^2 + m^2) > 0 \text{ and } x^2 < \beta^2 (y^2 + z^2) \quad (64)$$

Supersonic leading edge and the mach cone envelope. With a supersonic leading edge,  $1 - \beta^2 (a^2 + m^2) < 0$ , all functions in the equations for  $\phi$  will be shown to go to zero for points on the Mach cone from the origin, except for a region on this Mach cone which borders the envelope of Mach cones from the supersonic leading edge. Inside this envelope of Mach cones the functions F1, F2 and F3 will be shown to have constant values. However on the outer boundary of this envelope of Mach cones  $\phi$  will go to zero, and therefore all functions may be defined to be zero outside this envelope.

The envelope of Mach cones from the leading edge is illustrated on p-29 of Reference (55). The Mach cone from any point  $x_0, y_0, z_0$  can be written,

$$(x - x_0)^2 = \beta^2 [(y - y_0)^2 + (z - z_0)^2]$$

on the leading edge  $mx_0 = y_0$  and  $ax_0 = z_0$

Therefore

$$(mx - y_0)^2 = \beta^2 [m^2 (y - y_0)^2 + (mz - ay_0)^2] \quad (65)$$

The Mach cone envelope is determined by the maximum values for  $z$ , at a given  $x$  and  $y$ , obtained by a variation of  $y_0$ . Therefore differentiating (65) with respect to  $y_0$ , holding  $x$  and  $y$  constant, and setting  $dz/dy_0 = 0$

gives

$$(mx - y_0) = \beta^2 m^2 (y - y_0) + \beta^2 a (mz - ay_0)$$

or

$$y_0 [1 - \beta^2 (a^2 + m^2)] = m [x - \beta^2 (my + az)] \quad (66)$$

If  $y_0 = 0$  (65) and (66) give

$$x^2 = \beta^2 (y^2 + z^2) \text{ and } x - \beta^2 (my + az) = 0 \quad (67)$$

or

$$x^2 = \beta^2 y^2 + \beta^2 \left[ \frac{x - \beta^2 my}{\beta^2 a} \right]^2$$

$$\beta^2 a^2 x^2 = \beta^4 a^2 y^2 + x^2 - 2 \beta^2 mxy + \beta^4 m^2 y^2$$

$$\beta^4 (a^2 + m^2) y^2 - 2 \beta^2 mxy + x^2 (1 - \beta^2 a^2) = 0$$

or, if  $y_0 = 0$

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m \pm a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \quad (68)$$

where

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m - a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \quad z > 0$$

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m + a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \quad z < 0$$

For  $y_0 > 0$  we can eliminate  $y_0$  from (65) and (66).

From (65)

$$\left[ 1 - \beta^2 (a^2 + m^2) \right] y_0^2 - 2m \left[ x - \beta^2 (az + my) \right] y_0 + m^2 \left[ x^2 - \beta^2 (y^2 + z^2) \right] = 0$$

or

$$y_0 \left[ 1 - \beta^2 (a^2 + m^2) \right] = m \left[ x - \beta^2 (az + my) \right] \\ \pm m \sqrt{\left[ x - \beta^2 (az + my) \right]^2 - \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ x^2 - \beta^2 (y^2 + z^2) \right]}$$

Therefore from (66)

$$\left| x - \beta^2 (az + my) \right|^2 - \left| 1 - \beta^2 (a^2 + m^2) \right| \left| x^2 - \beta^2 (y^2 + z^2) \right| = 0$$

or

$$\beta^2 (a^2 + m^2) x^2 - 2 x \beta^2 (az + my) - \beta^4 (ay - mz)^2 + \beta^2 (y^2 + z^2) = 0$$

and solving for x

$$\begin{aligned} x &= \frac{(az + my) \pm \sqrt{(az + my)^2 + (a^2 + m^2) [(ay - mz)^2 \beta^2 - (y^2 + z^2)]}}{(a^2 + m^2)} \\ &= \frac{(az + my) \pm \sqrt{(ay - mz)^2 [\beta^2 (a^2 + m^2) - 1]}}{(a^2 + m^2)} \\ &= \frac{(az + my) \pm |mz - ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \end{aligned}$$

If we write

$$x = \frac{(az + my) \pm (mz - ay) \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)}$$

or

$$\frac{\partial z}{\partial y} \left[ a \pm m \sqrt{\beta^2 (a^2 + m^2) - 1} \right] = \left[ -m \pm a \sqrt{\beta^2 (a^2 + m^2) - 1} \right]$$

$$\frac{\partial z}{\partial y} (a^2 + m^2) [1 - \beta^2 m^2]$$

$$= (-a \pm m \sqrt{\beta^2 (a^2 + m^2) - 1}) (-m \pm a \sqrt{\beta^2 (a^2 + m^2) - 1})$$

$$= (a^2 + m^2) [-\beta^2 am \pm \sqrt{\beta^2 (a^2 + m^2) - 1}]$$

Therefore use

+ for  $mz > ay$

- for  $mz < ay$

or

$$x = \frac{(az + my) + |mz + ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \quad (69)$$

This is presented on page 29 of reference (55).

If we set

$$z = y \tan \theta \quad \text{on } x^2 = \beta^2 (y^2 + z^2)$$

then

$$x^2 = \beta^2 y^2 (1 + \tan^2 \theta)$$



or

$$x = \frac{\beta y}{\cos \theta} \quad \beta y = x \cos \theta \quad (70)$$

Therefore (67) gives for the two points where the envelope of Mach cones from the leading edge is on the Mach cone from the origin,

$$(y^2 + z^2) = \beta^2 (my + az)^2$$

or

$$[1 + \tan^2 \theta] = \beta^2 m^2 + 2 \beta^2 am \tan \theta + \beta^2 a^2 \tan^2 \theta$$

which means

$$(1 - \beta^2 a^2) \tan \theta = \beta^2 am \pm \sqrt{\beta^4 a^2 m^2 - (1 - \beta^2 a^2)(1 - \beta^2 m^2)}$$

or

$$(1 - \beta^2 a^2) \tan \theta = \beta^2 am \pm \sqrt{\beta^2 (a^2 + m^2) - 1} \quad (71)$$

but

$$x - \beta^2 (my + az) = 0$$

means

$$\frac{\beta a}{\cos \theta} - \beta^2 a (m + a \tan \theta) = 0 \quad (72)$$

Therefore the two points corresponding to (68) are

$$\begin{aligned}\tan \theta_1 &= \frac{\beta a}{\cos \theta_1} - \sqrt{\beta^2 (a^2 + m^2) - 1} \\ \tan \theta_2 &= \frac{\beta a}{\cos \theta_2} + \sqrt{\beta^2 (a^2 + m^2) - 1}\end{aligned}\tag{73}$$

(assume  $a > 0$  and  $\theta_2 > \theta_1$ )

From (56) and (57) the value of F1 or F3 as we approach the Mach cone from the origin depends on the sign of the denominator of its argument.

For

F 1 on  $x^2 = \beta^2 (y^2 + z^2)$  we can write

$$\begin{aligned}D1 &= \frac{1}{y^2} \left\{ y (y - mx) - \beta^2 a (ay - mz) + (z - ax)^2 \right\} \\ &= (1 - \beta^2 a^2) - \frac{\beta m}{\cos \theta} + \beta^2 a m \tan \theta + \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right]^2 \\ &= 1 - \beta^2 (a^2 + m^2) + \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right]^2 - \beta m \left[ \beta m + \frac{1}{\cos \theta} - \beta a \tan \theta \right] \\ &= 0 \text{ if } \theta = \theta_1, \text{ or } \theta = \theta_2 \text{ from (72) and (73)}\end{aligned}$$

If we differentiate this with respect to  $\theta$  we get

$$\begin{aligned}\frac{dD1}{d\theta} &= 2 \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left[ \sec^2 \theta - \frac{\beta a \sin \theta}{\cos^2 \theta} \right] + \beta m \left[ \beta a \sec^2 \theta - \frac{\sin \theta}{\cos^2 \theta} \right] \\ &= \frac{1}{\cos^2 \theta} \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left| 2 (1 - \beta a \sin \theta) - \beta m \cos \theta \right| \\ &= \frac{1}{\cos^2 \theta} \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left| 2 (1 - \beta a \sin \theta - \beta m \cos \theta) + \beta m \cos \theta \right|\end{aligned}$$

From (72)

$$1 - \beta a \sin \theta - \beta m \cos \theta = 0 \quad \text{at} \quad \theta = \theta_1, \theta_2$$

and since

$$\cos \theta_1 > 0, \cos \theta_2 > 0 \text{ and } \beta m > 0$$

and using (73) we get

$$\frac{dD1}{d\theta} < 0 \quad \theta = \theta_1$$

$$\frac{dD1}{d\theta} > 0 \quad \theta = \theta_2$$

and therefore using (57) if

$$1 - \beta^2 (a^2 + m^2) < 0$$

$$\lim_{x^2 \rightarrow \beta^2 (y^2 + z^2)} F1 = \begin{matrix} 0 & \theta > \theta_2 & \theta < \theta_1 \\ \pi & z > ax & \theta_1 < \theta < \theta_2 \\ -\pi & z < zx & \theta_1 < \theta < \theta_2 \end{matrix} \quad (74)$$

For

$$F3 \text{ on } x^2 = \beta^2 (y^2 + z^2)$$

let

$$D3 = z (z - ax) - y (mx - y)$$

$$= y^2 (1 + \tan^2 \theta) - yx (m + a \tan \theta)$$

$$= \frac{x^2}{\beta^2} \left[ 1 - \beta \cos \theta (m + a \tan \theta) \right]$$

$$= \frac{x^2}{\beta^2} \left[ 1 - (\beta m \cos \theta + \beta a \sin \theta) \right]$$

$$= 0 \text{ if } \theta = \theta_1 \text{ or } \theta = \theta_2 \text{ from (72)}$$

$$\frac{d}{d\theta} D3 = \frac{x^2}{\beta} [m \sin \theta - a \cos \theta]$$

$$= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} [m (1 - \beta^2 a^2) \tan \theta - a (1 - \beta^2 a^2)]$$

$$= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} [a \beta^2 m^2 - a (1 - \beta^2 a^2) \pm m \sqrt{\beta^2 (a^2 + m^2) - 1}]$$

$$= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \left\{ -a [1 - \beta^2 (a^2 + m^2)] \pm m \sqrt{\beta^2 (a^2 + m^2) - 1} \right\}$$

$$= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \sqrt{\beta^2 (a^2 + m^2) - 1} \left\{ a \sqrt{\beta^2 (a^2 + m^2) - 1} \pm m \right\}$$

$$> 0 \text{ for } \theta_2 (+m) [\text{see (73)}]$$

$$< 0 \text{ for } \theta_1 (-m) (\text{assuming } a > 0)$$

because

$$m^2 > a^2 [\beta^2 (a^2 + m^2) - 1]$$

since

$$(a^2 + m^2) (1 - \beta^2 a^2) > 0$$

Therefore since

$$\frac{d}{d\theta} D3 \begin{matrix} > 0 & \theta = \theta_2 \\ < 0 & \theta = \theta_1 \end{matrix}$$

and therefore

$$D3 \begin{matrix} > 0 \\ < 0 \end{matrix} \quad \begin{matrix} \theta > \theta_2 \text{ or } \theta < \theta_1 \\ \theta_1 < \theta < \theta_2 \end{matrix} \quad (75)$$

We can say

$$\lim_{x^2 \rightarrow \beta^2} \frac{F3}{(y^2 + z^2)} = \begin{matrix} 0 \\ \pi \\ -\pi \end{matrix} \quad \begin{matrix} \theta > \theta_2 \text{ or } \theta < \theta_1 \\ mz > ay \quad \theta_1 < \theta < \theta_2 \\ mz < ay \quad \theta_1 < \theta < \theta_2 \end{matrix} \quad (76)$$

Also on the plane

$$mz = ay$$

$$D3 = z(z - ax) - y(mx - y) = y^2 \left[ 1 + \frac{a^2}{m^2} \right] - xy \left[ m + \frac{a^2}{m} \right]$$

$$= \frac{y(y - mx)(a^2 + m^2)}{m^2}$$

and therefore

$$\lim_{(ay - mz) \rightarrow 0} F3 = \begin{matrix} 0 & y < 0 & y > mx \\ \pi & 0 < y < mx & mz > ay \\ -\pi & 0 < y < mx & mz < ay \end{matrix} \quad (77)$$

To find the limit of F2 as  $x^2 \rightarrow \beta^2 (y^2 + z^2)$  when  $1 - \beta^2 (a^2 + m^2) < 0$   
[see (53) for F2] we note that

$$\lim_{x^2 \rightarrow \beta^2 (y^2 + z^2)} \left| \frac{x - \beta^2 (my + az)}{\beta \sqrt{(mx-y)^2 + (z-ax)^2 - \beta^2 (ay-mz)^2}} \right| = 1$$

because

$$\begin{aligned} \beta^2 [(mx-y)^2 + (z-ax)^2 - \beta^2 (ay-mz)^2] &= \beta^2 (a^2 + m^2)x^2 + \beta^2 (y^2 + z^2) - 2x\beta^2 (my + az) + 2\beta^4 amy - \beta^2 a^2 y^2 - \beta^4 m^2 z^2 \\ &= [\beta^2 (a^2 + m^2) - 1] [x^2 - \beta^2 (y^2 + z^2)] + x^2 - 2x\beta^2 (my + az) + 2\beta^4 amy + \beta^4 m^2 y^2 + \beta^4 a^2 z^2 \\ &= [\beta^2 (a^2 + m^2) - 1] [x^2 - \beta^2 (y^2 + z^2)] + [x - \beta^2 (my + az)]^2 \end{aligned}$$

Now

$$x - \beta^2 (my + az) = x \left[ 1 - \beta^2 m \frac{y}{x} - \beta^2 a \frac{z}{x} \tan \theta \right]$$

$$= x [1 - \beta m \cos \theta - \beta a \sin \theta] = \frac{\beta^2}{x} D3$$

where D3 is defined by (75). Therefore from (75)

$$\lim_{x^2 \rightarrow \beta^2 (y^2 + z^2)} F2 = \frac{0}{\pi \sqrt{\beta^2 (a^2 + m^2) - 1}} \begin{cases} \theta_1 < \theta & \theta > \theta_2 \\ \theta_1 < \theta < \theta_1 \end{cases} \quad (78)$$

Therefore combining these results

$$F1 = \tan^{-1} \frac{m (z-ax) \sqrt{x^2 - \beta^2 (y^2 + z^2)}}{y [(y-mx) - \beta^2 a (ay-mz)] + (z-ax)^2} \quad x^2 > \beta^2 (y^2 + z^2)$$

If

$$1 - \beta^2 (a^2 + m^2) > 0$$

$$F_1 = 0 \quad x^2 \leq \beta^2 (y^2 + z^2) \quad (79)$$

$$\lim_{z \rightarrow ax} F_1 = \begin{matrix} 0 & y < 0 & y > mx \\ \pi & 0 < y < mx & z > ax \\ \pi & 0 < y < mx & z < ax \end{matrix}$$

Then from (69) and (74) if

$$1 - \beta^2 (a^2 + m^2) < 0$$

and

$$x^2 \leq \beta^2 (a^2 + m^2)$$

$$F_1 = 0 \quad \theta > \theta_2 \quad \theta < \theta_1 \quad (80)$$

$$F_1 = \pi \operatorname{sgn} (z - ax)$$

if

$$\theta_1 < \theta < \theta_2$$

and

$$x > \frac{(az + my) + |mz + ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} = 0$$



and

$$F1 = 0$$

if

$$\theta_1 < \theta < \theta_2$$

and

$$x > \frac{(az + my) + mz + ay}{(a^2 + m^2)} \sqrt{\beta^2 (a^2 + m^2) - 1}$$

and where from (71)

$$(1 - \beta^2 a^2) \tan \theta_1 = \beta^2 am - \sqrt{\beta^2 (a^2 + m^2) - 1}$$

$$(1 - \beta^2 a^2) \tan \theta_2 = \beta^2 am + \sqrt{\beta^2 (a^2 + m^2) - 1}$$

For

$$x^2 > \beta^2 (y^2 + z^2)$$

from (53), (54), and (56)

$$\frac{1}{2\sqrt{1-\beta^2(a^2+m^2)}} \log \frac{x-\beta^2(az+my) + \sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my) - \sqrt{[1-\beta^2(a^2+m^2)][x^2-\beta^2(y^2+z^2)]}} \quad 1-\beta^2(a^2+m^2) > 0$$

$$F2 = \frac{\sqrt{x^2 - \beta^2(y^2 + z^2)}}{x - \beta^2(az + my)} \quad 1 - \beta^2(a^2 + m^2) = 0$$

$$\frac{-1}{\sqrt{\beta^2(a^2+m^2)-1}} \tan^{-1} \frac{\sqrt{[\beta^2(a^2+m^2)-1][x^2-\beta^2(y^2+z^2)]}}{x-\beta^2(az+my)} \quad 1-\beta^2(a^2+m^2) < 0$$

(81)

and from (78)

$$\begin{aligned}
 F2 &= 0 & x^2 \leq \beta^2 (y^2+z^2) & & 1 - \beta^2 (a^2+m^2) > 0 \\
 &= 0 & x^2 \leq \beta^2 (y^2+z^2) & & \theta > \theta_2, \theta > \theta_1 \\
 &= \pi & x^2 \leq \beta^2 (y^2+z^2) & & \theta_1 < \theta < \theta_2 & & 1 - \beta^2 (a^2+m^2) < 0 \\
 &= 0 & \text{outside envelope of Mach cones.} & & & & (82)
 \end{aligned}$$

For

$$x^2 > \beta^2 (y^2+z^2)$$

From (56) and (58)

$$F3 = \tan^{-1} \frac{(mz-ay) \sqrt{x^2 - \beta^2 (y^2+z^2)}}{z(z-ax) - y(mx-y)} \quad (83)$$

and

$$\lim_{(mz-ay) \rightarrow 0} F3 = \begin{array}{lll} 0 & y < 0. & y > mx \\ \pi & 0 < y < mx & mz > ay \\ -\pi & 0 < y < mx & mz < ay \end{array}$$

and for

$$x^2 \leq \beta^2 (y^2+z^2)$$

from (76)

$$F3 = 0 \quad \theta < \theta_1 \quad \theta > \theta_2$$

$$= \pi \quad mz > ay \quad \theta_1 < \theta < \theta_2$$

$$= -\pi \quad mz < ay \quad \theta_1 < \theta < \theta_2$$

$$= 0 \quad \text{outside envelope of Mach cones.} \quad (84)$$

Value of  $\phi$  on the envelope of mach cones. For a supersonic leading edge [ $1 - \beta^2 (a^2 + m^2) < 0$ ], in the region inside the envelope of mach cones from the leading edge and outside the mach cone from the origin, we have for a surface distribution of sources [from (29), (79), (80), and (81)]

$$\begin{aligned} \sqrt{\beta^2 (a^2 + m^2) - 1} (1 - \beta^2 a^2) \frac{\bar{W} + \beta^2 a \bar{u}}{\pi} \phi_s &= (z - ax) \operatorname{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1} + y (1 - \beta^2 a^2) - m (x - \beta^2 az) \\ &= -x \left[ m + a \operatorname{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1} \right] + z \operatorname{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1} + y (1 - \beta^2 a^2) + \beta^2 amz \\ &= \frac{-x (a^2 + m^2) (1 - \beta^2 a^2) + [m - a \operatorname{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1}] [z \operatorname{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1} + y (1 - \beta^2 a^2) + \beta^2 amz]}{m - a \operatorname{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1}} \\ &= \frac{(1 - \beta^2 a^2) \left\{ -x (a^2 + m^2) + (my + az) + \operatorname{sgn} (z - ax) (mz - ay) \sqrt{\beta^2 (a^2 + m^2) - 1} \right\}}{m - a \operatorname{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1}} \end{aligned}$$

On the lines

$$x = \frac{(my + az) + |mz - ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)}$$

$$\operatorname{sgn} (z - ax) (mz - ay) = |mz - ay| \text{ [See figure in Woodward]}$$

Therefore on the envelope of mach cones from the leading edge

$$\phi_s = 0$$

In this same region for a constant pressure surface [from (71)] and

$$\begin{aligned} \phi_p \left[ \frac{1}{4\pi a} \left( \frac{\Delta P}{q_\infty} \right) \right]^{-1} (a^2 + m^2)^{-1} \\ = -a (ay - mz) \sqrt{\beta^2 (a^2 + m^2) - 1} + m (ay - mz) \operatorname{sgn} (mz - ay) + (a^2 + m^2) (z - ax) \operatorname{sgn} (z - ax) \end{aligned}$$

and in the region where

$$\text{sgn}(z - ax) = \text{sgn}(mz - ay)$$

$$= \frac{+ a |mz - ay| \sqrt{\beta^2(a^2 + m^2) - 1} + m(ay - mz) + (a^2 + m^2)(z - ax)}{\text{sgn}(z - ax)}$$

$$= \text{sgn}(z - ax) a \left| -x(a^2 + m^2) + (my + az) + |mz - ay| \sqrt{\beta^2(a^2 + m^2) - 1} \right|$$

$$= 0 \text{ on the lines } x = \frac{(my + az) + |mz - ay| \sqrt{\beta^2(a^2 + m^2) - 1}}{a^2 + m^2}$$

Therefore  $\phi_p = 0$  on the envelope of Mach cones from the leading edge.

Verification of the imposed boundary conditions. Now (8) can be verified for the case of a surface distribution of sources. From (29) and (79), on  $z = ax$ , since the  $\cosh^{-1}$  terms are continuous

$$\phi = \phi' \text{ or } \phi - \phi' = 0$$

For (8b) using (29) and (79) and the results of 2.6

$$u = -\frac{\bar{w} + \beta^2 \bar{a} \bar{u}}{\pi(1 - \beta^2 a^2)} \left| \pi a + mF2 \right| \quad 0 < y < mx \quad z > ax$$

$$u' = -\frac{\bar{w} + \beta^2 \bar{a} \bar{u}}{\pi(1 - \beta^2 a^2)} \left| -\pi a + mF2 \right| \quad 0 < y < mx \quad z < ax$$

$$w = \frac{\bar{w} + \beta^2 \bar{a} \bar{u}}{\pi(1 - \beta^2 a^2)} \left| \pi + \beta^2 a mF2 \right| \quad 0 < y < mx \quad z > ax$$

$$w' = \frac{\bar{w} + \beta^2 \bar{a} \bar{u}}{\pi(1 - \beta^2 a^2)} \left| -\pi + \beta^2 a mF2 \right| \quad 0 < y < mx \quad z < ax$$

Therefore

$$(w - w') + \beta^2 a(u - u') = 2 \frac{\bar{w} + \beta^2 \bar{a} \bar{u}}{\pi(1 - \beta^2 a^2)} \pi[1 - \beta^2 a^2] = 2[\bar{w} + \beta^2 \bar{a} \bar{u}] = \text{const}$$

which agrees with (8b)

For

$$y < 0$$

or

$$y > mx$$

where

$$f = m(z - ax) \sqrt{x^2 - \beta^2(y^2 + z^2)}$$

$$g = y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2$$

and since

$$f = 0 \text{ on } S$$

when

$$z = ax$$

$$\frac{\partial u}{\partial x} = \frac{-am \sqrt{x^2(1 - \beta^2 a^2)} - \beta^2 y^2}{y(y - mx)(1 - \beta^2 a^2)}$$

and likewise on

$$z = ax$$

$$\frac{\partial u}{\partial y} = 0$$

$$\frac{\partial u}{\partial z} = \frac{m \sqrt{x^2(1 - \beta^2 a^2)} - \beta^2 y^2}{y(y - mx)(1 - \beta^2 a^2)}$$

Therefore

$$\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \text{ and } \frac{\partial u}{\partial z}$$

are continuous on  $S$  (except possibly at the edges) and therefore

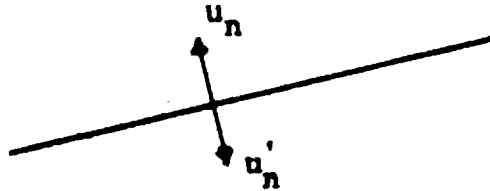
$$\frac{\partial}{\partial x} (u - u') = \frac{\partial}{\partial y} (u - u') = \frac{\partial}{\partial z} (u - u') = 0$$

and (30b) is verified.

Surface velocities on a constant pressure surface. - Flow normal to surface  $U_n$

$$u_n = \frac{1}{\sqrt{1+a^2}} (-au + w)$$

$$u'_n = \frac{1}{\sqrt{1+a^2}} (au' - w')$$



$$\text{Flow through surface} = \frac{1}{2} (u_n - u'_n) = \frac{\frac{1}{2}}{\sqrt{1+a^2}} \left\{ -a(u + u') + (w + w') \right\}$$

on

$$(z = ax) \quad u + u' = 0$$



from (51) and (79)

$$\frac{1}{2} (w + w') = \frac{1}{4\pi a} \left( \frac{\Delta P}{q_\infty} \right) \left\{ \frac{am}{(a^2 + m^2)} - \frac{1}{2} \log \frac{x + \sqrt{x^2(1-\beta^2 a^2) - \beta^2 y^2}}{x - \sqrt{x^2(1-\beta^2 a^2) - \beta^2 y^2}} \right.$$

$$\left. \frac{-am}{2(a^2 + m^2)} \sqrt{1 - \beta^2(a^2 + m^2)} \log \frac{x(1-\beta^2 a^2) - \beta^2 my + \sqrt{[1-\beta^2(a^2+m^2)][x^2(1-\beta^2 a^2) - \beta^2 y^2]}}{x(1-\beta^2 a^2) - \beta^2 my - \sqrt{[1-\beta^2(a^2+m^2)][x^2(1-\beta^2 a^2) - \beta^2 y^2]}} \right.$$

$$\left. \frac{-m^2}{a^2 + m^2} \tan^{-1} \frac{(mx - ay) \sqrt{x^2(1 - \beta^2 a^2) - \beta^2 y^2}}{-y(mx - y)} \right\}$$

$$= \frac{1}{2} \sqrt{1 + a^2} (u_n - u'_n)$$

$$\text{Source strength} = u_n + u'_n = \frac{1}{\sqrt{1 + a^2}} [-a(u - u') + (w - w')]$$

$$= \frac{(1 + a^2)}{\sqrt{1 + a^2}} \frac{2\Delta P}{4\pi a q_\infty} \pi$$

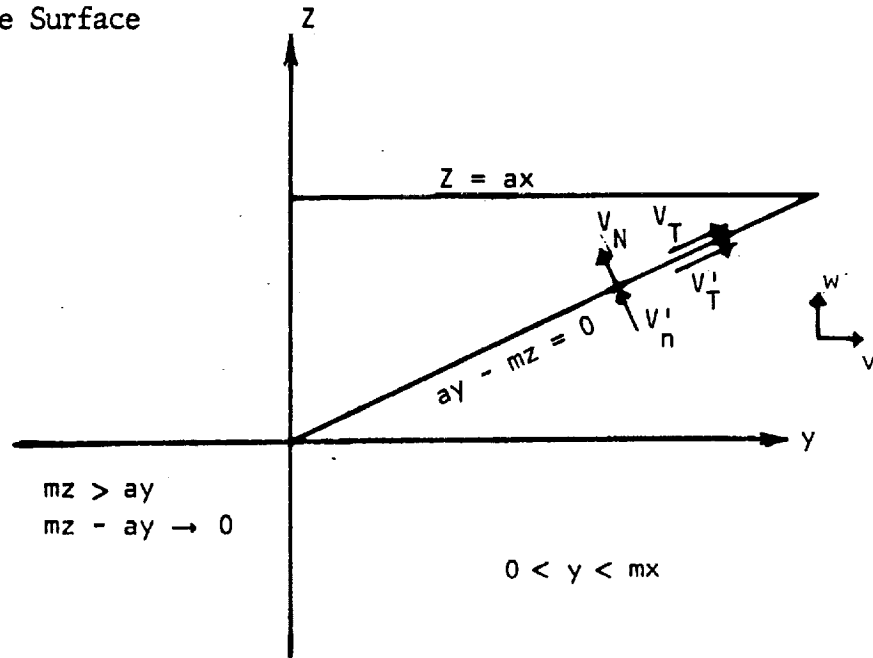
$$= 2 \frac{\sqrt{1 + a^2} \Delta P}{4a q_\infty}$$

(86)

Velocity discontinuity in the leading edge wake. -

Constant Pressure Surface

$$V_T = \frac{v + \frac{a}{m} w}{\sqrt{1 + \left(\frac{a}{m}\right)^2}}$$



$$V_n = \frac{-\frac{a}{m} v + w}{\sqrt{1 + \left(\frac{a}{m}\right)^2}}$$

$$V_n - V'_n = \frac{-\frac{a}{m}(v - v') + (w - w')}{\sqrt{1 + \left(\frac{a}{m}\right)^2}}$$

$$= \frac{1}{4\pi a \sqrt{1 + \left(\frac{a}{m}\right)^2}} \left( \frac{\Delta P}{q_\infty} \right) 2\pi \left[ \left( -\frac{a}{m} \right) am + (-m^2) \right] (a^2 + m^2)^{-1}$$

$$= \frac{-2m}{4a \sqrt{a^2 + m^2}} \left( \frac{\Delta P}{q_\infty} \right) \quad \text{Source Distribution! [see (86)]}$$

$$V_T - V'_T = \frac{2}{4\pi a \sqrt{1 + \left(\frac{a}{m}\right)^2}} \frac{\Delta P}{q_\infty} \pi \left| am - m^2 \frac{a}{m} \right| (a^2 + m^2)^{-1} = 0$$

Alternate sign choice of  $\tan^{-1}$  denominator. -

$$\frac{m(ay - mz)}{(a^2 + m^2)} \tan^{-1} \frac{(mz - ay) \sqrt{(mx - \eta)^2 - \beta^2 [(a\eta - mz)^2 + m^2 (\eta - y)^2]}}{[a(z - ax) - m(mx - y)]\eta + m[y(mx - y) - z(z - ax)]}$$

[see (45) and note sign change of denominator and since

$$[a(z - ax) - m(mx - y)]\eta_3 + m[y(mx - y) - z(z - ax)] < 0$$

The above term when evaluated at

$$\eta = \eta_3$$

is

$$\frac{m(ay - mz)}{(a^2 + m^2)} \pi \operatorname{sgn}(mz - ay)$$

Evaluated at  $\eta = 0$  the term becomes

$$\frac{m(ay - mz)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay) \sqrt{x^2 - \beta^2 (y^2 + z^2)}}{[y(mx - y) - z(z - ax)]}$$

If

$$mz = ay \quad y(mx - y) - z(z - ax) = \frac{y(mx - y)(a^2 + m^2)}{m^2}$$

which means that on  $mz = ay$  and  $\eta = 0$

$$\tan^{-1} = \begin{matrix} \pi \operatorname{sgn}(mz - ay) & y < 0 & y > mx \\ 0 & 0 < y < mx \end{matrix}$$

$$\tan^{-1} \left| \begin{matrix} \eta_3 \\ 0 \end{matrix} \right. = \begin{matrix} 0 & y < 0 & y > mx \\ \pi \operatorname{sgn}(mz - ay) & 0 < y < mx \end{matrix}$$

## Subappendix A - Woodward's Subsonic Equations

If  $\Omega(x, y, z)$  satisfies

$$\left\{ \beta^2 \frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2} \right\} \Omega(x, y, z) = 0 \quad \beta^2 = 1 - M^2$$

Then we can write the solution for  $\Omega(x, y, z)$  as

$$\Omega(x, y, z) = \iint_S \left[ \Omega(\xi, \eta, \zeta) \frac{\partial}{\partial \nu} - \frac{\partial \Omega(\xi, \eta, \zeta)}{\partial \nu} \right] \frac{1}{4\pi r} dS \quad (A1)$$

where

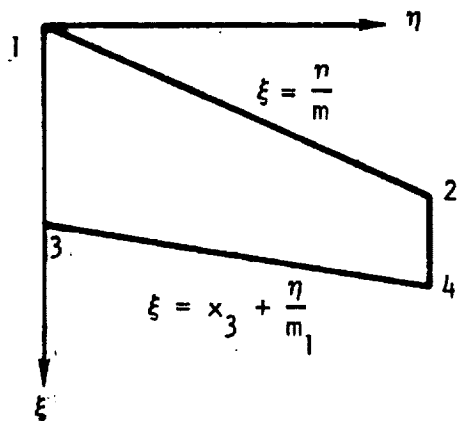
$$r = \sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - \zeta)^2}$$

and

$$\frac{\partial}{\partial \nu} = \left\{ \beta^2 n_\xi \frac{\partial}{\partial \xi} + n_\eta \frac{\partial}{\partial \eta} + n_\zeta \frac{\partial}{\partial \zeta} \right\}$$

$\vec{n} = (n_\xi, n_\eta, n_\zeta)$  is the unit normal on S

We will assume S is the surface  $\zeta = a\xi$  and



$$(x_1, y_1, z_1) = (0, 0, 0)$$

$$y_2 = y_4 = m_1 (x_4 - x_3) = m x_2$$

$$z_3 = a x_3 \quad z_2 = a x_2 \quad z_4 = a x_4$$

$$\therefore dS = \sqrt{1 + a^2} d\xi d\eta$$

Surface distribution of sources. - Let  $\Omega = \phi$  and as for supersonic flow assume that on S  $\sqrt{1 + a^2} \vec{\nu} = (-\beta^2 a, 0, 1)$

$$a) \quad \phi = \phi'$$

$$b) \quad \frac{\partial \phi}{\partial \nu} + \frac{\partial \phi}{\partial \nu'} = \frac{2}{\sqrt{1 + a^2}} (\bar{w} - \beta^2 a \bar{u}) = \text{const.} \quad \bar{w} = w - w^1$$

the primed quantities refer to  $z < ax$  or  $\xi < a\xi$

Therefore since on S  $\xi = a\xi$  we get

$$\phi(x, y, z) = \frac{\bar{w} - \beta^2 a \bar{u}}{2\pi} \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{d\xi}{\sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a\xi)^2}} d\eta \quad (A2)$$

This is the same as (9) for the supersonic case except for the limits, a factor of 1/2, and the fact that  $\beta^2$  is replaced by  $-\beta^2$ . Therefore the integral over  $\xi$  may be performed by using the equation below (9) for the supersonic case, and replacing  $\beta^2$  by  $-\beta^2$ .

$$\int \frac{d\xi}{\sqrt{(x-\xi)^2 + \beta^2(y-\eta)^2 + \beta^2(z-a\xi)^2}} = \quad (A3)$$

$$= \frac{-1}{2\sqrt{1+\beta^2 a^2}} \log \frac{\sqrt{(1+\beta^2 a^2)[(x-\xi)^2 + \beta^2(y-\eta)^2 + \beta^2(z-a\xi)^2]} + (x-\xi) + \beta^2 a(z-a\xi)}{\sqrt{(1+\beta^2 a^2)[(x-\xi)^2 + \beta^2(y-\eta)^2 + \beta^2(z-a\xi)^2]} - (x-\xi) - \beta^2 a(z-a\xi)}$$

A negative sign was included in the argument of the logarithm. This was possible since the derivative of  $\log(-1)$  is zero. Therefore

$$\phi(x, y, z) = \frac{\bar{w} - \beta^2 a \bar{u}}{4\pi \sqrt{1 + \beta^2 a^2}} \int_0^{y_2} \log \frac{\sqrt{(1+\beta^2 a^2)[(mx-\eta)^2 + \beta^2 m^2(\eta-y)^2 + \beta^2(mz-a\eta)^2]} + (mx-\eta) + \beta^2 a(mz-a\eta)}{\sqrt{(1+\beta^2 a^2)[(mx-\eta)^2 + \beta^2 m^2(\eta-y)^2 + \beta^2(mz-a\eta)^2]} - (mx-\eta) - \beta^2 a(mz-a\eta)} d\eta \quad (A4)$$

$$+ \frac{\bar{w} - \beta^2 a \bar{u}}{4\pi \sqrt{1 + \beta^2 a^2}} \int_0^{y_2} \log \frac{\sqrt{(1+\beta^2 a^2)[(m\hat{x}-\eta)^2 + \beta^2 m^2(\eta-y)^2 + \beta^2(m\hat{z}-a\eta)^2]} + (m\hat{x}-\eta) + \beta^2 a(m\hat{z}-a\eta)}{\sqrt{(1+\beta^2 a^2)[(m\hat{x}-\eta)^2 + \beta^2 m^2(\eta-y)^2 + \beta^2(m\hat{z}-a\eta)^2]} - (m\hat{x}-\eta) - \beta^2 a(m\hat{z}-a\eta)} d\eta$$

where  $\hat{x} = x - x_3$  and  $\hat{z} = z - z_3 = z - ax_3$

The second integral is the same as the first except that  $x$ ,  $z$  and  $m$  are replaced by  $x-x_3$ ,  $z-z_3$  and  $m_1$ . These integrals are the same as for supersonic flow, given by equation (21), if  $\beta^2$  is replaced by  $-\beta^2$  and a factor of 1/2 is added.

When the limit  $\eta = y_2$  is calculated it can be seen that it will be the same as when  $\eta = 0$  if  $x, y$  and  $z$  are replaced by  $x - x_2, y - y_2$  and  $z - z_2$ .

Therefore

$$\begin{aligned} \phi_0(x, y, z, m) = & \frac{\bar{w} - \beta^2 \bar{au}}{2\pi} \left\{ \frac{z - ax}{1 + \beta^2 a^2} \tan^{-1} \frac{m(z - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{y[(y - mx) + \beta^2 a(ay - mz)] + (z - ax)^2} \right. \\ & + \frac{y(1 + \beta^2 a^2) - m(x + \beta^2 az)}{2(1 + \beta^2 a^2) \sqrt{1 + \beta^2(a^2 + m^2)}} \log \frac{\sqrt{[1 + \beta^2(a^2 + m^2)][x^2 + \beta^2(y^2 + z^2)]} + [x + \beta^2(az + my)]}{\sqrt{[1 + \beta^2(a^2 + m^2)][x^2 + \beta^2(y^2 + z^2)]} - [x + \beta^2(az + my)]} \\ & \left. - \frac{y}{2\sqrt{1 + \beta^2 a^2}} \log \frac{\sqrt{(1 + \beta^2 a^2)[x^2 + \beta^2(y^2 + z^2)]} + (x + \beta^2 az)}{\sqrt{(1 + \beta^2 a^2)[x^2 + \beta^2(y^2 + z^2)]} - (x + \beta^2 az)} \right\} \quad (A5) \end{aligned}$$

Now we can write the result of (A4) as

$$\begin{aligned} \phi(x, y, z) = & \phi_0(x, y, z, m) - \phi_0(x - x_2, y - y_2, z - z_2, m) \\ & - \phi_0(x - x_3, y - y_3, z - z_3, m_1) + \phi_0(x - x_4, y - y_4, z - z_4, m_1) \end{aligned}$$

This means that (A5) may be interpreted as the velocity potential for an infinite panel with leading edge slope  $y = mx$ , and at angle of attack  $a$ .



Constant pressure surface. - In (A1) we will use  $u(x, y, z)$   
 $= \Omega(x, y, z)$  and

$$a) \frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} = 0 \quad \text{on } S$$

$$b) u - u' = \text{const} = \Delta u \quad \text{on } S$$

and where  $S$  is the same as previously defined.

Therefore (A1) becomes

$$u(x, y, z) = \frac{\Delta u}{4\pi} \int_S \int \frac{-\beta^2 a(x - \xi) + \beta^2(z - a\xi)}{[(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2]^{3/2}} d\xi d\eta$$

$$= -\frac{\Delta u}{4\pi} \frac{\partial}{\partial x} \int_S \int \frac{-\beta^2 a - \frac{(x - \xi)(z - a\xi)}{(y - \eta)^2 + (z - a\xi)^2}}{\sqrt{(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2}} d\xi d\eta \quad (A6)$$

Now we can write, using  $\Delta P/2q_\infty = -\Delta u$

$$\phi(x, y, z) = \int_{-\infty}^x u(x', y, z) dx$$

$$= -\frac{\Delta P}{8\pi q_\infty} \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{-\beta^2 a - \frac{(x - \xi)(z - a\xi)}{(y - \eta)^2 + (z - a\xi)^2}}{\sqrt{(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2}} d\xi d\eta$$

(A7)

$$\therefore -\frac{\Delta P}{8\pi q_\infty} \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{(z - a\xi)}{(y - \eta)^2 + (z - a\xi)^2} d\xi d\eta$$

The first term, except for the limits of integration and an additional factor of 1/2, is the same as the result for supersonic flow, which is given by the first equation of the section covering the evaluation of the integral over  $\xi$  for the constant pressure surface, with  $-\beta^2$  in place of  $\beta^2$ . Therefore, as in the case of a constant distribution of sources, we can use the supersonic result for subsonic flow if we substitute  $-\beta^2$  for  $\beta^2$ .

The second integral is new and does not occur for supersonic flow. This integral will now be evaluated.

$$\int \frac{(z - a\xi)d\xi}{(y - \eta)^2 + (z - a\xi)^2} = -\frac{1}{2a} \log [(y - \eta)^2 + (z - a\xi)^2]$$

(A8)

Therefore

$$\int_0^{y_2} \int_{\frac{y_1}{m_1} + x_3}^{\frac{y_1}{m_1} + x_3} \frac{(z - a\xi) d\xi}{(y - \eta)^2 + (z - a\xi)^2} d\eta = \frac{1}{2a} \int_0^{y_2} \log \left\{ \left[ m^2 (y - \eta)^2 + (mz - a\eta)^2 \right] \frac{1}{m^2} \right\} d\eta$$

$$\frac{1}{2a} \int_0^{y_2} \log \left\{ \left[ m_1^2 (y - \eta)^2 + \left( m_1 [z - z_3] - a\eta \right)^2 \right] \frac{1}{m_1^2} \right\} d\eta$$

note that:

$$mz - a\eta = (mz - ay) - a(\eta - y) = m(z - ax) + a(mx - y) - a(\eta - y) \quad (A9)$$

$$\begin{aligned} & \int \log \left\{ \frac{m^2 (\eta - y)^2 + (a\eta - mz)^2}{m^2} \right\} d\eta \\ &= \left[ \eta - \frac{m(my + az)}{a^2 + m^2} \right] \left[ \log \left\{ \frac{m^2 (\eta - y)^2 + (a\eta - mz)^2}{m^2} \right\} - 2 \right] \\ & \quad - \frac{2a(mz - ay)}{a^2 + m^2} \tan^{-1} \frac{m(mz - ay)}{(\eta - y)(a^2 + m^2) - a(mz - ay)} \end{aligned} \quad (A10)$$

When this is evaluated at  $\eta = 0$  we get from (A7), (A9), and (A10)

$$\phi_{00}(x, y, z, m) = \frac{\Delta P}{8\pi q_{\infty} a} \left\{ \frac{-m(az + my)}{(a^2 + m^2)} \left[ \frac{1}{2} \log(y^2 + z^2) - 1 \right] \right. \\ \left. - \frac{m(mz - ay)}{(a^2 + m^2)} \tan^{-1} \frac{(mz - ay)m}{-a(mz - ay) - (a^2 + m^2)y} \right\} \quad (A11)$$

Since all of the terms in (A10) may be written using only the variables,  $(mx - y)$ ,  $(z - ax)$ ,  $m$ , and  $(\eta - y)$ , the additional term which must be added for subsonic flow is,

$$\phi_1(x, y, z) = \phi_{00}(x, y, z, m) - \phi_{00}(x - x_2, y - y_2, z - z_2, m) \\ - \phi_{00}(x - x_3, y - y_3, z - z_3, m_1) \quad (A12) \\ + \phi_{00}(x - x_4, y - y_4, z - z_4, m_1)$$

Therefore using (51) we can write:

$$\begin{aligned} \phi(x, y, z) = & \frac{\Lambda p}{8\pi q_0 a} \left\{ \frac{m(az + my)}{a^2 + m^2} - \frac{1}{2} \log \frac{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2(y^2 + z^2)} - x} \right. \\ & + \frac{a(ay - mz)}{2(a^2 + m^2)} \sqrt{1 + \beta^2(a^2 + m^2)} \log \frac{\sqrt{[1 + \beta^2(a^2 + m^2)][x^2 + \beta^2(y^2 + z^2)]} + [x + \beta^2(az + my)]}{\sqrt{[1 + \beta^2(a^2 + m^2)][x^2 + \beta^2(y^2 + z^2)]} - [x + \beta^2(az + my)]} \\ & - \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)} \\ & + (z - ax) \tan^{-1} \frac{m(z - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{y[(y - mx) + \beta^2 a(ay - mz)] + (z - ax)^2} \\ & - y \sqrt{1 + \beta^2 a^2} \frac{1}{2} \log \frac{\sqrt{(1 + \beta^2 a^2) [x^2 + \beta^2(y^2 + z^2)]} + (x + \beta^2 az)}{\sqrt{(1 + \beta^2 a^2) [x^2 + \beta^2(y^2 + z^2)]} - (x + \beta^2 az)} \\ & - \frac{m(az + my)}{(a^2 + m^2)} \left[ \frac{1}{2} \log(y^2 + z^2) - 1 \right] \\ & \left. - \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay)m}{a(ay - mz) - y(a^2 + m^2)} \right\} \end{aligned}$$

So find the limiting form of  $\phi$  as  $a \rightarrow 0$   
 some of the terms must be combined. First,  
 consider the terms

$$\lim_{a \rightarrow 0} \frac{y}{2a} \left\{ \log \frac{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2(y^2 + z^2)} - x} \right. \\ \left. - \sqrt{1 + \beta^2 a^2} \log \frac{\sqrt{(1 + \beta^2 a^2)[x^2 + \beta^2(y^2 + z^2)]} + (x + \beta^2 a z)}{\sqrt{(1 + \beta^2 a^2)[x^2 + \beta^2(y^2 + z^2)]} - (x + \beta^2 a z)} \right\}$$

and since only terms up to the first power in  $a$  must  
 be considered inside the brackets, this becomes

$$\lim_{a \rightarrow 0} -\frac{y}{2a} \left\{ \log \frac{1 + \frac{\beta^2 a z}{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x}}{1 - \frac{\beta^2 a z}{\sqrt{x^2 + \beta^2(y^2 + z^2)} - x}} \right\} = -y \frac{z \sqrt{x^2 + \beta^2(y^2 + z^2)}}{y^2 + z^2}$$

since for  $\alpha \ll 1$   $\log(1 + \alpha) = \alpha$

Next consider

$$\lim_{a \rightarrow 0} z \left\{ \tan^{-1} \frac{m(z - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{y[(y - mx) + \beta^2 a(ay - mz)] + (z - ax)^2} \right. \\ \left. - \tan^{-1} \frac{(mz - ay) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)} \right\}$$

For small  $a$  we can expand

$$\begin{aligned} \tan^{-1} \frac{f(a)}{g(a)} &= \tan^{-1} \frac{f(0)}{g(0)} + a \frac{\partial}{\partial a} \tan^{-1} \frac{f(a)}{g(a)} \bigg|_{a=0} \\ &= \tan^{-1} \frac{f(0)}{g(0)} + a \frac{\frac{f'}{g} - \frac{f g'}{g^2}}{1 + \left(\frac{f}{g}\right)^2} \bigg|_{a=0} \\ &= \tan^{-1} \frac{f(0)}{g(0)} + \frac{a [f'g - fg']}{(g^2 + f^2)} \bigg|_{a=0} \end{aligned}$$

Now let

$$f = m(z - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

$$g = y[(y - mx) + \beta^2 a(ay - mz)] + (z - ax)^2$$

$$f(0) = mz \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

$$f'(0) = -mx \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

$$g(0) = y(y - mx) + z^2$$

$$g'(0) = -\beta^2 m y z - 2xz$$

$$\begin{aligned} f(0)^2 + g(0)^2 &= m^2 z^2 [x^2 + \beta^2(y^2 + z^2)] + y^2(mx - y)^2 + 2zy(y - mx) + z^4 \\ &= y^2 [(mx - y)^2 + (1 + \beta^2 m^2) z^2] + z^2 [m^2 x^2 + \beta^2 m^2 z^2 + y^2 - 2mxy + z^2] \\ &= (y^2 + z^2) [(mx - y)^2 + (1 + \beta^2 m^2) z^2] \end{aligned}$$

$$f'g - fg' = \sqrt{x^2 + \beta^2(y^2 + z^2)} \left\{ -[z^2 + y(y - mx)]mx + mz^2[\beta^2 my + x] \right\}$$

Now for the second term let

$$f = (mz - ay) \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

$$g = z(z - ax) - y(mx - y)$$

$$f(0) = mz \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

$$g(0) = z^2 - y(mx - y)$$

$$f'(0) = -y \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

$$g'(0) = -xz$$

$$f^2 + g^2 = (y^2 + z^2) [(mx - y)^2 + (1 + \beta^2 m^2) z^2]$$

$$f'g - fg' = \sqrt{x^2 + \beta^2(y^2 + z^2)} \left\{ -y[z^2 - y(mx - y)] + mz^2 x \right\}$$

Therefore

$$\lim_{a \rightarrow 0} z \left\{ \tan^{-1} \frac{m(z - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{y[(y - mx) + \beta^2 a(y - mx)] + (z - ax)^2} - \tan^{-1} \frac{(mz - ay) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)} \right\}$$

$$= z \left\{ \frac{(y - mx)[z^2 + y(y - mx)] + mz^2(\beta^2 my + x)}{(y^2 + z^2) [(mx - y)^2 + (1 + \beta^2 m^2) z^2]} \right\} \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

$$= z \left\{ \frac{y \sqrt{x^2 + \beta^2(y^2 + z^2)}}{(y^2 + z^2)} \right\}$$



To evaluate the additional term  $\phi_1$ , as  $a \rightarrow 0$ , we must combine points 1 and 3 and 2 and 4 from (A15)

$$\begin{aligned}
 & \lim_{a \rightarrow 0} \left\{ \phi_{00}(x, y, z, 0) - \phi_{00}(x-x_3, y, z-ax_3, m_1) \right\} \\
 &= \lim_{a \rightarrow 0} \left\{ -\frac{az+my}{m} \left[ \frac{1}{2} \log(y^2+z^2) - 1 \right] + \frac{a(z-ax_3) + m_1 y}{m_1} \left[ \frac{1}{2} \log(y^2+(z-ax_3)^2) - 1 \right] \right. \\
 &\quad \left. - \frac{mz-zy}{m} \tan^{-1} \frac{mz-ay}{(my+az)} + \frac{m_1(z-ax_3)-ay}{m_1} \tan^{-1} \frac{m_1(z-ax_3)-ay}{[m_1 y + a(z-ax_3)]} \right\} \frac{\Delta P}{8\pi q_\infty a} \\
 &= \frac{\Delta P}{8\pi q_\infty a} \left\{ \left[ -\frac{az}{m} + \frac{az}{m_1} \right] \left[ \frac{1}{2} \log(y^2+z^2) - 1 \right] - \frac{ax_3 zy}{y^2+z^2} \right. \\
 &\quad \left. + \left[ \frac{ay}{m} - \frac{a(y+m_1 x_3)}{m_1} \right] \tan^{-1} \frac{z}{-y} - \frac{az \left( 1 + \frac{z^2}{y^2} \right)}{m \left( 1 + \frac{z^2}{y^2} \right)} + \frac{az \left( 1 + \frac{z^2}{y^2} + \frac{m_1 x_3}{y} \right)}{m_1 \left( 1 + \frac{z^2}{y^2} \right)} \right\} \\
 &= \frac{\Delta P}{8\pi q_\infty} \left\{ \left[ -\frac{z}{m} + \frac{z}{m_1} \right] \left[ \frac{1}{2} \log(y^2+z^2) \right] - \left[ \frac{(mx-y)}{m} - \frac{[m_1(x-x_3)-y]}{m_1} \right] \tan^{-1} \frac{z}{-y} \right\}
 \end{aligned}$$

where we have used

$$\tan^{-1}(\alpha + \Delta\alpha) = \tan^{-1} \alpha + \frac{\Delta\alpha}{1+\alpha^2}$$

ORIGINAL PAGE IS  
OF POOR QUALITY

Since we can write

$$\tan^{-1} \frac{z}{-y} = \pi - \tan^{-1} \frac{-y}{z} = \pi + \tan^{-1} \frac{y}{z}$$

Therefore if we let

$$\hat{\phi}_{00}(x, y, z, m) = \frac{-\Delta P}{8\pi q_{\infty}} \left\{ \frac{(mx-y)}{m} \tan^{-1} \frac{y}{z} + \frac{z}{m} \frac{1}{2} \log(y^2+z^2) \right\} \quad (A15)$$

Then since  $\pi(mx-y)$  will cancel for a complete panel,

$$\begin{aligned} \lim_{a \rightarrow 0} \phi_1(x, y, z) &= \hat{\phi}_{00}(x, y, z, m) - \hat{\phi}_{00}(x-x_2, y-y_2, z, m) \\ &\quad - \hat{\phi}_{00}(x-x_3, y, z, m_1) + \hat{\phi}_{00}(x-x_4, y-y_4, z, m_1) \end{aligned}$$

And therefore for subsonic flow with  $a = 0$

$$\phi(x, y, z) = \frac{-\Delta P}{8\pi q_{\infty} m} \left\{ (mx-y) \tan^{-1} \frac{mz \sqrt{x^2 + \beta^2(y^2+z^2)}}{y(y-mx) + z^2} - \frac{z}{2} \log \frac{\sqrt{x^2 + \beta^2(y^2+z^2)} + x}{\sqrt{x^2 + \beta^2(y^2+z^2)} - x} \right.$$

$$\left. - y \frac{z \sqrt{x^2 + \beta^2(y^2+z^2)}}{y^2+z^2} + z \sqrt{1+\beta^2 m^2} \frac{1}{2} \log \frac{\sqrt{(1+\beta^2 m^2)[x^2 + \beta^2(y^2+z^2)]} + (x+\beta^2 my)}{\sqrt{(1+\beta^2 m^2)[x^2 + \beta^2(y^2+z^2)]} - (x+\beta^2 my)} \right.$$

$$\left. + z \frac{y \sqrt{x^2 + \beta^2(y^2+z^2)}}{y^2+z^2} + (mx-y) \tan^{-1} \frac{y}{z} + \frac{z}{2} \log(y^2+z^2) \right\}$$

In the form the above equation is written, derivatives with respect to  $x$ ,  $y$ , or  $z$  of all but the last two terms may be obtained by differentiating only the coefficients of each term. This was discussed previously when obtaining the velocity components for supersonic flow. If we use  $\beta^2 = 1 - M^2$  the above equation can also be used for  $M > 1$  if we multiply the expression by 2, take absolute values of the log arguments, and omit the last two terms. Therefore

$$\phi(x, y, z) = \frac{-\Delta P}{8\pi q_\infty m} k \left\{ (mx - y) \tan^{-1} \frac{mz \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y(y - mx) + z^2} - \frac{z}{2} \log \frac{\sqrt{x^2 + \beta^2 (y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2 (y^2 + z^2)} - x} \right. \\ + z \sqrt{1 + \beta^2 m^2} \frac{1}{2} \log \frac{\sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]} + (x + \beta^2 my)}{\sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]} - (x + \beta^2 my)} \quad y \frac{z \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y^2 + z^2} \\ \left. + z \frac{y \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y^2 + z^2} + k \left[ (mx - y) \tan^{-1} \frac{y}{z} + \frac{z}{2} \log (y^2 + z^2) \right] \right\}$$

where we take only the real part and

$$k = \begin{cases} 1 & M \leq 1 \\ 2 & M > 1 \end{cases}$$

$$k_0 = \begin{cases} 1 & M \leq 1 \\ 0 & M > 1 \end{cases}$$

$$\beta^2 = M^2 - 1$$

This is equivalent to the form derived by Woodward if it is noted that

$$\log \frac{x + \beta^2 my + \sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]}}{\beta \sqrt{(mx - y)^2 + (1 + \beta^2 m^2) z^2}}$$

$$= \frac{1}{2} \log \frac{\sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]} + (x + \beta^2 my)}{\sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]} - (x + \beta^2 my)}$$

and

$$\frac{1}{2} \log \frac{\sqrt{x^2 + \beta^2 (y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2 (y^2 + z^2)} - x} = \log \frac{x + \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{\beta \sqrt{y^2 + z^2}}$$

where  $\beta = \sqrt{|\beta^2|}$

If we set  $a = 0$  in (A10), the additional term which occurs in subsonic flow may be easily obtained. For  $a = 0$ ,

$$\begin{aligned}\phi_1(x, y, z) &= \frac{\Delta P}{8\pi q_\infty} \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{z}{(\eta - y)^2 + z^2} d\xi d\eta \\&= \int_0^{y_2} \int_{\frac{\eta}{m}}^{\frac{\eta}{m_1} + x_3} \frac{z}{(\eta - y)^2 + z^2} d\xi d\eta = -\frac{1}{m} \int_0^{y_2} \frac{\eta z}{(\eta - y)^2 + z^2} d\eta + \frac{1}{m_1} \int_0^{y_2} \frac{(\eta + m_1 x_3) z}{(\eta - y)^2 + z^2} d\eta \\&= -\frac{z}{m} \left\{ \frac{1}{2} \log [(\eta - y)^2 + z^2] - \frac{y}{z} \tan^{-1} \frac{z}{\eta - y} \right\} \bigg|_0^{y_2} \\&\quad + \frac{z}{m_1} \left\{ \frac{1}{2} \log [(\eta - y)^2 + z^2] - \frac{y + m_1 x_3}{z} \tan^{-1} \frac{z}{\eta - y} \right\} \bigg|_0^{y_2} \\&= \left\{ \frac{z}{2m} \log (y^2 + z^2) - \frac{y}{m} \tan^{-1} \frac{z}{-y} \right\}\end{aligned}$$

$$\phi_1(x, y, z) = \left\{ \frac{z}{2m_1} \log(y^2 + z^2) - \frac{y+m_1x_3}{m_1} \tan^{-1} \frac{z}{-y} \right\}$$

$$- \left\{ \frac{z}{2m} \log[(y-y_2)^2 + z^2] - \frac{y}{m} \tan^{-1} \frac{z}{(y-y_2)} \right\}$$

$$\left\{ \frac{z}{2m_1} \log[(y-y_2)^2 + z^2] - \frac{y+m_1x_3}{m_1} \tan^{-1} \frac{z}{(y-y_2)} \right\}$$

$$\frac{1}{m} \left\{ \frac{z}{2} \log(y^2 + z^2) + (mx-y) \tan^{-1} \frac{z}{-y} \right\}$$

$$- \frac{1}{m_1} \left\{ \frac{z}{2} \log(y^2 + z^2) + [m_1(x-x_3)-y] \tan^{-1} \frac{z}{-y} \right\}$$

$$- \frac{1}{m} \left\{ \frac{z}{2} \log[(y-y_2)^2 + z^2] + [m(x-x_2) - y-y_2] \tan^{-1} \frac{z}{(y-y_2)} \right\}$$

$$- \frac{1}{m_1} \left\{ \frac{z}{2} \log[(y-y_2)^2 + z^2] + [m_1(x-x_4)] - (y-y_4) \tan^{-1} \frac{z}{(y-y_4)} \right\}$$

The above is true since

$$y_2 = mx_2$$

$$y_4 = y_2 = m_1(x_4 - x_3)$$

ORIGINAL PAGE IS  
OF POOR QUALITY

Therefore, analogous to (A14) or (A18) when  $a = 0$

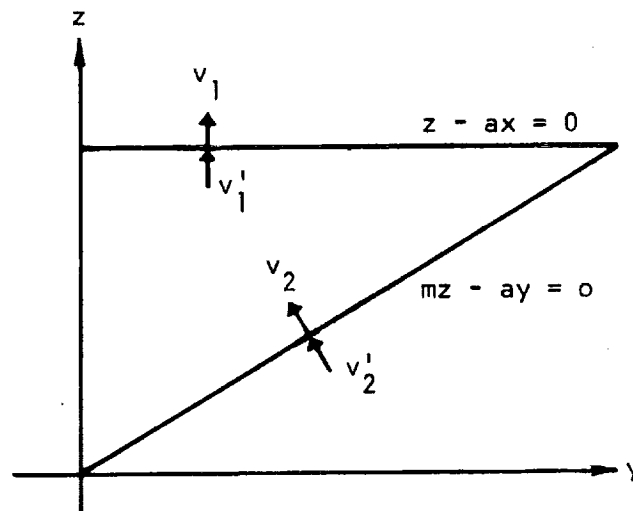
$$\hat{\phi}_{00}(x,y,z) = \frac{-\Delta P}{8\pi q_{\infty} m} \left\{ (mx-y) \tan^{-1} \frac{z}{-y} + \frac{z}{2} \log (y^2+z^2) \right\}$$

which is equivalent to [A18] since we can replace

$$\tan^{-1} \frac{z}{-y} \text{ by } \pi + \tan^{-1} \frac{y}{z} \text{ or by } \tan^{-1} \frac{y}{z}$$

since  $\pi(mx-y)$ , when the contributions from each corner are added, gives zero contribution.

Total Source Strength



$$v_1 - v_1' = K \frac{1}{a}$$

$$v_2 - v_2' = \frac{K}{a} \left\{ \frac{-a}{[a^2+m^2]^{1/2}} \frac{am}{a^2+m^2} + \frac{m}{[a^2+m^2]^{1/2}} \frac{-m^2}{(a^2+m^2)} \right\} = K \frac{m}{a (a^2+m^2)^{1/2}}$$

$$L_1 = y_0$$

$$L_2 = y_0 (a^2+m^2)^{1/2} \frac{1}{m}$$

$$(v_1 - v_1') L_1 + (v_2 - v_2') L_2 = y_0 K \left[ \frac{1}{a} - \frac{1}{a} \right] = 0$$

Point Source at  $\xi, \eta, \zeta$

$$r^2 = (x-\xi)^2 + \beta^2 [(y-\eta)^2 + (z-\zeta)^2]$$

$$\phi(x, y, z) = \frac{m}{4\pi r}$$

Doublet at  $\xi, \eta, \zeta$  in  $\vec{e}$  direction

$$\phi(x, y, z) = \vec{e} \cdot \nabla_{\beta} \left[ \frac{A}{4\pi r} \right] \quad \nabla_{\beta} = \beta^2 \frac{\partial}{\partial \xi} \vec{e}_{\xi} + \frac{\partial}{\partial \eta} \vec{e}_{\eta} + \frac{\partial}{\partial \zeta} \vec{e}_{\zeta}$$

If we integrate a row of these doublets in the direction of  $\vec{e}$  we get a source at one end and a sink at the other. Now let  $\vec{e}$  be



$$\frac{1}{\sqrt{1+a^2}} [-a \vec{e}_\xi + \vec{e}_\zeta]$$

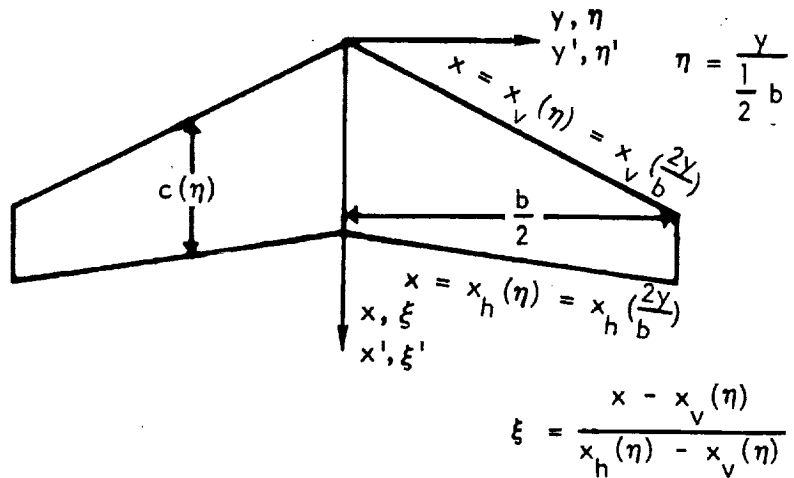
and integrate in the x direction.

$$\begin{aligned} \phi(x, y, z) &= \frac{A}{4\pi\sqrt{1+a^2}} \int_{\xi_0}^{\infty} \left[ \frac{\beta^2 (z-\zeta)}{[(x-\xi)^2 + \beta^2 [(y-\eta)^2 + (z-\zeta)^2]]^{3/2}} - \beta^2 a \frac{\partial}{\partial \xi} \frac{1}{\sqrt{(x-\xi)^2 + \beta^2 [(y-\eta)^2 + (z-\zeta)^2]}} \right] d\xi \\ &= \frac{+A}{4\pi\sqrt{1+a^2}} \frac{(z-\zeta)}{(y+\eta)^2 + (z-\zeta)^2} \left[ 1 + \frac{x-\xi_0}{\sqrt{(x-\xi_0)^2 + \beta^2 (y-\eta)^2 + \beta^2 (z-\zeta)^2}} \right] + \frac{A}{4\pi\sqrt{1+a^2}} \frac{\beta^2 a}{\sqrt{(x-\xi_0)^2 + \beta^2 (y-\eta)^2 + \beta^2 (z-\zeta)^2}} \end{aligned}$$

If we integrate this over S we obtain (A.7). Therefore (A.7) corresponds to a volume of doublets which means there will be a surface of sinks on one side and sources on the other.

Days of Rain (X)	Days of Sunshine (Y)
0	10
1	9
2	8
3	7
4	6
5	5
6	4
7	3
8	2
9	1
10	0

$$w(x,y,z) = \frac{\partial}{\partial z} \Phi(x,y,z)$$



$$\Phi(x, y, z) = V_{\infty} \iint_S \frac{z \, k(x', y')}{4\pi[(y-y')^2 + z^2]} \left\{ 1 + \frac{x-x'}{\sqrt{(x-x')^2 + (y-y')^2 + z^2}} \right\} dx' dy' \quad (1)$$



$$\eta' = \frac{y'}{\frac{1}{2}b}$$

$$y' = \frac{1}{2} \eta' b$$

$$\xi' = \frac{x' - x_V(\eta')}{x_h(\eta') - x_V(\eta')}$$

$$x' = \xi' c(\eta') + x_V(\eta')$$

$$dx' dy' = \begin{vmatrix} \frac{\partial x'}{\partial \xi'} & \frac{\partial x'}{\partial \eta'} \\ \frac{\partial y'}{\partial \xi'} & \frac{\partial y'}{\partial \eta'} \end{vmatrix} d\xi' d\eta' = \frac{1}{2} bc(\eta') d\xi' d\eta'$$

Therefore

$$\Phi(x, y, z) = \frac{V_{\infty} b}{8\pi} \int_{-1}^1 \int_0^1 \frac{z c(\eta') k[x'(\xi', \eta'), y'(\eta')]}{(y - \frac{1}{2} \eta' b)^2 + z^2} \left\{ 1 + \frac{x - x_V(\eta') - \xi' c(\eta')}{\sqrt{[x - x_V(\eta') - \xi' c(\eta')]^2 + (y - \frac{1}{2} \eta' b)^2 + z^2}} \right\} d\xi' d\eta' \quad (2)$$

Now if we define

$$\tilde{g}(x, y, z, \eta') = \frac{c(\eta')}{b} \int_0^1 k[x'(\xi', \eta'), y'(\eta')] \left\{ 1 + \frac{x - x_V(\eta') - \xi' c(\eta')}{\sqrt{[x - x_V(\eta') - \xi' c(\eta')]^2 + (y - \frac{1}{2} \eta' b)^2 + z^2}} \right\} d\xi'$$

Then

$$\Phi(x, y, z) = \frac{V_{\infty} b^2}{8\pi} \int_{-1}^1 \frac{z \tilde{g}(x, y, z, \eta')}{(y - \frac{1}{2} \eta' b)^2 + z^2} d\eta'$$

Near  $z = 0$  we can write

$$\Phi(x, y, z) = \Phi(x, y, 0) + z \frac{\partial}{\partial z} \Phi(x, y, z) \Big|_{z=0}$$

$$\tilde{g}(x, y, z, \eta') = \tilde{g}(x, y, 0, \eta') + O(z^2)$$

Therefore if we change variables and let (5)

$$\xi = \frac{x - x_V\left(\frac{2y}{b}\right)}{c\left(\frac{2y}{b}\right)}$$

$$\eta = \left(\frac{2y}{b}\right)$$

$$\xi = \left(\frac{2z}{b}\right)$$

$$\tilde{g}(x, y, 0, \eta') = g(\xi, \eta, \eta')$$

We can say

$$\frac{1}{V_\infty} \Phi(x, y, 0) = \varphi(\xi, \eta) = \frac{b}{4\pi} \lim_{\xi \rightarrow 0} \int_{-1}^1 \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \quad (6)$$

and

$$\lim_{z \rightarrow 0} \frac{-1}{V_\infty} \frac{\partial}{\partial z} \Phi(x, y, z) = \alpha(\xi, \eta) = -\frac{1}{2\pi} \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \quad (7)$$

Now integrate (6) by parts [Provided  $\frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')$  is continuous]

$$u = g(\xi, \eta, \eta')$$

$$dv = \frac{\xi d\eta'}{(\eta' - \eta)^2 + \xi^2}$$

$$du = \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')$$

$$v = -\tan^{-1} \frac{\xi}{\eta' - \eta}$$

$$\begin{aligned} \int_{-1}^1 \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' &= -g(\xi, \eta, \eta') \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right] \Bigg|_{-1}^1 \\ &+ \int_{-1}^1 \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right] \left[ \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \right] d\eta' \\ &= \int_{-1}^1 \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right] \left[ \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \right] d\eta' \end{aligned} \quad (8)$$

where the fact that  $g(\xi, \eta, \pm 1) = 0$ , because the loading goes to zero at  $\eta' = \pm 1$ , was used and since

$$\lim_{\xi \rightarrow 0} \tan^{-1} \frac{\xi}{\eta' - \eta} = \begin{cases} \pi \operatorname{sgn} \xi & \eta' - \eta < 0 \\ 0 & \eta' - \eta > 0 \end{cases} \quad (9)$$

$$\varphi(\xi, \eta) = \frac{b}{4} \operatorname{sgn} \xi \ g(\xi, \eta, \eta) \quad (10)$$

and

$$\begin{aligned} \alpha(\xi, \eta) &= \frac{1}{2\pi} \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right] \left[ \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \right] d\eta' \\ &= -\frac{1}{2\pi} \lim_{\xi \rightarrow 0} \int_{-1}^1 \frac{(\eta' - \eta) \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \quad (12) \end{aligned}$$

and from A1

$$= -\frac{1}{2\pi} \int_{-1}^1 \frac{\frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')}{\eta' - \eta} d\eta' = \frac{1}{2\pi} \left\{ \frac{2g(\xi, \eta, \eta)}{\epsilon} - \int_{-1}^1 \frac{g(\xi, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \right\}$$

If  $\frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')$  is not continuous we define

$$\alpha(\xi, \eta) = -\frac{1}{2\pi} \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' = -\frac{1}{2\pi} \int_{-1}^1 \frac{g(\xi, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \quad (13)$$

Now let

$$\frac{c(\eta')}{b} k[x'(\xi', \eta'), y'(\eta')] = - \sum_{n=0}^N f_n(\eta') h_n(\xi')$$

which means

$$g(\xi, \eta, \eta') = - \sum_{n=0}^N f_n(\eta') H_n(\xi, \eta, \eta')$$

where

$$h_n(\xi') = \frac{1}{\pi} \left[ \frac{1 - \xi'}{\xi'} \right]^{1/2} \left[ \frac{T_n(1 - 2\xi') + T_{n+1}(1 - 2\xi')}{1 - \xi'} \right]$$

or

$$\hat{h}_n(\Psi) = h_n \left[ \frac{1}{2} (1 - \cos \Psi) \right] = \frac{2}{\pi} \left[ \frac{\cos n\Psi + \cos (n+1)\Psi}{\sin \Psi} \right] = \frac{2}{\pi} \frac{\cos \left( n + \frac{1}{2} \right) \Psi}{\sin \frac{\Psi}{2}}$$

and

$$\begin{aligned} H_n(\xi, \eta, \eta') &= \int_0^1 h_n(\xi') \left\{ 1 + \frac{x - x_V(\eta') - \xi' c(\eta')}{\sqrt{\left[ x - x_V(\eta') - \xi' c(\eta') \right]^2 + \left[ y - \frac{1}{2} \eta' b \right]^2}} \right\} d\xi' \\ &= \int_0^1 h_n(\xi') \left\{ 1 + \frac{\frac{x - x_V(\eta')}{c(\eta')} - \xi'}{\sqrt{\left[ \frac{x - x_V(\eta')}{c(\eta')} - \xi' \right]^2 + \left[ \frac{(\eta - \eta') \frac{1}{2} b}{c(\eta')} \right]^2}} \right\} d\xi' \quad (15) \end{aligned}$$

and from (5)

$$= \int_0^1 h_n(\xi') \left\{ 1 + \frac{\frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi'}{\sqrt{\left[ \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi' \right]^2 + \left[ \frac{(\eta - \eta') \frac{1}{2} b}{c(\eta')} \right]^2}} \right\} d\xi'$$

Now, from (7), for any  $g(\xi, \eta, \eta')$  differentiable or not we can write

$$\begin{aligned} \alpha(\xi, \eta) &= -\frac{1}{2\pi} \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \\ &= \frac{1}{2\pi} \sum_{n=0}^N \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi f_n(\eta') [H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta')]}{(\eta - \eta')^2 + \xi^2} d\eta' \end{aligned} \quad (16)$$

where we define

$$H_n(\xi, \eta, \eta') \equiv H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta') \quad (17)$$

or

$$\alpha(\xi, \eta) = \frac{1}{2\pi} \sum_{n=0}^N \left\{ H_n(\xi, \eta, \eta) \int_{-1}^1 \frac{\frac{\partial}{\partial \eta'} f_n(\eta')}{(\eta - \eta')} d\eta' + \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi f_n(\eta') K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \right\} \quad (18)$$



where if  $\frac{\partial}{\partial \eta}$ ,  $K_n(\xi, \eta, \eta')$  is continuous, using (11) and (12)

$$\begin{aligned} \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi f_n(\eta') K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' &= \int_{-1}^1 \frac{\frac{\partial}{\partial \eta'} [f_n(\eta') K_n(\xi, \eta, \eta')]}{(\eta - \eta')} d\eta' \\ &= \int_{-1}^1 \frac{f_n(\eta') K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \end{aligned} \quad (19)$$

since  $f_n(+1) = 0$

Now referring to (15), let

$$\tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right\} d\xi'$$

Then, from (B5) for small  $(p - p_0)$  and small  $q$   $p_0 \neq 0, 1$

$$\tilde{H}_n(p, q) = \tilde{H}_n(p_0, 0) + 2 h_n(p_0) (p - p_0) - h'_n(p_0) q^2 \ln |q| \quad (20)$$

Now set [See (15)]

$$\begin{aligned} p &= \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} \\ p_0 &= \xi \\ q &= \frac{1/2 b(\eta' - \eta)}{c(\eta')} \end{aligned} \quad (21)$$

when

$$q = 0$$

$$\eta' = \eta$$

$$p = p_0$$

Then for  $(\eta' - \eta) \ll 1$

$$p - p_0 = \frac{\partial}{\partial \eta'} \left[ \frac{x_v(\eta) - \xi c(\eta) - x_v(\eta')}{c(\eta')} \right]_{\eta'=\eta} (\eta' - \eta) \equiv - \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)} \tan \phi \quad (21a)$$

where we have defined  $\phi(\xi, \eta)$  as the sweep of the constant percent chord lines

$$\tan \phi(\xi, \eta) \equiv - \frac{c(\eta)}{\frac{1}{2} b} \frac{\partial}{\partial \eta'} \left[ \frac{x_v(\eta) - \xi c(\eta) - x_v(\eta')}{c(\eta')} \right]_{\eta'=\eta} \quad (22)$$

Therefore for  $\xi \neq 0, 1$

$$\left| \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta')} \right| \ll 1$$

$$H_n(\xi, \eta, \eta') = H_n(\xi, \eta, \eta) - 2 \left[ \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)} \right] h_n(\xi) \tan \phi$$

$$- h'_n(\xi) \left[ \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)} \right]^2 \ln \left| \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)} \right|$$

Then for small  $(\eta' - \eta)$  and  $\xi \neq 0, 1$

$$K_n(\xi, \eta, \eta') = -2 \left[ \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \right] h_n(\xi) \tan \phi - h'_n(\xi) \left[ \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \right] \ln \left[ \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \right] \quad (23)$$

For  $\xi = 0$ , from (C2)

$$\tilde{H}_n(p, q) = \tilde{H}_n(0, 0) + \frac{8 \cos^2 \tilde{\phi}}{\pi} \left\{ I_1(\tilde{\phi}) (p^2 + q^2)^{1/4} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) (p^2 + q^2)^{3/4} \right\}$$

where  $I_1(\tilde{\phi})$  and  $I_2(\tilde{\phi})$  are defined in (C3) and (C4) and

$$\tilde{\phi} = \tan^{-1} \frac{-p}{q}$$

At the leading edge  $p_0 = 0$  and, from (21a)

$$-p = \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \tan \phi_{L.E.} = q \tan \phi_{L.E.}$$

and therefore

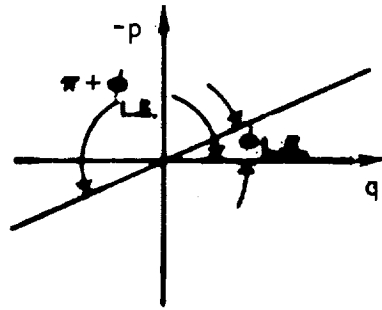
$$p^2 + q^2 = \frac{\frac{1}{2}b}{c(\eta)} \left[ 1 + \tan^2 \phi_{L.E.} \right] (\eta' - \eta)^2 = \frac{1}{\cos^2 \phi_{L.E.}} \left[ \frac{\frac{1}{2}b}{c(\eta)} \right]^2 (\eta' - \eta)^2$$

and

$$\tilde{\phi} = \tan^{-1} [\tan \phi_{\text{L.E.}}] = \phi_{\text{L.E.}}$$

$$q > 0 \text{ or } (\eta' - \eta) > 0$$

$$= \pi + \phi_{\text{L.E.}} \quad (\eta' - \eta) < 0$$



Therefore for  $(\eta' - \eta) \ll 1$

$$K_n(0, \eta, \eta') = \varphi_1(\phi) \left[ \frac{\frac{1}{2} b |\eta' - \eta|}{c(\eta)} \right]^{1/2} - \frac{1}{2} (1 + 4n + 4n^2) \varphi_2(\phi) \left[ \frac{\frac{1}{2} b |\eta' - \eta|}{c(\eta)} \right]^{3/2} \quad (24)$$

where

$$\varphi_1(\phi) = \frac{8 |\cos \phi|^{3/2}}{\pi} I_1(\phi)$$

$$\varphi_2(\phi) = \frac{8 |\cos \phi|^{1/2}}{3\pi} I_2(\phi)$$

and

$$\phi = \phi_{\text{L.E.}} \quad (\eta' - \eta) > 0$$

$$\phi = \pi + \phi_{\text{L.E.}} \quad (\eta' - \eta) < 0$$

Using a similar analysis for the trailing edge,  $\xi = 1$ , we get

$$K_n(1, \eta, \eta') = (-1)^n \varphi_2(\phi) (1 + 2n) \left[ \frac{\frac{1}{2} b |\eta' - \eta|}{c(\eta)} \right]^{5/2} (\eta' - \eta) \ll 1 \quad (25)$$

$$\phi = \phi_{T.E.} + \pi \quad (\eta' - \eta) > 0$$

$$\phi = \phi_{T.E.} \quad (\eta' - \eta) < 0$$

For  $\xi = 0$

$$\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')$$

is not continuous at  $\eta' = \eta$  due to the term involving  $(\eta' - \eta)^{1/2}$

Therefore, referring to (18), we must evaluate

$$\lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi f_n(\eta') K_n(0, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta'$$

Since the only discontinuity in

$$\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')$$

occurs at  $\eta' = \eta$  we can write, using (19)

$$\begin{aligned}
\lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{\xi f_n(\eta') K_n(0, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \\
= \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{\eta-\delta}^{\eta+\delta} \frac{\xi f_n(\eta') K_n(0, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \\
+ \int_{-1}^{\eta-\delta} \frac{f_n(\eta') K_n(0, \eta, \eta')}{(\eta' - \eta)^2} d\eta' + \int_{\eta+\delta}^1 \frac{f_n(\eta') K_n(0, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \quad (26)
\end{aligned}$$

The only term in  $K_n(0, \eta, \eta')$  which causes trouble is the term

$$\varphi_1(\phi_{L.E.}) \left[ \frac{\frac{1}{2} b(\eta' - \eta)}{c(\eta)} \right]^{1/2}$$

and if  $\delta$  is small enough we can approximate  $f_n(\eta')$  by  $f_n(\eta)$

Therefore we must evaluate the following expressions

$$\begin{aligned}
\int_0^\delta \frac{\xi \sqrt{s}}{s^2 + \xi^2} ds &= \sqrt{\xi} \int_0^{\delta/\xi} \frac{\sqrt{s}}{s^2 + 1} ds = \sqrt{\xi} \left\{ \int_0^\infty \frac{\sqrt{s}}{s^2 + 1} ds - \int_{\delta/\xi}^\infty \frac{\sqrt{s}}{s^2 + 1} ds \right\} = \sqrt{\xi} \left\{ \frac{\pi}{\sqrt{2}} - \int_{\delta/\xi}^\infty \frac{\sqrt{s}}{s^2 + 1} ds \right\} \\
&= \sqrt{\xi} \left\{ \frac{\pi}{\sqrt{2}} - \int_{\delta/\xi}^\infty s^{-3/2} \left[ 1 - \frac{1}{s^2} + \frac{1}{s^4} - \frac{1}{s^6} \dots \right] ds \right\} = \sqrt{\xi} \left\{ \frac{\pi}{\sqrt{2}} - 2 \left[ \frac{\xi}{\delta} \right]^{1/2} + \frac{2}{5} \left[ \frac{\xi}{\delta} \right]^{5/2} - \frac{2}{9} \left[ \frac{\xi}{\delta} \right]^{9/2} \dots \right\}
\end{aligned}$$

and,

$$2 \frac{\partial}{\partial \xi} \int_0^{\delta} \frac{\xi \sqrt{S}}{S^2 + \xi^2} dS = \left\{ \frac{\pi}{\sqrt{2\xi}} + \frac{2}{\sqrt{\delta}} \left[ -2 + \frac{\delta}{5} \frac{\xi^2}{\delta^2} - \frac{10}{9} \frac{\xi^4}{\delta^4} \dots \right] \right\}$$

$$\therefore \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_0^{\delta} \frac{\xi \sqrt{S}}{S^2 + \xi^2} dS = \infty \quad (26a)$$

Subappendices E and F consider the case where  $g(\xi, \eta, \eta')$  is considered to be a function of  $\xi$ , as in (3), but the expression (26a) is still infinite.

If  $\tan \phi$  is not continuous at some  $\eta = \eta'$  then, from (23) and (18) or (26), it can be seen that the integral of

$$\frac{K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2}$$

will give a logarithmic infinity. If  $\tan \phi$  is continuous the first term of (23) will give an odd function of  $(\eta' - \eta)$  which will give a finite value in the Cauchy principle value sense. Therefore kinks or cranks in the planform cannot be permitted because discontinuities in the angles of the constant percent chord lines will result. In fact subappendix G of this appendix shows that a logarithmic infinity occurs if the leading edge kink is considered as the limit of a hyperbola whose radius of curvature goes to zero. If the term in (18) involving  $\tan \phi$  is neglected for small  $(\eta' - \eta)$ , this has the effect of rounding the constant percent chord lines.

Now let  $\eta' = \cos \theta'$  and

$$f_n(\eta') = \sum_{m=1}^M f_{nm} \bar{S}_m(\theta) \quad (27)$$

where

$$\bar{S}_m(\theta') = \frac{2}{M+1} \sum_{\mu=1}^M \sin \mu \theta_m \sin \mu \theta' = \begin{cases} 1 & \theta' = \theta_m \\ \frac{1}{M+1} \frac{(-1)^{m+1} \sin \theta_m \sin(M+1) \theta'}{\cos \theta' - \cos \theta_m} & \theta' \neq \theta_m \end{cases}$$

$$\theta_m = \frac{m \pi}{M+1} \quad \text{discrete points}$$

Now the first part of (18) may be performed in closed form

$$\frac{1}{2\pi} \int_{-1}^1 \frac{\frac{\partial f_n(\eta')}{\partial \eta'}}{\partial \eta_\nu - \eta'} d\eta' = \sum_{m=1}^M f_{nm} b_{\nu m} \quad (28)$$

where

$$b_{\nu m} = \begin{cases} \frac{M+1}{4} \frac{1}{\sin \theta_\nu} & \theta_\nu = \theta_m \\ \frac{1}{M+1} \frac{1 - (-1)^{\nu+m}}{2} \frac{\sin \theta_m}{[\cos \theta_\nu - \cos \theta_m]^2} & \theta_\nu \neq \theta_m \end{cases} \quad (29)$$



Referring to (18), (19), (26), and (27) we define

$$A_{np\nu\nu} \equiv \frac{1}{2\pi} \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{\eta_\nu - \delta}^{\eta_\nu + \delta} \frac{\xi K_n(\xi p, \eta_\nu, \eta')}{(\eta_\nu - \eta')^2 + \xi^2} d\eta' \quad p = 0$$

$$= \frac{i}{2\pi} \int_{\eta_\nu - \delta}^{\eta_\nu + \delta} \frac{K_n(\xi p, \eta_\nu, \eta')}{(\eta_\nu - \eta')^2} d\eta' \quad p \neq 0 \quad (30)$$

$$B_{np\nu m} = \frac{1}{2\pi} \int_{\eta_\nu + \delta}^1 \frac{K_n(\xi p, \eta_\nu, \eta') \bar{S}_m[\cos^{-1} \eta']}{(\eta_\nu - \eta')^2} d\eta' \quad (31)$$

and letting

$$\eta' = \frac{1 + \eta_\nu + \delta}{2} + \frac{1 - \eta_\nu - \delta}{2} \cos \theta = \eta_1(\theta)$$

$$= \frac{1 - \eta_\nu - \delta}{4} \int_0^\pi \frac{K_n[\xi p, \eta_\nu, \eta_1(\theta)] \bar{S}_m[\cos^{-1} \eta_1(\theta)]}{[\eta_\nu - \eta_1(\theta)]^2} \sin \theta d\theta$$

$$= \frac{1 - \eta_\nu - \delta}{4(M_1 + 1)} \sum_{i=1}^{M_1} \frac{K_n[\xi p, \eta_\nu, \eta_1(\theta_i)] \bar{S}_m[\cos^{-1} \eta_1(\theta_i)]}{[\eta_\nu - \eta_1(\theta_i)]^2} \sin \theta_i$$

where

$$\theta_i = \frac{i\pi}{M_1 + 1} \quad i = 1, 2, \dots, M_1$$

$$C_{np\nu m} \equiv \frac{1}{2\pi} \int_{-1}^{\eta_\nu - \delta} \frac{K_n(\xi_p, \eta_\nu, \eta') \bar{S}_m[\cos^{-1} \eta']}{(\eta_\nu - \eta')^2} d\eta'$$

and letting

$$\eta' = \eta_2(\theta) = \frac{-1 + \eta_\nu - \delta}{2} + \frac{1 + \eta_\nu - \delta}{2} \cos \theta$$

$$C_{np\nu m} = \frac{1 - \eta_\nu - \delta}{4\pi} \int_0^\pi \frac{K_n[\xi_p, \eta_\nu, \eta_2(\theta)] \bar{S}_m(\theta) \sin \theta}{[\eta_\nu - \eta_2(\theta)]^2} d\theta$$

$$= \frac{1 - \eta_\nu - \delta}{4(M_2 + 1)} \sum_{i=1}^{M_2} \frac{K_n[\xi_p, \eta_\nu, \eta_2(\theta_i)] \bar{S}_m(\theta_i) \sin \theta_i}{[\eta_\nu - \eta_2(\theta_i)]^2} \quad \theta_i = \frac{i\pi}{M_2 + 1} \quad (32)$$

Therefore referring to (18), (19), (26), (28), (29), (30), (31), (32)

$$\alpha(\xi_p, \eta_\nu) = \sum_{n=0}^N \sum_{m=1}^M f_{nm} [H_{np\nu\nu} b_{\nu m} + \delta_{\nu m} A_{np\nu\nu} + B_{np\nu m} + C_{np\nu m}]$$

which is a set of linear equations to be solved for  $f_{nm}$

$$p = 0, 1, \dots, N$$

$$M \times (N+1)$$

$$\nu = 1, 2, \dots, M$$

$$H_{np\nu\nu} = H_n(\xi_p, \eta_\nu, \eta_\nu)$$

all of the  $H_n$ 's and  $K_n$ 's are computed numerically

# Subappendix A

Consider

$$\begin{aligned}
 & \lim_{z \rightarrow 0} \int_{-1}^1 \frac{(\eta' - \eta) F(\eta')}{(\eta' - \eta)^2 + z^2} d\eta' \\
 &= \lim_{z \rightarrow 0} \left\{ F(\eta) \int_{-1}^1 \frac{(\eta' - \eta)}{(\eta' - \eta)^2 + z^2} d\eta' + \int_{-1}^1 \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} d\eta' \right\} \\
 &= \lim_{z \rightarrow 0} \left\{ \frac{1}{2} F(\eta) \log [(\eta' - \eta)^2 + z^2] \right\}_{-1}^1 + \int_{-1}^1 \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} d\eta' \\
 &= F(\eta) \left\{ \log \frac{1-\eta}{\epsilon} + \log \frac{\epsilon}{1+\eta} \right\} + \lim_{z \rightarrow 0} \int_{-1}^1 \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} d\eta'
 \end{aligned}$$

If  $F(\eta')$  is differentiable at  $\eta' = \eta$  then  $F(\eta') - F(\eta) = O(\eta' - \eta)$  and we can write

$$\begin{aligned}
 & \lim_{z \rightarrow 0} \int_{-1}^1 \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} d\eta' \\
 &= \lim_{\epsilon \rightarrow 0} \left\{ \int_{\eta+\epsilon}^{1'} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)} d\eta' + \int_{-1}^{\eta-\epsilon} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)} d\eta' \right\}
 \end{aligned}$$

(A1)

Therefore

$$\begin{aligned} \lim_{z \rightarrow 0} \int_{-1}^1 \frac{(\eta' - \eta)F(\eta')}{(\eta' - \eta)^2 + z^2} d\eta' &= \int_{-1}^1 \frac{F(\eta')}{(\eta' - \eta)} d\eta' \\ &= \lim_{\epsilon \rightarrow 0} \left\{ \int_{\eta + \epsilon}^1 \frac{F(\eta')}{(\eta' - \eta)} d\eta' + \int_{-1}^{\eta - \epsilon} \frac{F(\eta')}{(\eta' - \eta)} d\eta' \right\} \end{aligned}$$

# Subappendix B

$$\begin{aligned}\tilde{H}_n(p, q) &= \tilde{H}_n(p_0, 0) + \oint \nabla \tilde{H}_n(p, q) \cdot d\vec{l} \\ &= \tilde{H}_n(p_0, 0) + \oint \left\{ \frac{\partial \tilde{H}_n(p, q)}{\partial p} dp + \frac{\partial \tilde{H}_n(p, q)}{\partial q} dq \right\}\end{aligned}\quad (B1)$$

where C is a contour integral in the  $p, q$  plane from  $(p_0, 0)$  to  $(p, q)$

$$\tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right\} d\xi'$$

$$\frac{\partial \tilde{H}_n(p, q)}{\partial p} = \int_0^1 k_n(\xi') \frac{q^2}{[(p - \xi')^2 + q^2]^{3/2}} d\xi'$$

if

$$p \neq 0, 1$$

we can write

$$h_n(\xi') = h_n(p) - h'_n(p)(p - \xi') + \frac{1}{2!} h''_n(p)(p - \xi')^2 + r_n(\xi')(p - \xi')^3 \quad (B2)$$

and since

$$\int \frac{x}{r^3} = -\frac{1}{r} \quad \int \frac{x^2}{r^3} dx = \frac{xr}{2} - \frac{a^2}{2} \log(x+r)$$

and

$$\int \frac{a^2}{r^3} = \frac{x}{r}$$

where  $r = \sqrt{x^2 + a^2}$

$$\frac{\partial}{\partial} \tilde{H}_n(p, q) = -h_n(p) \left. \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right|_{\xi'=0}^{\xi'=1} + O(q^2)$$

or

$$\lim_{q \rightarrow 0} \frac{\partial}{\partial p} \tilde{H}_n(p, q) = 2 h_n(p) \quad p \neq 0, 1 \quad (B3)$$

$$\frac{\partial}{\partial q} \tilde{H}_n(p, q) = -q \int_0^1 h_n(\xi') \frac{(p - \xi')}{[(p - \xi')^2 + q^2]^{3/2}} d\xi'$$

now write  $(p \neq 0, 1) \quad h_n(\xi') = h_n(p) - h'_n(p)(p - \xi') + r_n(\xi')(p - \xi')^2$

Then as  $q \rightarrow 0$

$$q \int_0^1 \frac{(p - \xi')}{[(p - \xi')^2 + q^2]^{3/2}} d\xi' = \frac{q}{[(p - \xi')^2 + q^2]^{3/2}} \Big|_0^1 = O(q)$$

and

$$q \int_0^1 \frac{r_n(\xi')(p - \xi')^3}{[(p - \xi')^2 + q^2]^{3/2}} d\xi' = O(q)$$

but

$$q \int_0^1 \frac{(p - \xi')^2}{[(p - \xi')^2 + q^2]^{3/2}} d\xi' = q \left\{ \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} + \log \left[ (\xi' - p) + \sqrt{(p - \xi')^2 + q^2} \right] \right\} \Big|_0^1$$

$$= q \left\{ -\frac{(1-p)}{\sqrt{(1-p)^2 + q^2}} - \frac{p}{\sqrt{p^2 + q^2}} + \log \left[ (1-p) + \sqrt{(1-p)^2 + q^2} \right] - \log \left[ -p + \sqrt{p^2 + q^2} \right] \right\}$$

$$= O(q) - q \log \left[ -p + \sqrt{p^2 + q^2} \right]$$

$$= O(q) - q \log \left[ -1 + \sqrt{1 + \frac{p^2}{q^2}} \right]$$

$$= O(q) - 2q \log |q|$$

Therefore as  $q \rightarrow 0$

$$\frac{\partial}{\partial q} \tilde{H}_n(p, q) = -2 h'_n(p) q \ln |q| + o(q) \quad (B4)$$

and since  $h_n(p)$  and  $h'_n(p)$  are slowly varying for  $p \neq 0, 1$  referring to (B1) and using (B3) and (B4) for  $(p-p_0) \ll 1$  and  $q \ll 1$

$$\oint \frac{\partial}{\partial p} \tilde{H}_n(p, q) dp = \int_{p_0}^p \frac{\partial}{\partial p} \tilde{H}_n(p, q) dp = 2 h_n(p_0) (p - p_0)$$

and

$$\oint \frac{\partial}{\partial q} \tilde{H}_n(p, q) dq = -2 h'_n(p_0) \int_0^q q \ln q dq = -h'_n(p_0) q^2 \ln q$$

or to lowest order in  $p - p_0$  and  $q$  (for  $p_0 \neq 0, 1$ )

$$\tilde{H}_n(p, q) = \tilde{H}_n(p_0, 0) + 2 h_n(p_0) (p - p_0) - h'_n(p_0) q^2 \ln q \quad (B5)$$



# Subappendix C

$$\tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right\} d\xi'$$

Leading Edge      Let

$$p = -\delta^2 \sin \tilde{\phi}$$

$$q = \delta^2 \cos \tilde{\phi}$$

or

$$\delta^2 = \sqrt{p^2 + q^2}$$

$$\tilde{\phi} = \tan^{-1} \frac{-p}{q}$$

and introduce a change of variables

$$\xi' = \sin^2 \sigma$$

$$d\xi' = 2 \sin \sigma \cos \sigma d\sigma$$

$$h_n(\sin^2 \sigma) = h_n \left[ \frac{1}{2} (1 - \cos 2\sigma) \right] = \hat{h}_n(2\sigma) = \frac{2}{\pi} \frac{\cos 2n\sigma + \cos 2(n+1)\sigma}{\sin 2\sigma}$$

$$= \frac{2}{\pi} \frac{\cos ([2n+1] - 1)\sigma + \cos ([2n+1] + 1)\sigma}{2 \sin \sigma \cos \sigma} = \frac{2}{\pi} \frac{\cos (2n+1)\sigma}{\sin \sigma}$$

$$\left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2}} \right\} = 1 + \frac{-(\delta^2 \sin \tilde{\phi} + \sin^2 \sigma)}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \tilde{\phi} \sin^2 \sigma + \delta^4}}$$

and

$$\tilde{H}_n(p, q) = \hat{H}_n(\delta, \tilde{\phi})$$

$$= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \left\{ 1 + \frac{-(\delta^2 \sin \tilde{\phi} + \sin^2 \sigma)}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \tilde{\phi} \sin^2 \sigma + \delta^4}} \right\} \cos (2n+1)\sigma \cos \sigma d\sigma$$

$$\begin{aligned} \frac{\partial \hat{H}_n}{\partial \delta} &= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{-2 \sin \tilde{\phi} \cos [(2n+1)\sigma] \cos \sigma d\sigma}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \tilde{\phi} \sin^2 \sigma + \delta^4}} d\sigma \\ &+ \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{2\delta [ + \sin \tilde{\phi} \sin^2 \sigma + \delta^2 ] (\delta^2 \sin \tilde{\phi} + \sin^2 \sigma) \cos [(2n+1)\sigma] \cos \sigma d\sigma}{[\sin^4 \sigma + 2 \delta^2 \sin \tilde{\phi} \sin^2 \sigma + \delta^4]^{3/2}} \\ &= \frac{8 \delta^3 \cos^2 \tilde{\phi}}{\pi} \int_0^{\frac{\pi}{2}} \frac{\cos [(2n+1)\sigma] \cos \sigma \sin^2 \sigma}{[\sin^4 \sigma + 2 \delta^2 \sin \tilde{\phi} \sin^2 \sigma + \delta^4]^{3/2}} d\sigma \end{aligned}$$

as  $\delta \rightarrow 0$  the only portion of the integrand which is important is in the region of  $\sigma = 0$

C-4

Therefore let  $x = \sin \sigma$   $dx = \cos \sigma d\sigma$  and expand

$$\begin{aligned}\cos [(2n+1)\sigma] &= \cos \left[ (2n+1) \left( x - \frac{x^3}{6} \right) \right] \\ &= 1 - \frac{1}{2} (2n+1)^2 x^2 = 1 - \frac{x^2}{2} (1 + 4n + 4n^2) + O(x^4)\end{aligned}$$

Therefore to first order in  $x^2$

$$\begin{aligned}\frac{\partial}{\partial \delta} \hat{H}_n(\delta, \tilde{\phi}) &= \frac{8 \delta^3 \cos^2 \tilde{\phi}}{\pi} \int_0^1 \frac{\left[ 1 - \frac{x^2}{2} (1 + 4n + 4n^2) \right] x^2}{\left[ x^4 + 2\delta^2 x^2 \sin \tilde{\phi} + \delta^4 \right]^{3/2}} dx \\ &= \frac{8 \cos^2 \tilde{\phi}}{\pi} \left\{ \int_0^{\frac{1}{\delta}} \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} - \frac{\delta^2}{2} \int_0^{\frac{1}{\delta}} \frac{(1 + 4n + 4n^2) x^4 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} \right\}\end{aligned}$$

Since

$$\begin{aligned}\int_0^{\frac{1}{\delta}} \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} &= \int_0^{\infty} \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} - \int_{\frac{1}{\delta}}^{\infty} \frac{dx}{x^4} \\ &= \int_0^{\infty} \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} + O(\delta^3)\end{aligned}$$

Near  $\delta = 0$  we can say

$$\frac{\partial}{\partial \delta} \hat{H}_n(\delta, \tilde{\phi}) = \frac{8 \cos^2 \tilde{\phi}}{\pi} \int_0^\infty \frac{x^2 \left[ 1 - \frac{\delta^2 x^2}{2} (1 + 4n + 4n^2) \right]}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}} dx + O(\delta^3)$$

integrating from 0 to  $\delta$  we get

$$\hat{H}_n(\delta, \tilde{\phi}) = \hat{H}_n(0, \tilde{\phi}) + \frac{8 \cos^2 \tilde{\phi}}{\pi} \left\{ I_1(\tilde{\phi}) \delta - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) \delta^3 \right\} \quad (C1)$$

or

$$\tilde{H}_n(p, q) = \tilde{H}_n(0, 0)$$

$$+ \frac{8 \cos^2 \tilde{\phi}}{\pi} \left\{ I_1(\tilde{\phi}) (p^2 + q^2)^{1/4} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) (p^2 + q^2)^{3/4} \right\} \quad (C2)$$

where

$$\tilde{\phi} = \tan^{-1} \frac{(-p)}{q}$$

and where

$$I_1(\tilde{\phi}) = \int_0^\infty \frac{x^2 dx}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}} = \frac{1}{\cos \tilde{\phi}} \frac{d}{d\tilde{\phi}} \int_0^\infty \frac{dx}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}} \quad (C3)$$

$$\begin{aligned}
 I_2(\tilde{\phi}) &= \int_0^{\infty} \frac{x^4 dx}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}} \\
 &= -\sin \tilde{\phi} I_1(\tilde{\phi}) + \int_0^{\infty} \frac{(x^4 + x^2 \sin \tilde{\phi})}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}} dx \quad (C4)
 \end{aligned}$$

$$u = x$$

$$du = dx$$

$$dv = \frac{(x^3 + x \sin \tilde{\phi}) dx}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}}$$

$$v = \frac{-\frac{1}{2}}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}}$$

Therefore

$$I_2(\tilde{\phi}) = -\sin \tilde{\phi} I_1(\tilde{\phi}) + \frac{1}{2} \int_0^{\infty} \frac{dx}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}} \quad (C5)$$

# Subappendix D

$$\tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2}} \right\} d\xi' \quad (D1)$$

Trailing Edge, let

$$p = 1 - \delta^2 \sin \theta$$

$$q = \delta^2 \cos \theta$$

or

$$\delta^2 = \sqrt{(1 - p)^2 + q^2}$$

$$\theta = \tan^{-1} \frac{(1 - p)}{q}$$

(D2)

and change variables in (D1)

$$\xi' = \sin^2 \sigma$$

$$d\xi' = 2 \sin \sigma \cos \sigma d\sigma$$

$$h_n(\sin^2 \sigma) = \frac{2}{\pi} \frac{\cos (2n + 1) \sigma}{\sin \sigma}$$

(see leading edge expansion)

$$\begin{aligned}\tilde{H}_n(p, q) &= \hat{H}_n(\delta, \theta) = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \left\{ 1 + \frac{\cos^2 \sigma - \delta^2 \sin \theta}{\sqrt{(\cos^2 \sigma - \delta^2 \sin \theta)^2 + \delta^4 \cos^2 \theta}} \right\} \cos (2n+1) \sigma \cos \sigma d\sigma \\ &= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \left\{ 1 + \frac{\sin^2 \sigma - \delta^2 \sin \theta}{\sqrt{\sin^4 \sigma - 2\delta^2 \sin^2 \sigma \sin \theta + \delta^4}} \right\} \cos \left[ (2n+1) \left( \frac{\pi}{2} - \sigma \right) \right] \sin \sigma d\sigma\end{aligned}$$

Now

$$\begin{aligned}\cos \left[ (2n+1) \left( \frac{\pi}{2} - \sigma \right) \right] &= \cos \left[ (2n+1) \frac{\pi}{2} \right] \cos \left[ (2n+1) \sigma \right] + \sin \left[ (2n+1) \frac{\pi}{2} \right] \sin \left[ (2n+1) \sigma \right] \\ &= \sin \left[ (2n+1) \frac{\pi}{2} \right] \sin \left[ (2n+1) \sigma \right] = (-1)^{n-1} \sin \left[ (2n+1) \sigma \right]\end{aligned}$$

The differentiation of the integrand with respect to  $\delta$  may be performed easily if it is compared with the leading edge case.

$$\left. \frac{\partial}{\partial \delta} \hat{H}_n(\delta, \theta) \right| = (-1)^n \frac{8\delta^3 \cos^2 \theta}{\pi} \int_0^{\frac{\pi}{2}} \frac{\sin \left[ (2n+1) \sigma \right] \sin^3 \sigma d\sigma}{\left[ \sin^4 \sigma - 2\delta^2 \sin \theta \sin^2 \sigma + \delta^4 \right]^{3/2}}$$

Changing variables of integration once again

$$x = \sin \sigma$$

$$dx = \cos \sigma d\sigma$$

$$\sin \left[ (2n+1) \sigma \right] = (1+2n)x + O(x^3)$$

and

$$\begin{aligned}
 \frac{\partial}{\partial \delta} H_n(\delta, \theta) &= (-1)^n (1 + 2n) \frac{8\delta^3 \cos^2 \theta}{\pi} \int_0^{\frac{1}{\delta}} \frac{x^4 dx}{[x^4 - 2\delta^2 x^2 \sin \theta + \delta^4]^{3/2}} \\
 &= (-1)^n (1 + 2n) \frac{8\delta^2 \cos^2 \theta}{\pi} \int_0^{\infty} \frac{x^4 dx}{[x^4 - 2x^2 \sin \theta + 1]^{5/2}} + O(\delta^4) \\
 &= (-1)^n (1 + 2n) \frac{8\delta^2 \cos^2 \theta}{\pi} I_1(\theta + \pi)
 \end{aligned} \tag{D3}$$

where

$$I_1(\theta) = \int_0^{\infty} \frac{x^4 dx}{[x^4 + 2x^2 \sin \theta + 1]^{3/2}} \tag{D4}$$

and integrating from 0 to  $\delta$  we get

$$\tilde{H}_n(p, q) = \tilde{H}_n(1, q) + (-1)^n (1 + 2n) \frac{8 \cos^2 \theta}{3\pi} I_1(\theta + \pi) \left[ (1 - p)^2 + q^2 \right]^{3/4} \tag{D5}$$

where

$$\theta = \tan^{-1} \frac{(1 - p)}{q} \tag{D6}$$



# Subappendix E

Suppose that  $k(x', y')$  may be written (as before)

$$\frac{c(\eta')}{b} k[x'(\xi', \eta'), y(\eta')] = \sum_{n=0}^N f_n(\eta') h_n(\xi')$$

where

$$h_n(\xi') = \frac{1}{\pi} \left[ \frac{1 - \xi'}{\xi'} \right]^{1/2} \left[ \frac{T_n(1 - 2\xi') + T_{n+1}(1 - 2\xi')}{1 - \xi'} \right]$$

then

$$\begin{aligned} \hat{h}_n(\Psi) &= h_n \left[ \frac{1}{2} (1 - \cos \Psi) \right] \\ &= \frac{2}{\pi} \left[ \frac{\cos n \Psi + \cos (n+1) \Psi}{\sin \Psi} \right] = \frac{2}{\pi} \frac{\cos \left( n + \frac{1}{2} \right) \Psi}{\sin \frac{\Psi}{2}} \end{aligned}$$

Then with

$$\xi = \frac{x - x_v \frac{2y}{b}}{c \frac{2y}{b}}$$

$$\eta = \frac{2y}{b}$$

$$\xi = \frac{2z}{b}$$

and from (3), (2) and (7) with  $\xi$  kept in  $g$

$$\Phi(x, y, z) = \phi(\xi, \eta, \xi) = \frac{b}{4\pi} \int_{-1}^1 \frac{\xi \hat{g}(\xi, \eta, \eta', \xi)}{(\eta' - \eta)^2 + \xi^2} d\eta'$$

$$\alpha(\xi, \eta) = \lim_{\xi \rightarrow 0} \frac{-1}{2\pi} \frac{\partial}{\partial \xi} \int_{-1}^1 \frac{g(\xi, \eta, \eta', \xi)}{(\eta' - \eta)^2 + \xi^2} d\eta'$$

where

$$g(\xi, \eta, \eta', \xi) = \sum_{n=0}^N f_n(\eta') \hat{H}_n(\xi, \eta, \eta', \xi)$$

and

$$\hat{H}_n(\xi, \eta, \eta', \xi')$$

$$= \int_0^1 h_n(\xi') \left\{ 1 + \frac{\frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi'}{\sqrt{\left[ \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi' \right]^2 + \left[ \frac{(\eta - \eta') \frac{1}{2} b}{c(\eta')} \right]^2 + \left[ \frac{\xi \frac{1}{2} b}{c(\eta')} \right]^2}} \right\} d\xi'$$

We can also define

$$\tilde{H}_n(p, q, \xi) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2 + \xi'^2}} \right\} d\xi'$$

For small  $\eta' - \eta$

$$p - p_0 = -\frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \tan \phi$$

$\phi$  = slope of constant percent chord lines

$$q = \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta')} \quad \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)}$$

$$c(\eta') = c(\eta) + \frac{\partial}{\partial \eta'} c(\eta') \bigg|_{\eta' = \eta} (\eta' - \eta)$$

$$\hat{\xi} = \frac{\xi(\frac{1}{2}b)}{c(\eta')} = \frac{z}{c(\eta')} = \frac{z}{c(\eta)}$$

$$\xi = \frac{z}{\frac{1}{2}b}$$

Leading Edge,  $P_0 = 0$ ,  $\phi = \phi_{LE}$  = leading edge sweep

$$\begin{aligned} p^2 + q^2 + \hat{\xi}^2 &= \left[ \frac{\frac{1}{2}b}{c(\eta)} \right]^2 \left\{ (\eta' - \eta)^2 [1 + \tan^2 \phi] + \left( \frac{z}{\frac{1}{2}b} \right)^2 \right\} \\ &= \frac{1}{\cos^2 \phi} \left[ \frac{\frac{1}{2}b}{c(\eta)} \right]^2 \left\{ (\eta' - \eta)^2 + \xi^2 \cos^2 \phi \right\} \end{aligned}$$

Therefore at the leading edge since  $\tilde{\phi} = \tan^{-1}\left(\frac{-p}{q}\right) = \begin{matrix} \phi_{LE} & \eta' > \eta \\ \phi_{LE} + \pi & \eta' < \eta \end{matrix}$

$$\begin{aligned} \hat{K}_n(\alpha, \eta, \eta', \xi) &= \frac{8(1 - \sin^2 \phi_{LE} \sin^2 \psi)}{\pi} \left\{ I_1(\tilde{\phi}, \psi) [p^2 + q^2 + \hat{\xi}^2]^{\frac{1}{4}} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}, \psi) [p^2 + q^2 + \hat{\xi}^2]^{\frac{3}{4}} \right\} \\ &= \frac{8(1 - \sin^2 \phi \sin^2 \psi)}{\pi} \left\{ I_1(\tilde{\phi}, \psi) \frac{1}{\cos^2 \phi} \left[ \frac{\frac{1}{2}b}{c(\eta)} \right]^{\frac{1}{2}} [(\eta' - \eta)^2 + \xi^2 \cos^2 \phi]^{\frac{1}{4}} \right. \\ &\quad \left. - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}, \psi) \left[ \frac{\frac{1}{2}b}{c(\eta)} \right]^{\frac{3}{2}} [(\eta' - \eta)^2 + \xi^2 \cos^2 \phi]^{\frac{1}{4}} \right\} \end{aligned}$$

$$\sin \psi = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \xi^2}} = \frac{|\eta' - \eta|}{\sqrt{(\eta' - \eta)^2 + \xi^2 \cos^2 \phi}} = \frac{|u|}{\sqrt{u^2 + \cos^2 \phi}}$$

where

$$u = \frac{\eta' - \eta}{\xi}$$

letting  $s = \eta' - \eta$  we must evaluate an expression of the form

$$\begin{aligned} \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_{-\delta}^{\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) I_1(\phi, \psi) \xi [s^2 + \xi^2 \cos^2 \phi]^{\frac{1}{4}}}{s^2 + \xi^2} ds \quad \phi = \begin{cases} \phi_{LE} & s > 0 \\ \phi_{LE} + \pi & s < 0 \end{cases} \\ \\ = \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \left\{ \sqrt{\xi} \int_0^{\frac{\delta}{\xi}} \frac{(1 - \sin^2 \phi_{LE} \sin^2 \hat{\phi}) [I_1(\phi_{LE}, \hat{\phi}) + I_1(\phi_{LE} + \pi, \hat{\phi})] [u^2 + \cos^2 \phi]^{\frac{1}{4}}}{u^2 + 1} du \right\} \\ \\ = \lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \left\{ \sqrt{\xi} \int_0^{\infty} \frac{(1 - \sin^2 \phi_{LE} \sin^2 \hat{\phi}) [I_1(\phi_{LE}, \psi) + I_1(\phi_{LE} + \pi, \hat{\phi})] [u^2 + \cos^2 \phi]^{\frac{1}{4}}}{u^2 + 1} du \right. \\ \left. - \sqrt{\xi} \int_{\frac{\delta}{\xi}}^{\infty} \frac{(1 - \sin^2 \phi_{LE} \sin^2 \psi) [I_1(\phi_{LE}, \hat{\phi}) + I_1(\phi_{LE} + \pi, \hat{\phi})] [u^2 + \cos^2 \phi]^{\frac{1}{4}}}{u^2 + 1} du \right\} \end{aligned}$$

where

$$\sin \hat{\phi} = \frac{u}{\sqrt{u^2 + \cos^2 \phi}} \quad (\text{no } \xi \text{ dependence!})$$

and

$$I_1(\phi, \psi) = \int_0^\infty \frac{x^2 dx}{[x^4 + 2x^2 \sin \phi \sin \psi + 1]^{3/2}}$$

The second integral goes to zero like  $\sqrt{\xi}$  and the first is independent of  $\xi$ . Therefore

$$\lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \sim \frac{1}{\sqrt{\xi}} \rightarrow \infty$$

# Subappendix F

$$\tilde{H}_n(p, q, z) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2 + z^2}} \right\} d\xi'$$

Leading edge, let

$$p = -\delta^2 \sin \hat{\phi} \sin \psi$$

$$q = \delta^2 \cos \hat{\phi} \sin \psi$$

$$z = \delta^2 \cos \psi$$

$$\delta^2 = \sqrt{p^2 + q^2 + z^2}$$

$$\hat{\phi} = \tan^{-1} \frac{-p}{q}$$

$$\psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{z}$$

introduce a change of variables in the integral

$$\xi' = \sin^2 \sigma$$

$$d\xi' = 2 \sin \sigma \cos \sigma d\sigma$$

$$h_n(\sin^2 \sigma) = h_n \left[ \frac{1}{2} (1 - \cos 2\sigma) \right] = \hat{h}_n(2\sigma) = \frac{2}{\pi} \frac{\cos 2n\sigma + \cos 2(n+1)\sigma}{\sin 2\sigma}$$

$$= \frac{2}{\pi} \frac{\cos [(2n+1) - 1]\sigma + \cos [(2n+1) + 1]\sigma}{2 \sin \sigma \cos \sigma} = \frac{2}{\pi} \frac{\cos (2n+1)\sigma}{\sin \sigma}$$

and

$$\left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2 + t^2}} \right\} = \left\{ 1 + \frac{-[\delta^2 \sin \hat{\phi} \sin \psi + \sin^2 \sigma]}{\sqrt{\sin^4 \sigma + 2\delta^2 \sin \hat{\phi} \sin \psi \sin^2 \sigma + \delta^4}} \right\}$$

$$\tilde{H}_n(p, q, t) = \hat{H}_n(\delta, \hat{\phi}, \psi) = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \left\{ 1 + \frac{-[\delta^2 \sin \hat{\phi} \sin \psi + \sin^2 \sigma]}{\sqrt{\sin^4 \sigma + 2\delta^2 \sin \hat{\phi} \sin \psi \sin^2 \sigma + \delta^4}} \right\} \cos [(2n+1)\sigma] \cos \sigma d\sigma$$

$$\frac{\partial \hat{H}_n(\delta, \hat{\phi}, \psi)}{\partial \delta} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{-2\delta \sin \hat{\phi} \sin \psi \cos [(2n+1)\sigma] \cos \sigma d\sigma}{\sqrt{\sin^4 \sigma + 2\delta^2 \sin \hat{\phi} \sin \psi \sin^2 \sigma + \delta^4}}$$

$$+ \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{2\delta [\sin \hat{\phi} \sin \psi \sin^2 \sigma + \delta^2] (\delta^2 \sin \hat{\phi} \sin \psi + \sin^2 \sigma) \cos [(2n+1)\sigma] \cos \sigma d\sigma}{[\sin^4 \sigma + 2\delta^2 \sin \hat{\phi} \sin \psi \sin^2 \sigma + \delta^4]^{3/2}}$$

$$= \frac{8\delta^3 [1 - \sin^2 \hat{\phi} \sin^2 \psi]}{\pi} \int_0^{\frac{\pi}{2}} \frac{\cos [(2n+1)\sigma] \cos \sigma \sin^2 \sigma d\sigma}{[\sin^4 \sigma + 2\delta^2 \sin \hat{\phi} \sin \psi \sin^2 \sigma + \delta^4]^{3/2}}$$



as  $\delta \rightarrow 0$  the only portion of the integrand which is important is in the region of  $\sigma = 0$ .

Therefore let  $x = \sin \sigma$ ,  $dx = \cos \sigma d\sigma$  and expand

$$\cos \left[ (2n+1) \sigma \right] = \cos \left[ (2n+1) \left( x - \frac{x^3}{6} \right) \right] = 1 - \frac{1}{2} (2n+1)^2 x^2$$

$$1 - \frac{x^2}{2} (1 + 4n + 4n^2) + O(x^4)$$

Therefore to first order in  $x^2$

$$\begin{aligned} \frac{\partial}{\partial \delta} \hat{H}_n(\delta, \hat{\theta}, \psi) &= \frac{8\delta^3 [1 - \sin^2 \hat{\theta} \sin^2 \psi]}{\pi} \int_0^1 \frac{\left[ 1 - \frac{x^2}{2} (1 + 4n + 4n^2) \right] x^2 dx}{\left[ x^4 + 2\delta^2 x^2 \sin \hat{\theta} \sin \psi + \delta^4 \right]^{3/2}} \\ &= \frac{8 [1 - \sin^2 \hat{\theta} \sin^2 \psi]}{\pi} \int_0^{\frac{1}{\delta}} \frac{x^2 - \frac{1}{2} \delta^2 x^4 (1 + 4n + 4n^2)}{\left[ x^4 + 2x^2 \sin \hat{\theta} \sin \psi + 1 \right]^{3/2}} dx \\ &= \frac{8 [1 - \sin^2 \hat{\theta} \sin^2 \psi]}{\pi} \int_0^{\infty} \frac{x^2 - \frac{1}{2} \delta^2 x^4 (1 + 4n + 4n^2)}{\left[ x^4 + 2x^2 \sin \hat{\theta} \sin \psi + 1 \right]^{3/2}} dx + O(\delta^3) \\ \hat{H}_n(\delta, \hat{\theta}, \psi) &= \hat{H}_n(0, \hat{\theta}, \psi) + \frac{8 (1 - \sin^2 \hat{\theta} \sin^2 \psi)}{\pi} \left[ I_1(\hat{\theta}, \psi) \delta - \frac{1}{6} (1 + 4n + 4n^2) I_2(\hat{\theta}, \psi) \delta^3 \right] \end{aligned}$$

Integrating from 0 to  $\delta$  we get

(C1)

and

$$\tilde{H}_n(p, q, \xi) = H_n(0, 0, 0) + \frac{8(1 - \sin^2 \hat{\phi} \sin^2 \psi)}{\pi} \left[ I_1(\hat{\phi}, \psi) [p^2 + q^2 + \xi^2]^{\frac{1}{4}} - \frac{1}{6}(1 + 4n + 4n^2) I_2(\hat{\phi}, \psi) (p^2 + q^2 + \xi^2)^{\frac{3}{4}} \right] \quad (C2)$$

where

$$\hat{\phi} = \tan^{-1} \frac{-p}{q}$$

$$\psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{\xi}$$

$$I_1(\hat{\phi}, \psi) = \int_0^\infty \frac{x^2 dx}{[x^4 + 2x^2 \sin \hat{\phi} \sin \psi + 1]^{3/2}} \quad (C3)$$

$$I_2(\hat{\phi}, \psi) = \int_0^\infty \frac{x^4 dx}{[x^4 + 2x^2 \sin \hat{\phi} \sin \psi + 1]^{3/2}} \quad (C4)$$

$$\sin \psi = \frac{\tan \psi}{\sqrt{1 + \tan^2 \psi}} = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \xi^2}} = \frac{\sqrt{\frac{p^2 + q^2}{\xi^2}}}{\sqrt{\frac{p^2 + q^2}{\xi^2} + 1}}$$

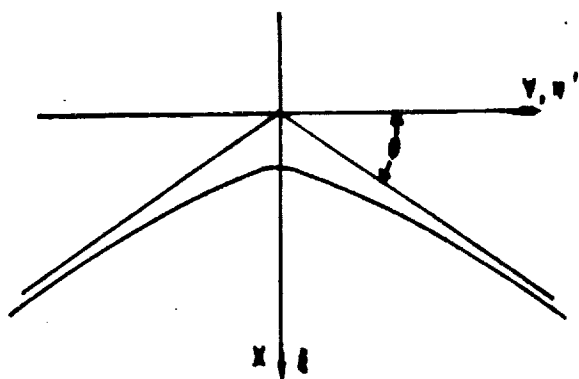
$$\sin \hat{\phi} = \frac{-p}{\sqrt{p^2 + q^2}}$$

$$\sin \hat{\phi} \sin \psi = \frac{-p}{\sqrt{p^2 + q^2 + \xi^2}} = \frac{\frac{-p}{\xi}}{\sqrt{\frac{p^2 + q^2}{\xi^2} + 1}}$$

# Subappendix G

$$\tilde{H}_n(p, q) = \tilde{H}_n(p_0, 0) + 2 h_n(p_0) (p - p_0) - h'_n(p_0) q^2 \ln |q|$$

Hyperbolic leading and trailing edges, constant chord.



$$[X_V(\eta')]^2 = c \tan^2 \theta [\eta'^2 + \rho^2 \tan^2 \theta]$$

$$X_V(\eta') = c \tan \theta \sqrt{\eta'^2 + \rho^2 \tan^2 \theta}$$

$$\eta' \gg 1 \quad \frac{1}{c} \frac{dX_V}{d\eta'} = \tan \theta$$

$$\text{Curvature at } \eta' = 0 \quad \frac{1}{c} \left. \frac{d^2 X_V}{d\eta'^2} \right|_{\eta'=0} = \frac{1}{\rho}$$

From (21) at  $\eta' = 0$

$$p - p_0 = \frac{X_V(0) - X_V(\eta')}{c}$$

or

$$p - p_0 = -\tan \theta \left[ \sqrt{\eta'^2 + \rho^2 \tan^2 \theta} - \rho \tan \theta \right]$$

Therefore from (15) and (20), for small  $\eta'$

$$H_n(\xi, 0, \eta') = H_n(\xi, 0, 0) - 2h_n(\xi) \tan \theta \left[ \sqrt{\eta'^2 + \rho^2 \tan^2 \theta} - \rho \tan \theta \right] \\ + h'_n(\xi) \eta'^2 \ln |\eta'|$$

Therefore referring to (17) and (18) we must evaluate

$$\lim_{\xi \rightarrow 0} \frac{\partial}{\partial \xi} \int_0^\delta \frac{\xi \left[ \sqrt{s^2 + a^2} - a \right]}{s^2 + \xi^2} ds \\ \int_0^\delta \frac{\xi \left[ \sqrt{s^2 + a^2} - a \right]}{s^2 + \xi^2} ds \\ = \xi \left\{ \log \left[ s + \sqrt{s^2 + a^2} \right] + \frac{\sqrt{a^2 - \xi^2}}{\xi} \tan^{-1} \frac{s \sqrt{a^2 - \xi^2}}{\xi \sqrt{s^2 + a^2}} - \frac{a}{\xi} \tan^{-1} \frac{s}{\xi} \right\} \Bigg|_0^\delta \\ = \xi \left\{ \log \left[ \delta + \sqrt{\delta^2 + a^2} \right] + \frac{\sqrt{a^2 - \xi^2}}{\xi} \tan^{-1} \frac{\delta \sqrt{a^2 - \xi^2}}{\xi \sqrt{\delta^2 + a^2}} - \frac{a}{\xi} \tan^{-1} \frac{\delta}{\xi} - \log a \right\}$$

and

$$\lim_{a \rightarrow 0} \lim_{\xi \rightarrow 0} \int_0^{\delta} \frac{\xi \left[ \sqrt{s^2 + a^2} - a \right]}{s^2 + \xi^2} ds = \infty.$$

and since  $a = \rho \tan \theta$ , there is a logarithmic infinity as the radius of curvature goes to zero and the leading edge becomes kinked.

APPENDIX F  
POTENTIAL FORM DRAG

The section load can be obtained by means of the Blasius theorem as follows;

$$F_X - i F_Y = \frac{1}{2} \rho i \int_C \left( \frac{dw}{dz} \right)^2 dz \quad (1)$$

where

$$\left( \frac{dw}{dz} \right)^2 = u^2 - 2uvi + (iv)^2 = u^2 - v^2 - 2uvi \quad (2)$$

Therefore;

$$F_X - i F_Y = \frac{1}{2} \rho \int_C [2uv + i(u^2 - v^2)] [dx + idy] \quad (3)$$

then

$$F_X = \frac{1}{2} \rho \int_C [2uv dx - (u^2 + v^2) dy] \quad (4)$$

$$F_Y = - \frac{1}{2} \rho \int_C [2uv dy + (u^2 - v^2) dx] \quad (5)$$

Since all of the singularities are on the chordal plane, for the panels, equations (4) and (5) reduce to;

$$F_x = -\rho \sum_{i=1}^{N_i} (u_{u_i} v_{u_i} - u_{L_i} v_{L_i}) \Delta x_i \quad (6)$$

$$F_y = \rho/2 \sum_{i=1}^{N_i} \left[ (u_{u_i}^2 - v_{u_i}^2) - (u_{L_i}^2 - v_{L_i}^2) \right] \Delta x_i \quad (7)$$

where the subscripts u and L indicate upper and lower surfaces, respectively.

For the two-dimensional lifting case;

$$u_{u_i} = \frac{1}{2} \gamma_i + V_\infty \cos \alpha \quad (8)$$

$$u_{L_i} = -\frac{1}{2} \gamma_i + V_\infty \cos \alpha \quad (9)$$

$$v_{u_i} = v_{L_i} = -\frac{1}{2\pi} \sum_{K=1}^{N_i} \frac{\Gamma_K}{x_i - x_K} + V_\infty \sin \alpha \quad (10)$$

Therefore,

$$F_x = -\rho \sum_{i=1}^{N_i} \Gamma_i V_\infty \sin \alpha + \frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_K} \frac{\Gamma_i \Gamma_K}{x_i - x_K} \quad (11)$$

Since

$$\frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_i} \frac{\Gamma_i \Gamma_K}{x_i - x_K} = 0 \quad (12)$$

$$F_x = -\rho V_{\infty} \sin \alpha \sum_{i=1}^{N_i} \Gamma_i = -L \sin \alpha \quad (13)$$

where  $L$  is the lift and  $\Gamma = \gamma \Delta x$  is the local vortex strength.

Also; from equations (7), (8), (9), and (10)

$$F_y = \rho \sum_{i=1}^{N_i} \Gamma_i V_{\infty} \cos \alpha = L \cos \alpha \quad (14)$$

which demonstrates that the discrete vortex lattice always gives the correct chord force  $F_x$  and normal force  $F_y$ , provided the correct circulations is obtained.

Similarly, for the two-dimensional thickness case;

$$u_{u_i} = u_{L_i} = \frac{1}{2\pi} \sum_{K=1}^{N_i} \frac{\Sigma_K}{x_i - x_K} + V_{\infty} \quad (15)$$

$$v_{u_i} = V_{\infty} \left( \frac{dz_t}{dx} \right)_i \quad (16)$$



$$v_{L_i} = - V_{\infty} \left( \frac{dz_t}{dx} \right)_i \quad (17)$$

and

$$\Sigma_i = 2 V_{\infty} \left( \frac{dz_t}{dx} \right)_i \Delta x_i \quad (18)$$

Therefore;

$$F_x = - \frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_i} \frac{\Sigma_i \Sigma_K}{x_i - x_K} - \rho V_{\infty} \sum_{i=1}^{N_i} \Sigma_i = 0 \quad (19)$$

since

$$\sum_{i=1}^{N_i} \Sigma_i = 0$$

if the airfoil is closed.

Also;  $F_y = 0$  from equation (7), (15), (16), and (17). This demonstrates that the discrete source lattice also gives the correct chord force  $F_x$  and normal force  $F_y$ .

The above equations can be generalized to compute the section potential form drag on a finite wing due to lift and thickness by evaluating the component of force in the free stream direction and by using the three dimensional influence equations.

The section potential form drag due to lift is computed as follows;

$$d_{L_i} = -\rho \sum_{i=1}^{N_i} \Gamma_i w_i \quad (20)$$

where  $i$  is summed over the section of the panel and  $w_i$  is the total velocity normal to the panel chordal surface at the quarter chord of the  $i^{\text{th}}$  sub-panel. The section induced drag coefficient

$$\frac{C_{d_{L_i}} C}{C_{\text{AVG.}}}$$

is then given by;

$$\frac{C_{d_{L_i}} C}{C_{\text{AVG.}}} = -\frac{2AR}{b} \sum_{i=1}^{N_i} \left( \frac{\Gamma_i}{V_\infty} \right) \left( \frac{w_i}{V_\infty} \right) \quad (21)$$

If  $w_i$  is computed at the three-quarter chord of the  $i^{\text{th}}$  subpanel, instead of the quarter chord of the  $i^{\text{th}}$  subpanel as is done in equation (21), the section zero percent suction drag coefficient

$$\frac{C_{d_{T=0}} C}{C_{\text{AVG.}}}$$

is obtained. The section leading edge thrust coefficient  $(C_T C)/(C_{\text{AVG.}})$  is equal to;

$$\frac{C_T C}{C_{\text{AVG.}}} = \frac{C_{d_{T=0}} C}{C_{\text{AVG.}}} - \frac{C_{d_{L_i}} C}{C_{\text{AVG.}}} \quad (22)$$

The section potential form drag due to thickness is computed by a similar procedure.

$$d_{T_i} = -\rho \sum_{i=1}^{2N_i} \Sigma_i u_i - \rho V_\infty \sum_{i=1}^{2N_i} \left( \frac{V_{x_i}}{V_\infty} \right) \Sigma_i \quad (23)$$

where  $i$  is summed over the section for both the quarter and three-quarter chord stations of each subpanel.  $\Sigma_i$  and  $u_i$  are the source strength and total velocity in the free stream direction, respectively, at either the quarter or three-quarter chord point of the subpanel. The section induced drag coefficient

$$\frac{C_{d_{T_i}}}{C_{AVG.}}$$

is then given by;

$$\frac{C_{d_{T_i}}}{C_{AVG.}} = -\frac{2AR}{b} \sum_{i=1}^{2N_i} \sqrt{1 + \tan^2 \Lambda_i} \left( \frac{\Sigma_i}{V_\infty} \right) \left( \frac{U_i}{V_\infty} \right) - \frac{2AR}{b} \sum_{i=1}^{2N_i} \sqrt{1 + \tan^2 \Lambda_i} \left( \frac{V_{x_i}}{V_\infty} \right) \left( \frac{\Sigma_i}{V_\infty} \right) \quad (24)$$

For the special case where the chordal surfaces of the panels are planar and parallel to each other the integral of the section induced drag

$$\frac{C_{d_{L_i}}}{C_{AVG.}}$$

over the span can be shown to be identical with the value of induced drag as computed in the far field, provided all of the lifting elements (bound vortices) are parallel and the lateral widths of the horseshoe vortices are equal for the complete system.

The total drag of a wing as computed in the near field is given by;

$$C_{D_i} = -\frac{AR}{b} \sum_{j=1}^N \sum_{K=1}^N \left( \frac{w}{V_\infty} \right)_{jk} \left( \frac{\Gamma}{V_\infty} \right)_j \Delta \eta_j \quad (25)$$

where

$$\begin{aligned} \left( \frac{w}{V_\infty} \right)_{jK} = & \frac{\left( \frac{\Gamma}{V_\infty} \right)_K}{4\pi} \left\{ \frac{\beta^2 Y_{jK} + X_{jK} T + (T^2 + \beta^2) y_\nu}{(X_{jK} - T Y_{jK}) \sqrt{(X_{jK} + T y_\nu)^2 + \beta^2 (Y_{jK} + y_\nu)^2}} \right. \\ & - \frac{\beta^2 Y_{jK} + X_{jK} T - (T^2 + \beta^2) y_\nu}{(X_{jK} - T Y_{jK}) \sqrt{(X_{jK} - T y_\nu)^2 + \beta^2 (Y_{jK} - y_\nu)^2}} \\ & + \frac{1}{(Y_{jK} + y_\nu)} \left[ 1 + \frac{X_{jK} + T y_\nu}{\sqrt{(X_{jK} + T y_\nu)^2 + \beta^2 (Y_{jK} + y_\nu)^2}} \right] \\ & \left. - \frac{1}{(Y_{jK} - y_\nu)} \left[ 1 + \frac{X_{jK} - T y_\nu}{\sqrt{(X_{jK} - T y_\nu)^2 + \beta^2 (Y_{jK} - y_\nu)^2}} \right] \right\} \quad (26) \end{aligned}$$

and

$$X_{jK} = X_j - X_K$$

$$Y_{jK} = (Y_j - Y_K)$$

$(X_K, Y_K)$  is the influencing point

$(X_j, Y_j)$  is the point being influenced

$y_\nu$  is half of the spanwise lattice spacing

$T$  is the tangent of the vortex line sweep

$\beta^2$  is  $1 - M_\infty^2$

Equation (26) can be divided into two parts; that due to the near field stagger of the lifting element and that due to the limit of integration at infinity.

Therefore;

$$\left(\frac{w}{V_\infty}\right)_{jK} = \frac{\left(\frac{\Gamma}{V_\infty}\right)_K}{4\pi} \left[ E_{S_{jK}} + E_{\infty_{jK}} \right] \quad (27)$$

The contribution to equation (26) from the near field limits of integration or lifting element stagger is given by  $E_{S_{jK}}$ ,

$$E_{S_{jK}} = \left\{ \frac{\beta^2 Y_{jK} + TX_{jK} + (T^2 + \beta^2) y_\nu}{(X_{jK} - Ty_{jK}) \sqrt{(X_{jK} + Ty_\nu)^2 + \beta^2 (Y_{jK} + y_\nu)^2}} - \frac{\beta^2 Y_{jK} + TX_{jK} - (T^2 + \beta^2) y_\nu}{(X_{jK} - Ty_{jK}) \sqrt{(X_{jK} - Ty_\nu)^2 + \beta^2 (Y_{jK} - y_\nu)^2}} \right. \\ \left. + \frac{X_{jK} + Ty_\nu}{(Y_{jK} + y_\nu) \sqrt{(X_{jK} + Ty_\nu)^2 + \beta^2 (Y_{jK} + y_\nu)^2}} - \frac{X_{jK} - Ty_\nu}{(Y_{jK} - y_\nu) \sqrt{(X_{jK} - Ty_\nu)^2 + \beta^2 (Y_{jK} - y_\nu)^2}} \right\} \quad (28)$$

That due to the limits of integration at infinity is given by  $E_{\infty jK}$

$$E_{\infty jK} = \left\{ \frac{1}{Y_{jK} + y_{\nu}} - \frac{1}{Y_{jK} - y_{\nu}} \right\} \quad (29)$$

The contribution to the total drag  $C_{Di}$ , given by equation (25), from  $E_{SjK}$  is seen to be exactly zero for all planar wings and loadings provided both  $T$  and  $y_{\nu}$  are constant everywhere on the wing. This is due to the fact that there is no contribution to the drag from  $E_{SjK}$  when  $X_{jj} = Y_{jj} = 0$  or when  $X_{KK} = Y_{KK} = 0$  due to taking the Cauchy principal value. Also, when  $j \neq K$  the drag from  $E_{SjK}$  is zero because the mutual interference drag due to the stagger is zero. This is seen by interchanging the influencing point and the point being influenced and observing that  $E_{SjK} = -E_{SKj}$ .

Therefore;

$$C_{Di} = \frac{AR}{b} \sum_{j=1}^N \sum_{K=1}^N \left[ \frac{\left(\frac{\Gamma}{V_{\infty}}\right)_K}{4\pi} E_{\infty jK} \right] \left(\frac{\Gamma}{V_{\infty}}\right)_j \Delta \eta_j \quad (30)$$

$$C_{Di} = \frac{AR}{4\pi b} \sum_{j=1}^N \sum_{K=1}^N \left(\frac{\Gamma}{V_{\infty}}\right)_K \left(\frac{\Gamma}{V_{\infty}}\right)_j \left( \frac{1}{Y_{jK} + y_{\nu}} - \frac{1}{Y_{jK} - y_{\nu}} \right) \Delta \eta_j \quad (31)$$

This equation is identical to that obtained from the standard Trefftz plane analysis. From reference (47);

$$D_i = -\frac{\rho V_{\infty}^2}{2} \iint_{S_{WAKE}} \phi \frac{\partial \phi}{\partial N} dS \quad (32)$$

where  $\phi$  is the velocity potential in the far field.

In the case of a planar wing the vorticity trace in the Trefftz plane can be replaced by a slit and equation (32) replaced by;

$$D_i = -\frac{\rho V_\infty^2}{2} \int_{-b/2}^{b/2} \Delta \phi(Y) \frac{\partial \phi}{\partial N}(Y) dY \quad (33)$$

where

$$\Delta \phi(Y) = \frac{K}{V_\infty}(Y) \quad (34)$$

$$\frac{\partial \phi}{\partial N}(Y) = \mp \frac{1}{2\pi} \int_{-b/2}^{b/2} \frac{\frac{d}{dY_1} \frac{K}{V_\infty}(Y_1) dY_1}{(Y - Y_1)} \quad (35)$$

and  $K/V_\infty(Y)$  is the total circulation at a given lateral station. Therefore,

$$\frac{K}{V_\infty} = \sum_{i=1}^{N_i} \left( \frac{\Gamma}{V_\infty} \right)_i \quad (36)$$

and  $N_i$  is the number of vortices per chord. Also;

$$\frac{\partial \phi}{\partial Y}(Y) = \mp \frac{1}{2\pi} \sum_{n=1}^{N_N} \left( \frac{K}{V_\infty} \right)_n \left( \frac{1}{Y - Y_n + y_\nu} - \frac{1}{Y - Y_n - y_\nu} \right) \quad (37)$$

where  $N_N$  is the number of vortices per span.

After substituting equations (34), (35), (36), and (37) into equation (33) and letting  $N = N_i \times N_n$ ;

$$D_i = \frac{\rho V_\infty^2}{4\pi} \sum_{j=1}^N \sum_{K=1}^N \left( \frac{\Gamma}{V_\infty} \right)_K \left( \frac{\Gamma}{V_\infty} \right)_j \left( \frac{1}{Y_j - Y_K + y_\nu} - \frac{1}{Y_j - Y_K - y_\nu} \right) \Delta Y_j \quad (38)$$

Since  $\Delta Y_j = b/2 \Delta \eta_j$

$$C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^N \sum_{K=1}^N \left( \frac{\Gamma}{V_\infty} \right)_K \left( \frac{\Gamma}{V_\infty} \right)_j \left( \frac{1}{Y_{jK} + y_\nu} - \frac{1}{Y_{jK} - y_\nu} \right) \Delta \eta_j \quad (39)$$

which is the same as equation (31).

The far field calculation of induced drag for a complete configuration composed of lifting bodies and thick lifting panels is done by representing the wake, from all of the bodies and panels, by an equivalent horseshoe vortex system where the bound segment of the horseshoe vortex is tangent to the trace of the wake in the Trefftz plane and the trailing legs are in the free stream direction. The section drag associated with the equivalent system, which in general is not equal to the actual configuration section induced drag is given by;

$$\bar{d}_j = \rho \bar{W}_j \times \bar{\Gamma}_j \Delta S_j \quad (40)$$

where

$$\bar{W}_j = V_j \hat{j} + W_j \hat{k} \quad (41)$$

and

$$\bar{\Gamma}_j = \Gamma_j (T_{Y_j} \hat{j} + T_{Z_j} \hat{k}) \quad (42)$$

where  $T_{Y_j}$  and  $T_{Z_j}$  are components of the unit vector tangent to the trace of the wake at the  $j^{\text{th}}$  section.



$$V_j = \sum_{K=1}^N \frac{K_k}{4\pi} \left( E_{V_{jk}} T_{Y_k} + E_{W_{jk}} N_{Y_k} \right) \quad (43)$$

$$W_j = \sum_{K=1}^N \frac{K_k}{4\pi} \left( E_{V_{jk}} T_{Z_k} + E_{W_{jk}} N_{Z_k} \right) \quad (44)$$

where  $T_{Y_k}$  and  $T_{Z_k}$  are the components of the unit vector tangent to the

trace of the wake at  $K^{th}$  station. Also,  $N_{Y_k}$  and  $N_{Z_k}$  are components

of the unit vector normal to the trace of the wake at the  $K^{th}$  section.  $N$  is the total number of sections along the trace of the wake for the complete configuration.

$$E_{V_{jk}} = \frac{\bar{Z}_{jk}}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} + \frac{1}{2} \Delta S_k)^2} - \frac{\bar{Z}_{jk}}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} - \frac{1}{2} \Delta S_k)^2} \quad (45)$$

$$E_{W_{jk}} = \frac{\bar{Y}_{jk} - \frac{1}{2} \Delta S_k}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} - \frac{1}{2} \Delta S_k)^2} - \frac{\bar{Y}_{jk} + \frac{1}{2} \Delta S_k}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} + \frac{1}{2} \Delta S_k)^2} \quad (46)$$

where

$$\bar{Y}_{jk} = (Y_k - Y_j) N_{Z_k} - (Z_k - Z_j) N_{Y_k} \quad (47)$$

$$\bar{Z}_{jk} = -(Y_k - Y_j) T_{Z_k} + (Z_k - Z_j) T_{Y_k} \quad (48)$$

and the indices  $j$  and  $k$  refer to the section being influenced and the influencing section, respectively.

therefore;

$$d_j = \rho (V_j \Gamma_j T_{Z_j} - W_j \Gamma_j T_{Y_j}) \Delta S_j \quad (49)$$

the total configuration induced drag is then given by;

$$D_i = \frac{\rho}{4\pi} \sum_{j=1}^N \sum_{k=1}^N \Gamma_j \Gamma_k \left( (E_{V_{jk}} T_{Y_k} + E_{W_{jk}} N_{Y_k}) T_{Z_j} - (E_{V_{jk}} T_{Z_k} + E_{W_{jk}} N_{Z_k}) T_{Y_j} \right) \Delta S_j \quad (50)$$

therefore;

$$C_{D_i} = \frac{AR}{2\pi b^2} \sum_{j=1}^N \sum_{k=1}^N \left( \frac{\Gamma}{v_\infty} \right)_j \left( \frac{\Gamma}{v_\infty} \right)_k \left( (E_{V_{jk}} T_{Y_k} + E_{W_{jk}} N_{Y_k}) T_{Z_j} - (E_{V_{jk}} T_{Z_k} + E_{W_{jk}} N_{Z_k}) T_{Y_j} \right) \Delta S_j \quad (51)$$

NOTE; For the planar wing equation (51) reduces to

$$C_{D_i} = \frac{AR}{2\pi b^2} \sum_{j=1}^N \sum_{k=1}^N \left( \frac{\Gamma}{v_\infty} \right)_j \left( \frac{\Gamma}{v_\infty} \right)_k E_{W_{jk}} \Delta S_j \quad (52)$$

$$C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^N \sum_{k=1}^N \left( \frac{\Gamma}{v_\infty} \right)_k \left( \frac{\Gamma}{v_\infty} \right)_j \left( \frac{1}{Y_{jk} + y_\nu} - \frac{1}{Y_{jk} - y_\nu} \right) \Delta \eta_j \quad (53)$$

which is the same as equations (31) and (30). This demonstrates that the same induced drag is obtained whether the equality of work and kinetic-energy increment, the equivalent far field horseshoe system, or the near field horseshoe vortices with the Kutta-Joukowski theorem is used.

APPENDIX G

COMPUTER PROGRAM LISTING

```

PROTRAN DERIV(INPUT=201, OUTPUT , TAPE5=INPUT, TAPE6=OUTPUT,
1 TAPE18 , TAPE19 , TAPE20=1002, TAPE21=1002, TAPE23
2 TAPE24 , TAPE10=1002, TAPE11 , TAPE12
COMMON DA(5100)
COMMON/PANATT/DUM1( 230)
COMMON/NUMBER/ DUM2(135)
COMMON/PANEL /DUM3( 34)
COMMON/BODY/DUM4(31550)
COMMON/CONPTS/ DUM5(7920)
COMMON/SCRAT/ DUM7(25000)
COMMON/INDEX /DUM10(7)
COMMON /PANINF/ PANSYM(10), DUM11(600), PANREF(10), PCHORD(10)
COMMON/CONTRV/ DUM13(240)
COMMON/SLOPE/DUM14(2000)
COMMON /CONPRS/ BETAM
CALL FTNBIN(1,0,0)
DO 1 I=1,5100
DA(I)=0.0
OVL=3HOVL
CALL ATTACH(DUM1(1),DUM1(2),DUM1(3),DUM1(4),123)
CALL OVERLAY(OVL,1,0)
CALL OVERLAY(OVL,2,0)
CALL OVERLAY(OVL,3,0)
CALL OVERLAY(OVL,4,0)
CALL OVERLAY(OVL,5,0)
CALL OVERLAY(OVL,6,0)
CALL OVERLAY(OVL,7,0)
END

```

1

```

SUBROUTINE ATTACH(XV,YV,ZV,NR,KODE)
C FOR KODE = -1, THIS SUBROUTINE IS SETTING XATT FROM BODY INPUTS.
C FOR KODE = 0, THIS SUBROUTINE IS SETTING XATT FROM PANEL INPUTS.
C FOR KODE = 1, THIS SUBROUTINE SETS XV,YV,ZV, FROM XATT ARRAY.
DIMENSION XV(NR,1), YV(NR,1), ZV(NR,1)
COMMON DA(5000)
1 ,NX,NXTH,LNVOR,LIVOR,NTVV,NRVV,NTV,NXTHV,NRV,NTH(49)
2 ,LNDIV,LTDIV,LNPTS,LTPTS
COMMON/PANATT/ NATT(30),XATT(200)
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1 ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON/PANEL/ NPAN,IPSYM,IMC,NBVVP,NTVVP,LNCFP,LTCFP,LNCFP,LTCFP
1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NMCTXC,NMCTET,NTHXC
2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NATT
3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
EQUIVALENCE (DA(2),PANS)
DATA L/O/,L1/O/,L2/O/
IF(KODE.EQ.123) RETURN
NPANS=PANS
NB1=NBVV+1
IF(KODE) 1,2,200
NSA=NB
1 GO TO 3
2 NSA=NPAN+NBODS
3 CONTINUE
DO 100 I=1,NPANS
I1=(I-1)*3+2
IF(NATT(I1).EQ.NSA) GO TO 5
GO TO 100
5 I2=NATT(I1+1)
GO TO 70
10 L=L+1
XATT(L)=-NATT(I1)
XATT(L+1)= I2
L=L+2
XATT(L)=NB1
NA=(I2-1)*LTDIV+1

```

```

IF(KODE.EQ.0) GO TO 50
DO 20 K=1,NB1
  L=L+1
  XATT(L)=XV(K,NA)
  XATT(L+1)=YV(K,NA)
  L=L+2
  XATT(L)=ZV(K,NA)
  GO TO 65
20  NA=NATT(I1)
  DO 60 K=1,NB1
    L=L+1
    XATT(L)=XV(NA,K)
    XATT(L+1)=YV(NA,K)
    L=L+2
    XATT(L)=ZV(NA,K)
    L1=NATT(I1)
    L2=I2
    GO TO 100
60  IF(L1.EQ.NATT(I1).AND.L2.EQ.I2) GO TO 100
65  GO TO 10
100 CONTINUE
101 WRITE(6,101)(XATT(I),I=1,L)
    FORMAT(27H0IN SUB. ATTACH, XATT ARRAY/(1P8E13.4))
    RETURN
C  HERE, SUB. ATTACH FINDS WHICH SET OF X,Y,Z ARE FOR THE ATTACH LINE.
200 AN1=NATTCH
    AN2=NTRATT
    DO 300 I=1,200
      IF(XATT(I).EQ.-AN1.AND.XATT(I+1).EQ.AN2) GO TO 201
      GO TO 300
201 J=I+2
    NB1=XATT(J)+0.01
    DO 205 K=1,NB1
      J=J+1
      XV(K,1)=XATT(J)
      YV(K,1)=XATT(J+1)
      J=J+2
205  ZV(K,1)=XATT(J)
      MP3=NB1

```

1070  
1080  
1090  
1100  
1110  
1120  
1130  
1140

C  
C  
C  
C  
C  
C  
C  
C

```

WRITE(5,206)(XV(K,1),Y(K,1),ZV(K,1),K=1,NB1)
FORMAT(43H0IN SUB. ATTACH, AT,YT,ET, FOR ATTACH. LINE/(15A513.4))
RETURN
CONTINUE
WRITE(5,305) 'PAN'
FORMAT(35H0NO ATTACHMENT LINE FOUND FOR PANEL,I2,I3HIN SUB ATTACH)
RETURN
END

```

206  
300  
305

ORIGINAL PAGE IS  
OF POOR QUALITY

```

SUBROUTINE DECRD(DATA,IUNIT)
  DIMENSION DATA(1),ADATA(5),IDATA(17),IIDATA(8)
  DATA IBLANK/10H
15 READ(IUNIT,16)IIDATA
16 FORMAT(8A10)
      C
      IF (EOF(IUNIT).EQ. 0) GOTO 19
      IF (EOF, IUNIT) 199, 19
199 CONTINUE
      IUNIT = -IUNIT
      RETURN
19 DECODE(72,17,IIDATA)IADD,ADATA
17 FORMAT(112,5G12.0)
      DECODE(80,18,IIDATA)IDATA
18 FORMAT(12X,17A4)
      J=IADD
      IF(IADD) 22,40,24
22 J=-J
24 DO 30 I=1,5
      L=3*I
      K=L-2
      DO 26 M=K,L
      IF(IDATA(M)-IBLANK)28,26,28
26 CONTINUE
      GO TO 30
28 DATA(J)=ADATA(I)
30 J=J+1
      IF(IADD)100,40,15
40 WRITE(6,50)IADD,ADATA
50 FORMAT(17H0DECRD ER. CARD=(,1112,17A4,2H).)
      CALL EXIT
100 RETURN
      END

```

0	1150
0	1160
0	1170
0	1180
0	1190
0	1200
0	1210
0	1220
0	1230
0	1240
0	1250
0	1260
0	1270
0	1280
0	1290
0	1300
0	1310
0	1320
0	1330
0	1340
0	1350
0	1360
0	1370
0	1380
0	1390
0	1400
0	1410
0	1420
0	1430
0	1440
0	1450
0	1460



```

C
C****
C      A CONTROLLED DEVIATION INTERPOLATION METHOD
C
C      DIMENSION  XI(1)      ,YI(1)      ,T(1)      ,ANS(1)
C
C      XK=1.0
C      N=NI
C      DO 910 IE=1,NA
C      X=T(IE)
C      100 IF(N-2)110,120,200
C      110 Y = YI(N)
C      GO TO 900
C      120 Y = (YI(2)-YI(1))/(XI(2)-XI(1))* (X-XI(1)) +YI(1)
C      GO TO 900
C      200 J = 1
C      210 IF(XI(J)-X)230,220,250
C      220 Y =YI(J)
C      GO TO 900
C      230 J = J+1
C      IF(J-N)210,210,250
C      250 IF(J-2)120,155,260
C      155 J = 3
C      JJ = 1
C      GO TO 285
C      260 IF(J-N)280,265,270
C      265 J = N-1
C      JJ = 2
C      GO TO 285
C      270 Y= (YI(N)-YI(N-1))/(XI(N)-XI(N-1))* (X-XI(N-1))+YI(N-1)
C      GO TO 900
C      280 JJ = 3
C      285 IF(N-3)290,290,295
C      290 J = 3
C      295 K = J-1
C      M = K-1
C      L = J+1
C      A1 = X-XI(M)

```

1630  
 1640  
 1650  
 1660  
 1670  
 1680  
 1690  
 1700  
 1710  
 1720  
 1730  
 1740  
 1750  
 1760  
 1770  
 1780  
 1790  
 1800  
 1810  
 1820  
 1830  
 1840  
 1850  
 1860  
 1870  
 1880  
 1890  
 1900  
 1910  
 1920  
 1930  
 1940  
 1950  
 1960  
 1970  
 1980  
 1990  
 2000  
 2010

A2 = X-XI(K)	0	2020
A3 = X-XI(J)	0	2030
AL = (X-XI(K))/(XI(J)-XI(K))	0	2040
S = AL*YI(J)+(1.0-AL)*YI(K)	0	2050
C1= A3*A2/((XI(M)-XI(K))*(XI(M)-XI(J)))	0	2060
C2= A1*A3/((XI(K)-XI(M))*(XI(K)-XI(J)))	0	2070
C3= A2*A1/((XI(J)-XI(M))*(XI(J)-XI(K)))	0	2080
P1 = C1*YI(M)+C2*YI(K)+C3*YI(J)	0	2090
IF(N-3)305,305,310	0	2100
305 P2 = P1	0	2110
GO TO 315	0	2120
310 A4 = X-XI(L)	0	2130
C4= A4*A3/((XI(K)-XI(J))*(XI(K)-XI(L)))	0	2140
C5= A2*A4/((XI(J)-XI(K))*(XI(J)-XI(L)))	0	2150
C6= A3*A2/((XI(L)-XI(K))*(XI(L)-XI(J)))	0	2160
P2 = C4*YI(K)+C5*YI(J)+C6*YI(L)	0	2170
315 GO TO (320,330,350),JJ	0	2180
320 P2 = P1	0	2190
AL = (X-XI(1))/(XI(2)-XI(1))	0	2200
S = AL*YI(2)+(1.0-AL)*YI(1)	0	2210
P1= S + XK*(P2-S)	0	2220
GO TO 350	0	2230
330 P1 = P2	0	2240
AL = (X-XI(N-1))/(XI(N)-XI(N-1))	0	2250
S = AL*YI(N)+(1.0-AL)*YI(N-1)	0	2260
P2 = S + XK*(P1-S)	0	2270
350 E1 = ABS(P1-S)	0	2280
E2 = ABS(P2-S)	0	2290
IF(E1+E2)400,400,410	0	2300
400 Y = S	0	2310
GO TO 900	0	2320
410 BT = (E1*AL)/(E1*AL+(1.0-AL)*E2)	0	2330
Y = BT*P2+(1.0-BT)*P1	0	2340
900 ANS(IE)=Y	0	2350
910 CONTINUE	0	2360
RETURN	0	2370
END	0	2380

FUNCTION COSD(X)  
Y=0.017453293\*X  
COSD=COS(Y)  
RETURN  
END

0 2390  
0 2400  
0 2410  
0 2420  
0 2430

```

SUBROUTINE DATAWR(DA)
  DIMENSION DA(1)
  DO 200 I=1,1000
    ID=(I-1)*5
    LDATA=0
    DO 100 J=1,5
      IJ=ID+J
      IF(DA(IJ).NE.0.0) LDATA=1
    CONTINUE
    IF(LDATA.EQ.0) GO TO 200
    ID1=ID+1
    WRITE(6,150) ID1,IJ,(DA(K),K=ID1,IJ)
    FORMAT(5H0DA( ,I4,13H ) THRU DA( ,I4,4H ) =1P5E15.6)
    CONTINUE
    WRITE(6,250)
    FORMAT(52H0ALL VALUES OF DA NOT GIVEN ABOVE ARE EQUAL TO ZERO.)
    RETURN
  END

```

0	2440
0	2450
0	2460
0	2470
0	2480
0	2490
0	2500
0	2510
0	2520
0	2530
0	2540
0	2550
0	2560
0	2570
0	2580
0	2590
0	2600
0	2610

```

SUBROUTINE NOR1(X,Y,Z)
  D=SQRT(X**2 + Y**2 + Z**2)
  X=X/D
  Y=Y/D
  Z=Z/D
  RETURN
  END

```

0	2620
0	2630
0	2640
0	2650
0	2660
0	2670
0	2680

```

FUNCTION DOT(X1,Y1,Z1,X2,Y2,Z2)
DOT=X1*X2+Y1*Y2+Z1*Z2
RETURN
END

```

```

0 2690
0 2700
0 2710
0 2720

```

```

FUNCTION ARCOS(X)
ARCOS=ACOS(X)
RETURN
END

```

```

0 2730
0 2740
0 2750
0 2760

```

```

FUNCTION COTAN(X)
COTAN=COS(X)/SIN(X)
RETURN
END

```

```

0 2770
0 2780
0 2790
0 2800

```

ORIGINAL PAGE IS  
OF POOR QUALITY

FUNCTION	CODIM1 (X,XI,YI,N,XK)	0	2810
		0	2820
CALLING SEQUENCE.....		0	2830
X	INDEPENDENT VARIABLE.....ABSCISSA REQUESTED	0	2840
XI	ARRAY OF GIVEN ABSCISSAS	0	2850
YI	ARRAY OF GIVEN ORDINATES	0	2860
N	NUMBER OF GIVEN POINTS DESCRIBING THE CURVE	0	2870
XK	END INTERVAL INTERPOLATION CONTROL CONSTANT	0	2880
	XK=0	0	2890
	XK=+1	0	2900
	XK BETWEEN	0	2910
	0 AND 1	0	2920
	XK=-1	0	2930
	PROGRAM WILL COMPUTE END INTERPOLATION	0	2940
	CONTROL CONSTANT	0	2950
		0	2960
		0	2970
		0	2980
		0	2990
		0	3000
		0	3010
		0	3020
		0	3030
		0	3040
		0	3050
		0	3060
		0	3070
		0	3080
		0	3090
		0	3100
		0	3110
		0	3120
		0	3130
		0	3140
		0	3150
		0	3160
		0	3170
		0	3180
		0	3190

DIMENSION	XI(1), YI(1)	
W = X		
N1 = N		
DETERMINE THE NUMBER OF POINTS GIVEN		
IF (N1-2)100,200,300		
ONE POINT GIVEN		
100 IF (XI(1)-W)130,175,130		
130 WRITE	( 6,150)	
150 FORMAT (99H- ONLY ONE POINT WAS GIVEN FOR ARRAY XI IN CODIM1....THO		
IF ABSCISSA ARGUMENT IS NOT THE SAME AS THE /16H0 GIVEN ABSCISSA)		
165 CALL DUMP		
175 CODIM1 = YI(1)		
GO TO 1700		
TWO POINT STRAIGHT LINE COMPUTATION		
200 N1 = 2		
TEST IF ABSCISSAS ARE IDENTICAL		
225 IF (XI(N1-1)-XI(N1))250,275,250		



C	GO TO 1700	0	3590
C	ABSCISSAS ARE DECREASING IN VALUE ALGEBRAICALLY	0	3600
C	FIND NEXT SMALLEST ABSCISSA AFTER W	0	3610
	600 DO 650 J=1,N1	0	3620
	IF (XI(J)-W)800,475,650	0	3630
	650 CONTINUE	0	3640
C	W IS LESS THAN XI(N)	0	3650
	GO TO 225	0	3660
C		0	3670
C	TEST IF W LIES BETWEEN XI(1) AND XI(2)	0	3680
C	800 IF (J-2)200,825,850	0	3690
C		0	3700
C	W LIES BETWEEN XI(1) AND XI(2)	0	3710
C	825 J = 3	0	3720
	JJ = 1	0	3730
	GO TO 925	0	3740
C		0	3750
C	W OCCURS AFTER XI(2).....TEST IF W IS BETWEEN XI(N-1) AND XI(N)	0	3760
C	850 IF (J-N1)900,875,225	0	3770
C		0	3780
C	W LIES BETWEEN XI(N-1) AND XI(N)	0	3790
C	875 J = N1-1	0	3800
	JJ = 2	0	3810
	GO TO 925	0	3820
C		0	3830
C	W LIES BETWEEN XI(2) AND XI(N-1)	0	3840
C	900 JJ = 3	0	3850
C		0	3860
C	SETUP SUBSCRIPTS	0	3870
C	925 K = J-1	0	3880
	M = K-1	0	3890
	L = J+1	0	3900
	XIM = XI(M)	0	3910
	XIK = XI(K)	0	3920
	XIJ = XI(J)	0	3930
	XIL = XI(L)	0	3940
C		0	3950
C	TEST IF N=3.....IF SO ALTER M AND JT SO THAT DO LOOPS 1040 AND	0	3960
		0	3970



```

C      1120 TEST ONLY 3 POINTS VICF 4
      IF (VI-3)970,940,970
      940 IF (JJ-2)950,940,970
      950 JT = 2
      GO TO 1000
      960 M = 1
      970 JT = J
C
C      TEST IF ABSCISSA VALUES ARE INCREASING OR DECREASING ALGEBRAICALLY
      1000 IF (XI(1)-XI(2))1020, 325,1100
C
C      TEST IF ABSCISSA VALUES ARE ALL INCREASING ALGEBRAICALLY
      1020 DO 1040 IB=M,JT
      IF (XI(IB)-XI(IB+1))1040,1060,1060
      1040 CONTINUE
      GO TO 1175
      1060 IB1 = IB+1
      WRITE
      ( 6,1070)IB,XI(IB),IB1,XI(IB1)
      1070 FORMAT ( 79H- ALL ABSCISAS IN CODINI ARE NOT EITHER ALL INCREASING
      1/DECREASING ALGEBRAICALLY /10H0 ABSCISSA17,2H =E17.8,10X, #HABSCIS
      2SA17,2H =E17.8)
      GO TO 165
C
C      TEST IF ABSCISSA VALUES ARE ALL DECREASING ALGEBRAICALLY
      1100 DO 1120 IB=M,JT
      IF (XI(IB)-XI(IB+1))1060,1060,1120
      1120 CONTINUE
C
      1175 A1 = W-XIM
      A2 = W-XIK
      A3 = W-XIJ
      A4 = W-XIL
      AL = (W-XIK)/(XIJ-XIK)
      C1 = A3*A2/((XIM-XIK)*(XIM-XIJ))
      C2 = A1*A3/((XIK-XIM)*(XIK-XIJ))
      C3 = A2*A1/((XIJ-XIM)*(XIJ-XIK))
      C4 = A4*A3/((XIK-XIJ)*(XIK-XIL))
      C5 = A2*A4/((XIJ-XIK)*(XIJ-XIL))
      C6 = A3*A2/((XIL-XIK)*(XIL-XIJ))

```

S = AL*YI(J)+(1.-AL)*YI(K)	0	4370
P1 = C1*YI(M)+C2*YI(K)+C3*YI(J)	0	4380
P2 = C4*YI(K)+C5*YI(J)+C6*YI(L)	0	4390
GO TO (1200,1400,1500),JJ	0	4400
C	0	4410
W LIES BETWEEN XI(1) AND XI(2)	0	4420
1200 P2 = P1	0	4430
IF (XK)1230,1220,1220	0	4440
1220 XE = XK	0	4450
GO TO 1260	0	4460
C	0	4470
COMPUTE XK	0	4480
1230 SLOPE1 = ABS((YI(K)-YI(M))/(XIK-XIM))	0	4490
SLOPE2 = ABS((YI(K)-YI(J))/(XIK-XIJ))	0	4500
XE = 1.- (ABS(SLOPE1-SLOPE2)/(SLOPE1+SLOPE2))	0	4510
C	0	4520
1260 AL = (W-XI(1))/(XI(2)-XI(1))	0	4530
S = AL*YI(2)+(1.-AL)*YI(1)	0	4540
P1 = S+XE*(P2-S)	0	4550
GO TO 1500	0	4560
C	0	4570
W LIES BETWEEN XI(N-1) AND XI(N)	0	4580
1400 P1 = P2	0	4590
IF (XK)1430,1420,1420	0	4600
1420 XE = XK	0	4610
GO TO 1460	0	4620
1430 SLOPE1 = ABS((YI(J)-YI(L))/(XIJ-XIL))	0	4630
SLOPE2 = ABS((YI(J)-YI(K))/(XIJ-XIK))	0	4640
XE = 1.- (ABS(SLOPE1-SLOPE2)/(SLOPE1+SLOPE2))	0	4650
C	0	4660
1460 AL = (W-XIJ)/(XIL-XIJ)	0	4670
S = AL*YI(L)+(1.-AL)*YI(J)	0	4680
P2 = S+XE*(P1-S)	0	4690
C	0	4700
1500 E1 = ABS(P1-S)	0	4710
E2 = ABS(P2-S)	0	4720
IF (E1+E2)1530,1530,1560	0	4730
1530 CODIM1 = S	0	4740
GO TO 1700	0	4750
C	0	

1560 BT = (E1\*AL)/(E1\*AL+(1.-AL)\*P2)  
 COLIM1 = CT\*P2+(1.-BT)\*P1

C 1700 RETURN  
 END

0 0 0 0 0 0  
 4760  
 4770  
 4780  
 4790  
 4800

```

PROGRAM BODYGM
COMMON DA(5000)
1  ,NX,NXTH,LNVOR,LTIVOR,NIVV,NBVV,NIV,NXTHV,NBV,NTH(49)
2  ,LNDIV,LTDIV,LNPTS,LTPTS
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LIC(7),LNC(7)
1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON/BODY/ XVL(151,31),YV(151,31),ZV(151,31)
EQUIVALENCE(DA(1),BODIES),(DA(15),BODYNO)
EQUIVALENCE(DA(10),ALPHA),(DA(11),BETA)
COMMON /COMPRS/ BETAM
EQUIVALENCE(DA(3),FMACH)
C*****
C*****
COMMON/PANATT/NATT(30),XATT(200)
COMMON/SCRAT/ ATACH(50)
CALL NARDAP
REWIND 21
READ(21) ATACH
DO 5 I=1,10
K=(I-1)*5
J=(I-1)*3
NATT(J+1)=ATACH(K+1)
NATT(J+2)=ATACH(K+2)
NATT(J+3)=ATACH(K+3)
WRITE(6,205) NATT
205 FORMAT(*ONATT*/(3I5))
C*****
C*****
DO 1 I=1,14
1  DA(I)=0.0
MTAPE=19
NTAPE=20
C UNIT 18 - VORTEX COORDINATES
REWIND 18
C UNITS 19,20 - AX,AY,AZ MATRICES (COMPUTED IN SUBROUTINE INFL)
REWIND 19
REWIND 20

```

```

C UNIT 21 - COMPLETE INFLUENCE MATRIX :A: (COMPUTED IN SUB. MATA) 0400
REWIND 21 0410
C UNIT 23 - PHI,THETA MATRIX. 0420
REWIND 23 0430
C REWIND UNIT 12. UNIT 12 WILL HAVE DATA USED FOR FORCES. 0440
REWIND 12 0450
CALL DECRD(DA) 0460
NBODS=3BODIES 0470
ALPHA=ALPHA/57.29577951 0480
BETA=BETA/57.29577951 0490
BETAM = SQRT(1.0-FMACH**2) 0500
IF(NBODS.EQ.0) GO TO 105 0510
C NCT IS COUNTER FOR CONTROL POINTS. ACT IS INCREMENTED IN SUB. VPTS. 0520
NCT=C 0530
NCTV=0 0540
KCON=0 0550
KL=0 0560
KLT=0 0570
DO 100 I=1,NBODS 0580
DO 10 J=15,3419 0590
DA(J)=0.0 0600
CALL DECRD(DA) 0610
NB=BODYNO 0620
CALL SEIDAT 0630
CALL XYZVB 0640
CALL MATL(KL) 0650
CALL MATLT(KLT) 0660
CALL VPTS 0670
CALL ATTACH(XV1,YV,ZV,151,-1) 0680
NSEG=ISEG(I)+LSEG(I) 0690
KCON=KCON+NSEG 0700
IF(NSEG.EQ.0) GO TO 100 0710
CALL MATPT 0720
100 CONTINUE 0730
C 0740
C VORTEX POINTS (INCLUDING DIVISION PTS.) ARE ON UNIT 18. (ALL BODIES) 0750
C UNIT 23 HAS PHI,THETA MATRIX FOR ALL BODIES. 0760
C ALNGTH HAS L FOR ALL BODIES 0770
C 0780

```

105 CONTINUE  
END

1 0790  
1 0800

```

SUBROUTINE MATPI
  COMPUTE PHI,THETA CONSTRAINT TRANSFORMATION MATRIX.
  C
  COMMON DA(5000)
  1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
  2  ,LNDIV,LTDIV,LNPTS,LTPIS
  EQUIVALENCE (DA(17),CHORD)
  1  , (DA(3140),CSLN), (DA(3190),CSLT), (DA(3210),CFLN), (DA(3250),CFLT)
  2  , (DA(41),XS), (DA(34),FUNCLN), (DA(35),FUNCLT), (DA(36),SEGLN)
  3  , (DA(37),SEGLT), (DA(27),XBO)
  COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
  1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
  2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
  3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
  COMMON/SCRAT/ XVV(200),THVV(151,31)
  1  ,XVOR(100)
  DIMENSION CFLN(40),CFLT(40),CSLN(50),CSLT(20),ANORM(3),XS(49)
  DIMENSION PTH(5000)
  COMMON/BODY/ ARRAY(20000)
  EQUIVALENCE(ARRAY(15001),PTH)
  C DIVLDA IS NO. LONG. DIV. GIVEN IN DATA.
  EQUIVALENCE (DIVLDA,DA(321))
  LNFUNC=FUNCLN
  LTFUNC=FUNCLT
  LNSEG=SEGLN
  LTSEG=SEGLT
  PI=3.1415926
  C DETERMINE IF Y,Z AND NORMALS ARE REQUIRED FOR CONSTRAINT FUNCTIONS.
  C SET CODE=1 IF THEY ARE REQUIRED.
  CODE=0
  DO 1 I=1,2
    IF(CFLN(I),NE,2.0) GO TO 1
  CODE=1
  GO TO 3
  CONTINUE
  DO 2 I=1,3
    IF(CFLT(I),NE,2.0.AND,CFLT(I),NE,3.0) GO TO 2
  CODE=1
  GO TO 3
  1
  2
  3
  4
  5
  6
  7
  8
  9
  10
  11
  12
  13
  14
  15
  16
  17
  18
  19
  20
  21
  22
  23
  24
  25
  26
  27
  28
  29
  30
  31
  32
  33
  34
  35
  36
  37
  38
  39
  40
  41
  42
  43
  44
  45
  46
  47
  48
  49
  50
  51
  52
  53
  54
  55
  56
  57
  58
  59
  60
  61
  62
  63
  64
  65
  66
  67
  68
  69
  70
  71
  72
  73
  74
  75
  76
  77
  78
  79
  80
  81
  82
  83
  84
  85
  86
  87
  88
  89
  90
  91
  92
  93
  94
  95
  96
  97
  98
  99
  100
  101
  102
  103
  104
  105
  106
  107
  108
  109
  110
  111
  112
  113
  114
  115
  116
  117
  118
  119
  120
  121
  122
  123
  124
  125
  126
  127
  128
  129
  130
  131
  132
  133
  134
  135
  136
  137
  138
  139
  140
  141
  142
  143
  144
  145
  146
  147
  148
  149
  150
  151
  152
  153
  154
  155
  156
  157
  158
  159
  160
  161
  162
  163
  164
  165
  166
  167
  168
  169
  170
  171
  172
  173
  174
  175
  176
  177
  178
  179
  180
  181
  182
  183
  184
  185
  186
  187
  188
  189
  190
  191
  192
  193
  194
  195
  196
  197
  198
  199
  200
  201
  202
  203
  204
  205
  206
  207
  208
  209
  210
  211
  212
  213
  214
  215
  216
  217
  218
  219
  220
  221
  222
  223
  224
  225
  226
  227
  228
  229
  230
  231
  232
  233
  234
  235
  236
  237
  238
  239
  240
  241
  242
  243
  244
  245
  246
  247
  248
  249
  250
  251
  252
  253
  254
  255
  256
  257
  258
  259
  260
  261
  262
  263
  264
  265
  266
  267
  268
  269
  270
  271
  272
  273
  274
  275
  276
  277
  278
  279
  280
  281
  282
  283
  284
  285
  286
  287
  288
  289
  290
  291
  292
  293
  294
  295
  296
  297
  298
  299
  300
  301
  302
  303
  304
  305
  306
  307
  308
  309
  310
  311
  312
  313
  314
  315
  316
  317
  318
  319
  320
  321
  322
  323
  324
  325
  326
  327
  328
  329
  330
  331
  332
  333
  334
  335
  336
  337
  338
  339
  340
  341
  342
  343
  344
  345
  346
  347
  348
  349
  350
  351
  352
  353
  354
  355
  356
  357
  358
  359
  360
  361
  362
  363
  364
  365
  366
  367
  368
  369
  370
  371
  372
  373
  374
  375
  376
  377
  378
  379
  380
  381
  382
  383
  384
  385
  386
  387
  388
  389
  390
  391
  392
  393
  394
  395
  396
  397
  398
  399
  400
  401
  402
  403
  404
  405
  406
  407
  408
  409
  410
  411
  412
  413
  414
  415
  416
  417
  418
  419
  420
  421
  422
  423
  424
  425
  426
  427
  428
  429
  430
  431
  432
  433
  434
  435
  436
  437
  438
  439
  440
  441
  442
  443
  444
  445
  446
  447
  448
  449
  450
  451
  452
  453
  454
  455
  456
  457
  458
  459
  460
  461
  462
  463
  464
  465
  466
  467
  468
  469
  470
  471
  472
  473
  474
  475
  476
  477
  478
  479
  480
  481
  482
  483
  484
  485
  486
  487
  488
  489
  490
  491
  492
  493
  494
  495
  496
  497
  498
  499
  500
  501
  502
  503
  504
  505
  506
  507
  508
  509
  510
  511
  512
  513
  514
  515
  516
  517
  518
  519
  520
  521
  522
  523
  524
  525
  526
  527
  528
  529
  530
  531
  532
  533
  534
  535
  536
  537
  538
  539
  540
  541
  542
  543
  544
  545
  546
  547
  548
  549
  550
  551
  552
  553
  554
  555
  556
  557
  558
  559
  560
  561
  562
  563
  564
  565
  566
  567
  568
  569
  570
  571
  572
  573
  574
  575
  576
  577
  578
  579
  580
  581
  582
  583
  584
  585
  586
  587
  588
  589
  590
  591
  592
  593
  594
  595
  596
  597
  598
  599
  600
  601
  602
  603
  604
  605
  606
  607
  608
  609
  610
  611
  612
  613
  614
  615
  616
  617
  618
  619
  620
  621
  622
  623
  624
  625
  626
  627
  628
  629
  630
  631
  632
  633
  634
  635
  636
  637
  638
  639
  640
  641
  642
  643
  644
  645
  646
  647
  648
  649
  650
  651
  652
  653
  654
  655
  656
  657
  658
  659
  660
  661
  662
  663
  664
  665
  666
  667
  668
  669
  670
  671
  672
  673
  674
  675
  676
  677
  678
  679
  680
  681
  682
  683
  684
  685
  686
  687
  688
  689
  690
  691
  692
  693
  694
  695
  696
  697
  698
  699
  700
  701
  702
  703
  704
  705
  706
  707
  708
  709
  710
  711
  712
  713
  714
  715
  716
  717
  718
  719
  720
  721
  722
  723
  724
  725
  726
  727
  728
  729
  730
  731
  732
  733
  734
  735
  736
  737
  738
  739
  740
  741
  742
  743
  744
  745
  746
  747
  748
  749
  750
  751
  752
  753
  754
  755
  756
  757
  758
  759
  760
  761
  762
  763
  764
  765
  766
  767
  768
  769
  770
  771
  772
  773
  774
  775
  776
  777
  778
  779
  780
  781
  782
  783
  784
  785
  786
  787
  788
  789
  790
  791
  792
  793
  794
  795
  796
  797
  798
  799
  800
  801
  802
  803
  804
  805
  806
  807
  808
  809
  810
  811
  812
  813
  814
  815
  816
  817
  818
  819
  820
  821
  822
  823
  824
  825
  826
  827
  828
  829
  830
  831
  832
  833
  834
  835
  836
  837
  838
  839
  840
  841
  842
  843
  844
  845
  846
  847
  848
  849
  850
  851
  852
  853
  854
  855
  856
  857
  858
  859
  860
  861
  862
  863
  864
  865
  866
  867
  868
  869
  870
  871
  872
  873
  874
  875
  876
  877
  878
  879
  880
  881
  882
  883
  884
  885
  886
  887
  888
  889
  890
  891
  892
  893
  894
  895
  896
  897
  898
  899
  900
  901
  902
  903
  904
  905
  906
  907
  908
  909
  910
  911
  912
  913
  914
  915
  916
  917
  918
  919
  920
  921
  922
  923
  924
  925
  926
  927
  928
  929
  930
  931
  932
  933
  934
  935
  936
  937
  938
  939
  940
  941
  942
  943
  944
  945
  946
  947
  948
  949
  950
  951
  952
  953
  954
  955
  956
  957
  958
  959
  960
  961
  962
  963
  964
  965
  966
  967
  968
  969
  970
  971
  972
  973
  974
  975
  976
  977
  978
  979
  980
  981
  982
  983
  984
  985
  986
  987
  988
  989
  990
  991
  992
  993
  994
  995
  996
  997
  998
  999
  1000

```

```

2  CONTINUE
3  CONTINUE
10 IF(LTFUNC) 10,10,11
   JTLIM=NTVV
   GO TO 12
11 JTLIM=LTFUNC
12 IF(LNFUNC) 13,13,14
13 JNLIM=NBVV
   GO TO 15
14 JNLIM=LNFUNC
15 CONTINUE
   KFUNC=0
   IF(LNFUNC.NE.0) GO TO 25
   IF(LTFUNC.EQ.0) GO TO 25
   KFUNC=1
   JTS=JTLIM
   JTLIM=JNLIM
   JNLIM=JTS
   CONTINUE
25 ITAPE=0
   DO 5000 JTI=1,JTLIM
   JT=JTI
   IF(KFUNC.NE.1) GO TO 26
   JN=JTI
   DO 5000 JNN=1,JNLIM
   ITAPE=ITAPE+1
   IF(KFUNC.EQ.0) GO TO 261
   JT=JNN
   GO TO 27
261 JN=JNN
27 IF(LTFUNC.EQ.0) GO TO 28
   LTCF=CFLT(JT)
28 IF(LNFUNC.EQ.0) GO TO 29
   LNCF=CFLN(JN)
29 CONTINUE
   KC=0
   DO 1000 J=1,NTVV
   DO 1000 I=1,NBVV
   KC=KC+1

```

```

1 1200
1 1210
1 1220
1 1230
1 1240
1 1250
1 1260
1 1270
1 1280
1 1290
1 1300
1 1310
1 1320
1 1330
1 1340
1 1350
1 1360
1 1370
1 1380
1 1390
1 1400
1 1410
1 1420
1 1430
1 1440
1 1450
1 1460
1 1470
1 1480
1 1490
1 1500
1 1510
1 1520
1 1530
1 1540
1 1550
1 1560
1 1570
1 1580

```



IF(LNCF.EQ.1) GO TO 75	1	1590
IF(LNCF.EQ.0) GO TO 75	1	1600
DO 50 II=1,LNSEQ	1	1610
LNCS=CSLN(II)	1	1620
IF(I.LE.LNCS) GO TO 55	1	1630
CONTINUE	1	1640
IF(II.NE.1) GO TO 60	1	1650
XO=XS(1)-XB0	1	1660
PHIO=ARCOS(1.0-2.0*XO/CHORD)	1	1670
GO TO 65	1	1680
III=CSLN(II-1)	1	1690
IX1=III	1	1700
IX2=IX1+1	1	1710
XO=0.75*XVOR(IX2)-0.25*XVOR(IX1)	1	1720
XO=XO-XB0	1	1730
GO TO 58	1	1740
III=CSLN(II)	1	1750
IF(III.LT.NBVV) GO TO 66	1	1760
XF=CHORD	1	1770
GO TO 68	1	1780
IX1=III	1	1790
IX2=IX1+1	1	1800
XF=0.75*XVOR(IX2)-0.25*XVOR(IX1)	1	1810
XF=XF-XB0	1	1820
IF(XF.GT.CHORD) XF=CHORD	1	1830
PHIF=ARCOS(1.0-2.0*XF/CHORD)	1	1840
X=XVOR(1)	1	1850
X=X-XB0	1	1860
PHI =ARCOS(1.0-2.0*X /CHORD)	1	1870
PRAT=PI*(PHI-PHIO)/(PHIF-PHIO)	1	1880
IF(LTCF.EQ.1) GO TO 85	1	1890
IF(LTCF.EQ.0) GO TO 85	1	1900
DO 80 JJ=1,LTSEG	1	1910
LTC=CSLT(JJ)	1	1920
IF(J.LE.LTCS) GO TO 81	1	1930
CONTINUE	1	1940
JX1=(J-1)*LTDIV+1	1	1950
JX2=J*LTDIV+1	1	1960
II=(I-1)*DIVLDA+1	1	1970

THEIA=0.5\*(THVV(I1,JX1))+THVV(I1,JX2))

IF(JJ.NE.1) GO TO 82

THEIO=THVV(I1,1)

GO TO 83

82 JX1=(CSLT(JJ-1))\*LTDIV+1.01

THEIO=THVV(I1,JX1)

83 JX2=LICS\*LTDIV+1

THEIF=THVV(I1,JX2)

IRAI=PI\*(THEIA-THIO)/(THEIF-THIO)

85 CONTINUE

IF(KODE.EQ.0) GO TO 100

CALL VYZN(J,I,Y,Z,ANORM)

100 CONTINUE

LATF=1

LONF=1

C IF LATF=0, DO NOT COMPUTE GT.

C IF LONF=0, DO NOT COMPUTE GN.

IF(LIFUNC.NE.0) GO TO 105

LATF=0

IF(JT.NE.J) GO TO 30

GT=1.0

GO TO 105

30 GT=0.0

LONF=0

105 CONTINUE

IF(LNFUNC.NE.0) GO TO 106

LONF=0

IF(JN.NE.I) GO TO 251

GN=1.0

GO TO 106

251 GN=0.0

LATF=0

106 CONTINUE

IF(LATF.EQ.0) GO TO 300

GO TO(110,120,130,140,150,160,170,180,190,200),LTCF

110 GT=1.0

GO TO 300

120 GT=ANORM(3)/SQRT(ANORM(3)\*\*2+ANORM(2)\*\*2)

GO TO 300

1 1980

1 1990

1 2000

1 2010

1 2020

1 2030

1 2040

1 2050

1 2060

1 2070

1 2080

1 2090

1 2100

1 2110

1 2120

1 2130

1 2140

1 2150

1 2160

1 2170

1 2180

1 2190

1 2200

1 2210

1 2220

1 2230

1 2240

1 2250

1 2260

1 2270

1 2280

1 2290

1 2300

1 2310

1 2320

1 2330

1 2340

1 2350

1 2360

130	GT=ANORM(2)/SQRT(ANORM(3)**2+ANORM(2)**2)		2370
	GO TO 300		2380
140	GT=SIN(PI*TRAT)		2390
	GO TO 300		2400
150	GT=COS(PI*TRAT)		2410
	GO TO 300		2420
160	GT=TRAT		2430
	GO TO 300		2440
170	GT=SIN(2.0*PI*TRAT)		2450
	GO TO 300		2460
180	GT=COS(2.0*PI*TRAT)		2470
	GO TO 300		2480
190	GT=TRAT**2		2490
	GO TO 300		2500
200	GT=0.0		2510
300	CONTINUE		2520
	IF(LONF.EQ.0) GO TO 500		2530
	GO TO (310,320,330,340,350,360,370,380,390,400		2540
	1,403,404,405,406,407,408,409,410,411,412,413,414,415),LNCF		2550
310	GN=1.0		2560
	GO TO 500		2570
320	T1=Y*ANORM(1)/ANORM(2)+Z*ANORM(1)/ANORM(3)		2580
	GN=1.0/SQRT(1.0+(T1**2)/(Y**2+Z**2))		2590
	GO TO 500		2600
330	GN=COTAN(0.5*PI)		2610
	GO TO 500		2620
340	GN=COTAN(0.5*(PI-PI))		2630
	GO TO 500		2640
350	GN=SIN(PI*TRAT)		2650
	GO TO 500		2660
360	GN=COS(PI*TRAT)		2670
	GO TO 500		2680
370	GN=(X-X0)/(XF-X0)		2690
	GO TO 500		2700
380	GN=SIN(2.0*PI*TRAT)		2710
	GO TO 500		2720
390	GN=COS(2.0*PI*TRAT)		2730
	GO TO 500		2740
400	GN=((X-X0)/(XF-X0))**2		2750

```

      GO TO 500
403  GN=SIN(3.0*PRAT)
      GO TO 500
404  GN=SIN(4.0*PRAT)
      GO TO 500
405  GN=SIN(5.0*PRAT)
      GO TO 500
406  GN=SIN(6.0*PRAT)
      GO TO 500
407  GN=SIN(7.0*PRAT)
      GO TO 500
408  GN=SIN(8.0*PRAT)
      GO TO 500
409  GN=SIN(9.0*PRAT)
      GO TO 500
410  GN=SIN(10.0*PRAT)
      GO TO 500
411  GN=SIN(11.0*PRAT)
      GO TO 500
412  GN=SIN(12.0*PRAT)
      GO TO 500
413  GN=SIN(13.0*PRAT)
      GO TO 500
414  GN=SIN(14.0*PRAT)
      GO TO 500
415  GN=SIN(15.0*PRAT)
500  CONTINUE
1000 PTH(KC)=GN*GT
5000 WRITE(23)(PTH(K1),K1=1,KC)
      RETURN
      END

```

```

1 2760
1 2770
1 2780
1 2790
1 2800
1 2810
1 2820
1 2830
1 2840
1 2850
1 2860
1 2870
1 2880
1 2890
1 2900
1 2910
1 2920
1 2930
1 2940
1 2950
1 2960
1 2970
1 2980
1 2990
1 3000
1 3010
1 3020
1 3030
1 3040
1 3050
1 3060

```



```

4  CONTINUE
   NBI=NBVV+1
   SUBAX=0.0
   SUBAY=0.0
   SUBAZ=0.0
   K=0
   DO 50 J=1,NTV
     JI=J+1
     DO 50 I=1,NBV
       I1=I+1
       K=K+1
       X31=XV (I1,J1)-XV (I,J)
       Y31=YV(I1,J1)-YV(I,J)
       Z31=ZV(I1,J1)-ZV(I,J)
       X24=XV (I,J1)-XV (I1,J)
       Y24=YV(I,J1) -YV(I1,J)
       Z24=ZV(I,J1) -ZV(I1,J)
       XAREA(K) = (Y31*Z24-Z31*Y24)*0.5
       YAREA(K) = (Z31*X24-X31*Z24)*0.5
       ZAREA(K) = (X31*Y24-Y31*X24)*0.5
       SUBAX=SUBAX+XAREA(K)
       SUBAY=SUBAY+YAREA(K)
       SUBAZ=SUBAZ+ZAREA(K)
     WRITE(6,55) SUBAX,SUBAY,SUBAZ
     FORMAT(1H0/(1P8E15.6))
     DO 100 I=1,NBV
       K1=(I-1)*LNDIV+1
       K2=K1+LNDIV
       XVOR(I)=0.75*XVV(K1)+0.25*XVV(K2)
       WRITE(6,55)(XVOR(I),I=1,NBV)
       N3=NBV+1
       N2=NTV+1
       DO 300 I=1,LNPTS
         KC=FLNC(I)
         KC=(KC-1)*LNDIV+1
         XCON(I)= 0.25*XVV(KC)+0.75*XVV(KC+LNDIV)
       300
     C
     C WRITE AREAS ON UNIT 12 FOR USE IN CALCULATING FORCES.
     C

```

```

3850
3860
3870
3880
3890
3900
3910
3920
3930
3940
3950
3960
3970
3980
3990
4000
4010
4020
4030
4040
4050
4060
4070
4080
4090
4100
4110
4120
4130
4140
4150
4160
4170
4180
4190
4200
4210
4220
4230

WRITE(12) XAREA,YAREA,ZAREA
K=0
DO 60 J=1,NTV
J1=J+1
DO 60 I=1,NBV
I1=I+1
K=K+1
XPC(K)=0.25*(XV(I,J) + XV(I1,J) + XV(I,J1) + XV(I1,J1))
YPC(K)=0.25*(YV(I,J) + YV(I1,J) + YV(I,J1) + YV(I1,J1))
ZPC(K)=0.25*(ZV(I,J) + ZV(I1,J) + ZV(I,J1) + ZV(I1,J1))
DO 65 K=1,NB1
K1=(K-1)*LNDIV+1
XTL(K)=XVV(K1)

WRITE SUB-PANEL CENTERS AND X FOR PANEL EDGES ON UNIT 12 FOR FORCES.
C
C ALSO WRITE XCON=X COORDS FOR CONTROL POINTS AND OTHER NEEDED TERMS.
C
WRITE(12) XPC,YPC,ZPC,XTL,XCON,FLNC,FLTC,NBV,NTV,LNPTS,LTPTS
1
,LNDIV,LTDIV,NBV,NTVV,CHORD,XCG,YCG,ZCG,ALPHA,BREF
WRITE(6,55)(XCON(I),I=1,LNPTS)
NLAT1=LTDIV/2 + 1
JS=NCT
DO 500 J=1,LTPTS
K1=LTDIV*(FLTC(J)-1.0)+NLAT1
K2=K1+1
DO 400 I=1,NBV
I2=I1+1
XQ(I1)=0.25*(XV(I2,K1)+XV(I1,K1)+XV(I2,K2)+XV(I1,K2))
T1(I1)=XV(I2,K1)-XV(I1,K1)+XV(I2,K2)-XV(I1,K2)
T2(I1)=YV(I2,K1)-YV(I1,K1)+YV(I2,K2)-YV(I1,K2)
T3(I1)=ZV(I2,K1)-ZV(I1,K1)+ZV(I2,K2)-ZV(I1,K2)
CALL INTER(XQ,T11,NBV,XCON,T12,LNPTS)
CALL INTER(XQ,T21,NBV,XCON,T22,LNPTS)
CALL INTER(XQ,T31,NBV,XCON,T32,LNPTS)
DO 410 I=1,LNPTS
DENOM=SQRT(T12(I)**2+T22(I)**2+T32(I)**2)
T12(I)=T12(I)/DENOM
T22(I)=T22(I)/DENOM
T32(I)=T32(I)/DENOM
JSI=JS+1

```

410	TMX(JSI)=T12(I)	1	4240
	TMX(JSI)=T22(I)	1	4250
	TMZ(JSI)=T32(I)	1	4260
	DO 450 I1=1,NBV	1	4270
	I2=I1+1	1	4280
	T11(I1)=XV(I1,K2)-XV(I1,K1)+XV(I2,K2)-XV(I2,K1)	1	4290
	T21(I1)=YV(I1,K2)-YV(I1,K1)+YV(I2,K2)-YV(I2,K1)	1	4300
450	T31(I1)=ZV(I1,K2)-ZV(I1,K1)+ZV(I2,K2)-ZV(I2,K1)	1	4310
	CALL INTER(XQ,T11,NBV,XCON,T12,LNPTS)	1	4320
	CALL INTER(XQ,T21,NBV,XCON,T22,LNPTS)	1	4330
	CALL INTER(XQ,T31,NBV,XCON,T32,LNPTS)	1	4340
	DO 460 I=1,LNPTS	1	4350
	DENOM=SQRT(T12(I)**2+T22(I)**2+T32(I)**2)	1	4360
	T12(I)=T12(I)/DENOM	1	4370
	T22(I)=T22(I)/DENOM	1	4380
	T32(I)=T32(I)/DENOM	1	4390
	JSI=JS+I	1	4400
	TTX(JSI)=T12(I)	1	4410
	TTY(JSI)=T22(I)	1	4420
460	TTZ(JSI)=T32(I)	1	4430
	JS=JS+LNPTS	1	4440
500	CONTINUE	1	4450
	JS=NCT	1	4460
	DO 600 K=1,LTPTS	1	4470
	K1=LTDIV*(FLTC(K)-1.0)+NLAT1	1	4480
	K2=K1+1	1	4490
	CALL INTER(XV(1,K1),YV(1,K1),N3,XCON,T11,LNPTS)	1	4500
	CALL INTER(XV(1,K2),YV(1,K2),N3,XCON,T12,LNPTS)	1	4510
	CALL INTER(XV(1,K1),ZV(1,K1),N3,XCON,T21,LNPTS)	1	4520
	CALL INTER(XV(1,K2),ZV(1,K2),N3,XCON,T22,LNPTS)	1	4530
	DO 550 I=1,LNPTS	1	4540
	JS=JS+1	1	4550
	XQ(JS)=XCON(I)	1	4560
	YQ(JS)=0.5*(T11(I)+T12(I))	1	4570
550	ZQ(JS)=0.5*(T21(I)+T22(I))	1	4580
600	CONTINUE	1	4590
	DO 250 J=1,N2	1	4600
	CALL INTER(XV(1,J),YV(1,J),N3,XVOR,T11,NBVV)	1	4610
	CALL INTER(XV(1,J),ZV(1,J),N3,XVGR,T12,NBVV)	1	4620



DO 250 I=1,NBVV	1	4630
YV(I,J)=T11(I)	1	4640
ZV(I,J)=T12(I)	1	4650
CONTINUE	1	4660
DO 650 I=1,NBVV	1	4670
DO 650 J=1,N2	1	4680
XV(I,J)=XVOR(I)	1	4690
DO 700 J=1,N2	1	4700
XV(NB1,J)=XVV(N3)	1	4710
YV(NB1,J)=YV(N3,J)	1	4720
ZV(NB1,J)=ZV(N3,J)	1	4730
DO 800 J=1,LTPTS	1	4740
DO 800 I=1,LNPTS	1	4750
NCT=NCT+1	1	4760
XN1=TMV(NCT)*TTZ(NCT)-TMZ(NCT)*TTY(NCT)	1	4770
YN1=TMZ(NCT)*TTX(NCT)-TMX(NCT)*TTZ(NCT)	1	4780
ZN1=TMX(NCT)*TTY(NCT)-TMY(NCT)*TTX(NCT)	1	4790
DENOM=XN1**2+YN1**2+ZN1**2	1	4800
XN(NCT)=XN1/DENOM	1	4810
YN(NCT)=YN1/DENOM	1	4820
ZN(NCT)=ZN1/DENOM	1	4830
CONTINUE	1	4840
WRITE(18) BODYWR	1	4850
WRITE(6,55) (XQ(I),I=1,NCT)	1	4860
WRITE(6,55) (YQ(I),I=1,NCT)	1	4870
WRITE(6,55) (ZQ(I),I=1,NCT)	1	4880
WRITE(6,55) (XN(I),I=1,NCT)	1	4890
WRITE(6,55) (YN(I),I=1,NCT)	1	4900
WRITE(6,55) (ZN(I),I=1,NCT)	1	4910
RETURN	1	4920
END	1	4930
	1	4940

```

SUBROUTINE MATLT(K)
C
C   BLNGTH GIVE LATERAL LENGTHS OF PANELS .....
C   FIRST LONGITUDINALLY -- THEN Laterally.
C
COMMON DA(5000)
1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
2  ,LNDIV,LTDIV,LNPTS,LTPTS
COMMON/BODY/ ARRAY(31000)
DIMENSION XV1(151,31),YV(151,31),ZV(151,31),BLNGTH(5000)
EQUIVALENCE(BLNGTH,ARRAY(15001)),(XV1,ARRAY(1))
1  , (YV,ARRAY(4682)),(ZV,ARRAY(9363))
REAL L2
LN=LNDIV/2+1
DO 100 J=1,NTVV
JT=(J-1)*LTDIV
DO 100 I=1,NBVV
II1=(I-1)*LNDIV+LN
II2=II1+1
L2=0.0
DO 50 JJ=1,LTDIV
JJ1=JT+JJ
JJ2=JJ1+1
X1=0.5*(XV1(II1,JJ1)+XV1(II2,JJ1))
X2=0.5*(XV1(II1,JJ2)+XV1(II2,JJ2))
DX=X2-X1
Y1=0.5*(YV(II1,JJ1)+YV(II2,JJ1))
Y2=0.5*(YV(II1,JJ2)+YV(II2,JJ2))
DY=Y2-Y1
Z1=0.5*(ZV(II1,JJ1)+ZV(II2,JJ1))
Z2=0.5*(ZV(II1,JJ2)+ZV(II2,JJ2))
DZ=Z2-Z1
50  L2=SQRT(DX**2+DY**2+DZ**2)+L2
K=K+1
100 BLNGTH(K)=L2
WRITE(6,200)(BLNGTH(I),I=1,K)
200 FORMAT(*OBLNGTH*/(1P8E15.6))
WRITE(18) BLNGTH
RETURN

```

1 5340

END

```

SUBROUTINE MATL(K)
COMMON DA(5000)
1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
2  ,LNDIV,LTDIV,LNPTS,LTPTS
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON/BODY/ ARRAY(31000)
DIMENSION XV1(151,31),YV(151,31),ZV(151,31),ALNGTH(5000)
EQUIVALENCE(ALNGTH,ARRAY(15001)),(XV1,ARRAY(1))
1  , (YV,ARRAY(4682)),(ZV,ARRAY(9363))
DO 10 J=1,NTVV
JJ1=(J-1)*LTDIV+LTDIV/2 + 1
JJ2=JJ1+1
DO 10 I=1,NBVV
K=K+1
VL=0.0
DO 4 II=1,LNDIV
II1=LNDIV*(I-1)+II
II2=II1+1
X1=0.5*(XV1(II1,JJ1)+XV1(II1,JJ2))
X2=0.5*(XV1(II2,JJ1)+XV1(II2,JJ2))
DX=X2-X1
Y1=0.5*(YV(II1,JJ1)+YV(II1,JJ2))
Y2=0.5*(YV(II2,JJ1)+YV(II2,JJ2))
DY=Y2-Y1
Z1=0.5*(ZV(II1,JJ1)+ZV(II1,JJ2))
Z2=0.5*(ZV(II2,JJ1)+ZV(II2,JJ2))
DZ=Z2-Z1
4  VL=SQRT(DX**2+DY**2+DZ**2)+VL
10  ALNGTH(K)=VL
200 WRITE(6,200)(ALNGTH(I),I=1,K)
FORMAT(*OALNGTH*/(1P8E15.6))
WRITE(18) ALNGTH
RETURN
END

```

```

SUBROUTINE XYZVB
COMMON DA(5000)
1  ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
EQUIVALENCE(DA(3),FMACH)
COMMON/COMPRS/ BETAM
EQUIVALENCE (DA(17),CHORD),
1  ,(DA(20),BODTAB),(DA(19),SYM),(DA(2151),XTHV),(DA(131),TH)
2  ,(DA(800),R),(DA(1850),XV),(DA(41),XS),(DA(86),XTH)
3  ,(DA(1600),CPTS),(DA(1601),XMF),(DA(1650),YMF),(DA(1700),ZMF)
4  ,(DA(1750),YCAM),(DA(1800),ZCAM),(DA(2200),THVS),(DA(22),CAMBI)
EQUIVALENCE (DA(21),BMULT)
COMMON/SCRAT/ XVV(200),THVV(151,31)
DIMENSION THV(31,151), THSS(151,31),
1  ,ST(31,31), RR(151,31),
2  ,YCP(200), ZCP(200), SINYC(200), COSYC(200)
3  ,SINZC(200), COSZC(200), SLAT(100), TH(1200)
4  ,XTHV(150), TH(669), R(800), XS(49)
5  ,XTH(49), XMF(49), YMF(49), ZMF(49)
6  ,YCAM(50), THVS(800), XV(150), ZCAM(50)
DIMENSION D(25000), THETA(200),
1  ,YM(200), ZM(200), YC(200),
2  ,YY(151,31), ZZ(151,31), Y(151,31), Z(151,31)
3  ,XM(200), XP(200)
EQUIVALENCE(D,ARRAY)
EQUIVALENCE(D(1),THV), {D(4682),THSS},
1  ,(D(10324),S), {D(15005),ST},
2  ,(D(20647),YCM),(D(20847),ZCM),
3  ,(D(21247),ZCP),(D(21447),SINYC),
4  ,(D(21847),SINZC),(D(22047),COSZC),
5  ,(D(22347),TH1), {D(22547),THETA},
6  ,(D(22797),Z1), {D(22847),YM},
7  ,(D(23247),YC), {D(23447),ZC}
8  ,(D(23647),XM), {D(23847),XP}

```

```

COMMON/BODY/ ARRAY(31000)
EQUIVALENCE(XV1,ARRAY(1)), (YV,ARRAY(4682)),(ZV,ARRAY(9363))
1      ,(XB00,ARRAY(26964)),(YB00,ARRAY(26969))
2      ,(ZB00,ARRAY(26974))
1      DIMENSION XV1(151,31),YV(151,31),ZV(151,31),XB00(5),YB00(5)
1      ,ZB00(5)
1      COMMON/COMPTS/XQ(1320),YQ(1320),ZQ(1320)
1      EQUIVALENCE (YY(1,1),THSS(1,1)),(ZZ(1,1),RS(1,1))
1      , (Y(1,1),YV(1,1)),(Z(1,1),ZV(1,1))
C
C      MCPTS=CPTS
C      TEST FOR (R,THETA) INPUT OR (Y,Z) INPUT
C
C      IF(BODTAB.NE.0.) GO TO 110
C      R,THETA INPUT
C
C      SET UP THV ARRAY
C      TEMPORARY CHANGE..... SET GRIDL=1.
C      THIS IS USED FOR EVEN DELTA THETA VORTEX INPUT,
C      AND ONLY ONE SET OF THETA'S CREATED IN SUBROUTINE SETDAT.
C      GRIDL=1.0
C
C      56 N1=NTVV+1
C      LOC=0
C      DO 65 I=1,NXTHV
C      DO 60 J=1,N1
C      LOC=LOC+1
C      TH1(J)=THVS(LOC)
C      58 CONTINUE
C      IF(GRIDL.GT.0.)LOC=0
C      65 CALL FILLDV(TH1,THV(1,1),N1,DIVLAT)
C      WRITE(6,500)(TH1(J),J=1,N1)
C      400 FORMAT(1H1/(1P8E15.6))
C      500 FORMAT(*OXYZVB*/(5X,1P6E15.6))
C      IF(BODTAB.EQ.1.0) GO TO 150
C
C      A ROW OF THV IS FOR A TRAILING VORTEX.
C
C      NOW INTERPOLATE IN THE THV ARRAY TO GET THVV AT XVV STATIONS.

```



```

WRITE(6,4000)((RS(I,J),J=1,NX),I=1,N2)
4000 FORMAT(1H0/(1P10E12.4))
C
C COLUMNS OF RS ARE R'S AT THSS'S.
C EACH COLUMN CORRESPONDS TO ONE XS STATION.
C
C ROWS OF RS ARE R VS. XS FOR SUCCESSIVE TRAILING VORTICES.
C THERE ARE N2 ROWS AND NX COLUMNS IN RS. (N2 * NX)
C
C INTERPOLATE ON RS VS. XS CURVES DEVELOPED IN ABOVE STEP
C TO OBTAIN R AT XVV STATIONS.
C
C DO 100 I=1,N2
C DO 95 J=1,NX
C 95 THETA(J)=RS(I,J)
C 100 CALL INTER(XS, THETA,NX,XVV, RR(1,I),N3)
C
C ARRAYS THVV(I,J) AND RR(I,J) I=1,N3 J=1,N2
C DESCRIBE LOCATION OF VORTEX POINTS.
C ROWS CORRESPOND TO BOUND VORTEX LINES.
C COLUMNS CORRESPOND TO TRAILING VORTEX LINES.
C
C GO TO 180
C
C Y,Z INPUT OR DELTA-S INPUT
C
110 CONTINUE
C DEVELOP Z VS. S AND Y VS. S AT INPUT STATIONS.
KSUB=1-NTH(1)
JSUB=KSUB-1
DO 120 I=1,NXTH
KSUB=KSUB+NTH(I)
JSUB=JSUB+NTH(I)+1
N=NTH(I)
IF(SYM.LT.0.) N=N+1
Y(1,I)=TH(JSUB)
Z(1,I)=R(KSUB)
DELS=0.0

```

```

1 6880
1 6890
1 6900
1 6910
1 6920
1 6930
1 6940
1 6950
1 6960
1 6970
1 6980
1 6990
1 7000
1 7010
1 7020
1 7030
1 7040
1 7050
1 7060
1 7070
1 7080
1 7090
1 7100
1 7110
1 7120
1 7130
1 7140
1 7150
1 7160
1 7170
1 7180
1 7190
1 7200
1 7210
1 7220
1 7230
1 7240
1 7250
1 7260

```



```

S(1,1)=0.0
DO 120 J=2,N
IF(J.EQ.N.AND.SYM.LT.0.) GO TO 115
K1=JSUB+J-1
K2=KSUB+J-1
Y(J,1)=TH(K1)
Z(J,1)=R(K2)
DELY=TH(K1)-TH(K1-1)
DELZ=R(K2)-R(K2-1)
DELS=SQRT(DELY**2+DELZ**2) + DELS
S(J,1)=DELS
GO TO 120
115 S(J,1)=SQRT((Y(J-1,1)-Y(1,1))**2 + (Z(J-1,1)-Z(1,1))**2) + DELS
Y(N,1)=Y(1,1)
Z(N,1)=Z(1,1)
120 CONTINUE
IF(THVS(1).NE.0.) GO TO 56
C
C Y,Z INPUT
C
DO 130 I=1,NXTH
N=NTH(I)
CALL FILLDV(S(1,I),SLAT,N,DIVLAT)
NLAT=DIVLAT*N + 1.01
CALL CODIM(S(1,I),Y(1,I),N,SLAT,YY(1,I),NLAT)
CALL CODIM(S(1,I),Z(1,I),N,SLAT,ZZ(1,I),NLAT)
130 C A COLUMN OF YY (OR ZZ) ARE VALUES OF Y (OR Z) AT INPUT XS STATIONS.
C NOW INTERPOLATE TO OBTAIN Y AND Z AT XVV'S.
C
132 N3=NBV+1
N2=NTV+1
DO 140 I=1,N2
DO 135 J=1,NXTH
Y1(J)=YY(I,J)
Z1(J)=ZZ(I,J)
135 CALL INTER(XS,Y1,NXTH,XVV,YV(1,I),N3)
140 CALL INTER(XS,Z1,NXTH,XVV,ZV(1,I),N3)
GO TO 220
150 CONTINUE

```

```

C      NOW A COLUMN OF THV CONTAINS S'S AT AN INPUT X-THETA STATION.
C      A ROW OF THV GIVES S'S FOR A TRAILING VORTEX (AT ALL INPUT X-THETA'S)
C      INTERPOLATE IN THE S VS. XTH PLOTS FOR S'S AT INPUT XS STATIONS.
C      N2=NTV+1
C      DO 160 I=1,N2
C      DO 155 J=1,NXTH
C      155 THETA(J)=THV(I,J)
C      160 CALL INTER(XTH,THETA,NXTH,XS,ST(I,I),NX)
C      INTERPOLATE IN THE Y VS. S AND Z VS. S ARRAYS AT XS STATIONS
C      FOR Y'S AND Z'S AT ST'S FOUND ABOVE.
C      DO 170 I=1,NX
C      DO 165 J=1,N2
C      165 THETA(J)=ST(I,J)
C      CALL CODIM(S(1,I),Y(1,I),N,THETA,YY(1,I),N2)
C      170 CALL CODIM(S(1,I),Z(1,I),N,THETA,ZZ(1,I),N2)
C      GO TO 132
C      180 CONTINUE
C      COMPUTE YY,ZZ FROM THVV AND RR.
C      DO 200 I=1,N3
C      DO 200 J=1,N2
C      THSS(I,J)=RR(I,J)*SIND(THVV(I,J))
C      200 S(I,J)=RR(I,J)*COSD(THVV(I,J))
C      THSS IS THE Y ARRAY. S IS THE Z ARRAY.
C      THESE WILL BE MULTIPLIED BY THE MULT. FACTORS TO GET YV,ZV.
C      220 CONTINUE
C      PERFORM MULTIPLICATION AND REVISION OF VORTEX POINT
C      COORDINATES DUE TO CAMBER
C      INTERPOLATE FOR MULTIPLICATION FACTORS AND CAMPER AT XVV'S.
C      IF(MCPTS.NE.0) GO TO 223
C      DO 222 I=1,N3

```

YC(I)=0.0	1	8050
ZC(I)=0.0	1	8060
YM(I)=1.0	1	8070
ZM(I)=1.0	1	8080
GO TO 236	1	8090
CONTINUE	1	8100
CALL INTER(XMF,YMF,MCPTS,XVV,YM,N3)	1	8110
IF(BMULT.EQ.1.0) GO TO 230	1	8120
DQ 225 I=1,N3	1	8130
ZM(I)=YM(I)	1	8140
GO TO 235	1	8150
CONTINUE	1	8160
CALL INTER(XMF,ZMF,MCPTS,XVV,ZM,N3)	1	8170
CONTINUE	1	8180
CALL INTER(XMF,YCAM,MCPTS,XVV,YC,N3)	1	8190
CALL INTER(XMF,ZCAM,MCPTS,XVV,ZC,N3)	1	8200
CONTINUE	1	8210
	1	8220
	1	8230
	1	8240
	1	8250
	1	8260
	1	8270
	1	8280
	1	8290
	1	8300
	1	8310
	1	8320
	1	8330
	1	8340
	1	8350
	1	8360
	1	8370
	1	8380
	1	8390
	1	8400
	1	8410
	1	8420
	1	8430

```

C
C   FOR CAMBI=0, COMPUTE ANGLES IN X-Z AND X-Y PLANES WHICH CAMBER LINES
C   MAKE AT XVV STATIONS.
C
      IF(CAMBI.NE.0.)GO TO 280
      DX=CHORD*0.001
      DO 240 I=1,N3
        XM(I)=XVV(I)+DX
        XP(I)=XVV(I)-DX
      240 CALL INTER(XVV,YC,N3,XM,YCM,N3)
          CALL INTER(XVV,ZC,N3,XM,ZCM,N3)
          CALL INTER(XVV,YC,N3,XP,YCP,N3)
          CALL INTER(XVV,ZC,N3,XP,ZCP,N3)
          DO 260 I=1,N3
            DY=YCP(I)-YCM(I)
            DZ=ZCP(I)-ZCM(I)
            YCA=ATAN(DY/(2.*DX))
            ZCA=ATAN(DZ/(2.*DX))
            SINYC(I)=SIN(YCA)
            COSYC(I)=COS(YCA)
            SINZC(I)=SIN(ZCA)
            COSZC(I)=COS(ZCA)
          260

```

GO TO 290		
280 DO 285 I=1,N3		8440
SINYC(I)=0.0		8450
COSYC(I)=1.0		8460
SINZC(I)=0.0		8470
COSZC(I)=1.0		8480
285 DO 300 J=1,N2		8490
DO 300 I=1,N3		8500
Y2=YM(I)*THSS(I,J)		8510
Z2=ZM(I)*S(I,J)		8520
XV1(I,J)=XVV(I)-Y2*SINYC(I)*COSZC(I)-Z2*SINZC(I) + XB00(NB)		8530
YV(I,J) =(YC(I)+Y2*COSYC(I) +YB00(NB))*BETAM		8540
300 ZV(I,J) =(ZC(I)-Y2*SINYC(I)*SINZC(I)+Z2*COSZC(I)+ZB00(NB))*BETAM		8550
WRITE(6,301) N2,N3		8560
WRITE(12)(YV(N3,J),ZV(N3,J),J=1,N2)		8570
WRITE(6,7003) (J,YV(N3,J),ZV(N3,J),J=1,N2)		8580
7003 FORMAT(*OBODY DRAG COORDS.*/(I5,2F15.5))		8590
301 FORMAT(*OXYZVB N2,N3=*2I5)		8600
RETURN		8610
END		8620
		8630



```

LNPTS=PTSLN
LTC(NB)=LTPTS
LNC(NB)=LNPTS
LSEG(NB)=SEGLN
TSEG(NB)=SEGLT
LFUNC(NB)=FUNCLN
TFUNC(NB)=FUNCLT
SYMM(NB)=SYM
NTVV=FLATV
C NBVV EQUALS NO. OF LONGITUDINAL PANELS.
C NTVV EQUALS NO. OF LATERAL PANELS.
NBVV=FLONGV
LNVOR=VORLN
LTVOR=VORLT
NXTHV=FNXTHV
XB00(NB)=XB0
YB00(NB)=YB0
ZB00(NB)=ZB0
C
C SET UP LONGITUDINAL VORTEX GRID (XVV ARRAY)
NB1=NBVV+1
IF(LNVOR) 15,20,30
15 DEL=CHORD/NBVV
XV(1)=0.0
GO TO 24
20 DEL=3.1415926/NBVV
XV(1)=0.0
24 DO 26 I=2,NB1
26 XV(I)=XV(I-1)+DEL
C
30 CONTINUE
C FOR LNVOR=1, XV IS GIVEN IN INPUT
NB1=NBVV+1
IF(LNDIV.NE.1) GO TO 40
DO 35 I=1,NB1
35 XVV(I)=XV(I)
GO TO 45
40 CALL FILLDV(XV,XVV,NB1,DIVLON)
45 CONTINUE

```

```

1 9030
1 9040
1 9050
1 9060
1 9070
1 9080
1 9090
1 9100
1 9110
1 9120
1 9130
1 9140
1 9150
1 9160
1 9170
1 9180
1 9190
1 9200
1 9210
1 9220
1 9230
1 9240
1 9250
1 9260
1 9270
1 9280
1 9290
1 9300
1 9310
1 9320
1 9330
1 9340
1 9350
1 9360
1 9370
1 9380
1 9390
1 9400
1 9410

```

```

          NVL(NB)=NBVV
          NVT(NB)=NTVV
          NTV=NTVV*DIVLAT+0.01
          NBV=NBVV*DIVLON+0.01
          NLN(NB)=NBV+1
          NLT(NB)=NTV+1
          NVPTS(NB)=NLN(NB)*NLT(NB)
          NCPTS(NB)=LTPIS*LNPTS
          WRITE(6,400)(NPP(I),I=1,48)
          FORMAT(1H0,10(3X,15))
          IF(LNVOR.NE.0) GO TO 50
          C2=0.5*CHORD
          N1=NBV+1
          C NBV IS TOTAL NO. OF LONGITUDINAL VORTICES.
          DO 48 I=1,N1
            48 XVV(I)=C2*(1.0-COS(XVV(I)))
            50 CONTINUE
          C
          C
          C SET-UP BODY LATERAL VORTEX PARAMETERS IF REQUIRED HERE.
          IF(LTVOR) 300,310,320
          C THETA'S SAME AS BODY DEFINITION STATIONS
          C FOR THIS OPTION THERE MUST BE (NTVV+1) THETA'S (THN'S) GIVEN AT X'S.1
          300 THETA1=0
             N1=NTVV+1
             M1=0
             M2=1
             DO 301 I=1,N1
                M1=M1+1
                M2=M2+1
                THVS(M1)=THN(M2)
                IF(I.EQ.N1) M2=M2+1
            301 CONTINUE
             GO TO 320
          C THIS OPTION IS FOR EVEN DELTA THETA'S. DETERMINE(NTVV+1) THVS'S.
          310 THETA1=1.0
             N1=NTVV+1
             IF(SYM) 311,312,311

```

```

9810
9820
9830
9840
9850
9860
9870
9880
9890
9900
9910
9920
9930
9940
9950
9960
9970
9980
9990
10000

```

```

1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1
1

```

```

311  TRANGE=360.0
    GO TO 312
312  TRANGE=180.0
313  DTH=TRANGE/NTVV
    DO 314 I=1,N1
314  THVS(I)=(I-1)*DTH
    C   FOR LIVOR = 1, THVS ARE GIVEN IN DATA.
320  CONTINUE
    IF(NXTHV.EQ.0) NXTHV=LONGV+1.01
    IF(LNVOR.NE.0) GO TO 58
    DO 54 I= 1,NXTHV
54   XV(I)=C2*(1.0- COS(XV(I)))
58   IF(FNXTH.NE.0) GO TO 65
    DO 60 I=1,NXTH
60   XTH(I)=XS(I)
65   CONTINUE
    C
    CALL DATAWR(DA)
    RETURN
    END

```



```

SUBROUTINE VYZN(J1,Y1,Z,AN)
COMMON DA(5000)
1  NX,NXTH,LNVOR,LIVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)
2  LNDIV,LTDIV,LNPTS,LTPTS
COMMON /BODY/ A(31000)
DIMENSION XV(151,31),YV(151,31),ZV(151,31)
EQUIVALENCE(XV,A(1)),(YV,A(4682)),(ZV,A(9363))
DIMENSION AN(1)
J1=LTDIV*(J-1)+LTDIV/2 + 1
J2=J1+1
IF(I.NE.1) GO TO 100
X3=XV (2,J1)
X4=XV (2,J2)
X1=2.0*XV (1,J1)-X3
X2=2.0*XV (1,J2)-X4
Y3=YV(2,J1)
Y4=YV(2,J2)
Y1=2.0*YV(1,J1)-Y3
Y2=2.0*YV(1,J2)-Y4
Z3=ZV(2,J1)
Z4=ZV(2,J2)
Z1=2.0*ZV(1,J1)-Z3
Z2=2.0*ZV(1,J2)-Z4
GO TO 200
100  I1=I-1
101  I2=I1+2
102  X1=XV (I1,J1)
103  X2=XV (I1,J2)
104  X3=XV (I2,J1)
105  X4=XV (I2,J2)
106  Y1=YV(I1,J1)
107  Y2=YV(I1,J2)
108  Y3=YV(I2,J1)
109  Y4=YV(I2,J2)
110  Z1=ZV(I1,J1)
111  Z2=ZV(I1,J2)
112  Z3=ZV(I2,J1)
113  Z4=ZV(I2,J2)
200  Y=0.25*(Y1+Y2+Y3+Y4)

```

```

1 10010
1 10020
1 10030
1 10040
1 10050
1 10060
1 10070
1 10080
1 10090
1 10100
1 10110
1 10120
1 10130
1 10140
1 10150
1 10160
1 10170
1 10180
1 10190
1 10200
1 10210
1 10220
1 10230
1 10240
1 10250
1 10260
1 10270
1 10280
1 10290
1 10300
1 10310
1 10320
1 10330
1 10340
1 10350
1 10360
1 10370
1 10380
1 10390

```

```

Z=0.25*(Z1+Z2+Z3+Z4)
TX=X3-X1+X4-X2
TY=Y3-Y1+Y4-Y2
TZ=Z3-Z1+Z4-Z2
D=SQRT(TX**2+TY**2+TZ**2)
T1=TX/D
T2=TY/D
T3=TZ/D
TX=X2-X1+X4-X3
TY=Y2-Y1+Y4-Y3
TZ=Z2-Z1+Z4-Z3
D=SQRT(TX**2+TY**2+TZ**2)
AN(1)=(T2*TZ-T3*TY)/D
AN(2)=(T3*TX-T1*TZ)/D
AN(3)=(T1*TY-T2*TX)/D
RETURN
END

```

```

1 10410
1 10415
1 10420
1 10430
1 10440
1 10450
1 10460
1 10470
1 10480
1 10490
1 10500
1 10510
1 10520
1 10530
1 10540
1 10550
1 10560

```

SUBROUTINE INTER (X,Y,N,X1,Y1,N1)		10570
DIMENSION X(1),Y(1),X1(1),Y1(1)		10580
IF(X(2)-X(1).GT.0.0) GO TO 20		10590
IM=1		10600
XMIN=X1(1)		10610
DO 5 I=2,N1		10620
IF(XMIN-X1(I).LE.0.0) GO TO 6		10630
XMIN=X1(I)		10640
IM=I		10650
CONTINUE		10660
NU=IM		10670
IM=1		10680
XMIN=X(1)		10690
DO 10 I=2,N		10700
IF(XMIN-X(I).LE.0.0) GO TO 11		10710
XMIN=X(I)		10720
IM=I		10730
CONTINUE		10740
CONTINUE		10750
CALL REVERS(X,IM)		10760
CALL REVERS(Y,IM)		10770
CALL REVERS(X1,NU)		10780
CALL CODIM(X,Y,IM,X1,Y1,HU)		10790
CALL REVERS(X,IM)		10800
CALL REVERS(Y,IM)		10810
CALL REVERS(X1,NU)		10820
CALL REVERS(Y1,NU)		10830
WRITE(6,100)(X(I),Y(I),I=1,IM),(X1(I),Y1(I),I=1,NU)		10840
FORMAT(10SJB, INTER/(1P8F16.6))		10850
CALL CODIM(X(IM),Y(IM),IM-IM+1, X1(NU+1),Y1(NU+1),NU-NU)		10860
NX=N-IM+1		10870
NY=N1-NU		10880
WRITE(6,100)(X(IM-1+I),Y(IM-1+I),I=1,NX),(X1(NU+1),Y1(NU+1),I=1,NY)		10890
1, I=1,NY)		10900
RETURN		10910
CALL CODIM(X,Y,N,X1,Y1,N1)		10920
RETURN		10930
END		10940

```

SUBROUTINE FILLDV(X,Y,N,DIV)
DIMENSION X(1),Y(1)
N1=N-1
NDIV=DIV+0.01
DO 10 I=1,N1
DEL=(X(I+1)-X(I))/DIV
II=(I-1)*NDIV
DO 10 J=1,NDIV
K=II+J
Y(K)=X(I)+(J-1)*DEL
Y(K+1)=X(N)
RETURN
END

```

10

```

1 10950
1 10960
1 10970
1 10980
1 10990
1 11000
1 11010
1 11020
1 11030
1 11040
1 11050
1 11060
1 11070

```

```

SUBROUTINE REVERS(X,N)
  DIMENSION X(1)
  NL=N/2
  DO 100 I=1,NL
    XHOLD=X(I)
    N1=N+1-I
    X(I)=X(N1)
    X(N1)=XHOLD
  100 RETURN
  END

```

```

1 11080
1 11090
1 11100
1 11110
1 11120
1 11130
1 11140
1 11150
1 11160
1 11170

```

```

PROGRAM PANEL5M
COMMON DA(5000)
COMMON/BOCY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1  ,YVO(10,20),ZVO(10,20),PL(500),PLT(500),YGBV(100),CHORD(100)
2  ,XVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
3  ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
4  ,ELE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
5  ,BDUMY(12130),XYZN(3000)
COMMON/PANEL/ NPAN,IPSYN,INC,NBVP,NVWP,LNCFP,LICFP,LNCFP,LICFP
1  ,NPERPT,NSPACE,NATCH,NIRATT,NPRCLN,NPRCLT,NCTXC,NWCTET,NTHAC
2  ,NTHET,NTHP,CHTIP,RCOT,OUTER,NNATT
3  ,MPI,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
COMMON/SCRAT/ DUMS(6000)
EQUIVALENCE(DA(2),PANS)
COMMON /PANINF/ PANSYN(10), DUN11(400), PANREF(10), PCHORD(10)
EQUIVALENCE (DA(2421),PREF), (DA(2422),PCH)
IF(PANS.EQ.0.0) GO TO 2
NPANS=PANS
DO 1 I=1,NPANS
DO 10 J=3420,5000
DA(J)=0.0
CALL DECRD(DA)
CALL PANDAT
PANSYN(I)=IPSYN
PCHORD(I) = PCH
PANREF(I) = PREF
CALL PANEL1
CALL ATTACH(XVR,YVR,ZVR,10,0)
CONTINUE
CONTINUE
END

```

10

1 2

ORIGINAL PAGE IS  
OF POOR QUALITY

```

SUBROUTINE PANELL1
C
C THIS SUBROUTINE STARTS GEOM. OF A PANEL.
C COMPUTES XJ,YJ,ZJ POINTS.(COORDINATES OF JUNCTURE)
C COMPUTES XR,YR,ZR POINTS.(COORDINATES OF OUTER EDGE OF ROOT OR PANEL
C HAVING NO ROOT SECTION.
COMMON DA(5000)
COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
2 ,XCV( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
5 ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
6 ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
7 ,XVS(100),YVS(100),ZVS(100)
COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LNCFPP,LTCFP,LTCPP
1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
COMMON/SCRAT/ XA(21),YA(21),ZA(21),SA(21),SJ(20)
1 ,DUMMY(1300)
COMMON /COMPRS/ BETAM
EQUIVALENCE(PERIM,DUMMY)
DIMENSION PERIM(400)
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1 ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,ITAPE,NCTV,ITAPE,JTAPE
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
EQUIVALENCE(DA(3450),PERP), (DA(4600),PXCV), (DA(4680),CPLN)
1 , (DA(4720),CPLT), (DA(4640),PETV)
EQUIVALENCE(DA(3432),XP0), (DA(3433),YPC), (DA(3434),ZP0)
DIMENSION PERP(150),PXCV(40),PETV(40),CPLN(40),CPLT(40)
LOGICAL ROOT,OUTER
NSPACE = 0
DO 11 I=1,NPERPT
IF(PERP(4*I).NE.0.0) GO TO 12
NSPACE=NSPACE + 1
CONTINUE
11 WRITE(6,13) NSPACE
12

```

ORIGINAL PAGE IS  
OF POOR QUALITY

```

13  FORMAT('XONSPACE =*,I3)
    ROOT=.FALSE.
    OUTER=.FALSE.
    NBPI =NDVVP+1
    IF(NSPACE.NE.0) ROOT=.TRUE.
    IF(.NOT.ROOT) OUTER=.TRUE.
    DO 1 I=1,NPERPT
    I1=NPERPT+1-I
    I2=I
    IF(PERP(4*I1).NE.0.0) GO TO 2
    CONTINUE
1   NTIP=NPERPT+1-I2
2   IF(OUTER) GO TO 3
    IF(PERP(4*(NTIP-1)).EQ.0.0) GO TO 3
    OUTER=.TRUE.
    CONTINUE
3   CALL PERGM(PERP,PERIM)
    DO 4 I=1,NTIP
    I1=4*(NPERPT-I)
    XTE(I)=PERIM(I1+1)
    YTE(I) = PERIM(I1+2) *BETAM
    ZTE(I) = PERIM(I1+3) *BETAM
    I4=4*(I-1)
    XLE(I)=PERIM(I4+1)
    YLE(I) = PERIM(I4+2) *BETAM
    ZLE(I) = PERIM(I4+3) *BETAM
4   YP0 = YP0 *BETAM
    ZP0 = ZP0 *BETAM

C   C   FIND MINIMUM XLE+XP0 (ROOT) AND SET=XVS(100).
    XMLE=XLE(1)+XP0
    IF(NPAN.EQ.1) GO TO 100
    IF(XMLE.LT.XMLEH) GO TO 100
    GO TO 110
100  XMLEH=XMLE
110  XVS(100)=XMLEH
    WRITE(6,120) NPAN,XMLE,XVS(100)
120  FORMAT(30HOPANEL1... NPAN,XMLE,XVS(100)=I5,1P2E20.6)
C

```



```

C
      WRITE(6,200)(I,XLE(I),YLE(I),ZLE(I),I=1,NTIP)
      WRITE(6,200)(I,XTE(I),YTE(I),ZTE(I),I=1,NTIP)
      FORMAT(1H,13,1P3E16.6)
200  DO 401 I=1,NTIP
      DO 401 I=1,NTIP
401  ETE(I)=SQRT((YLE(I)-YLE(1))**2+(ZLE(I)-ZLE(1))**2)
      1 /SQRT((YLE(NTIP)-YLE(1))**2+(ZLE(NTIP)-ZLE(1))**2)
      DLIS=0.0
      NTIP1=NTIP-1
      SLE(1)=0.0
      DO 5 I=1,NTIP1
      DY=YLE(I)-YLE(I+1)
      DZ=ZLE(I)-ZLE(I+1)
      DELS=DELS+SQRT(DY**2+DZ**2)
      SLE(I+1)=DELS
      IF(I.EQ.NSPACE) DSRROOT=DELS
      CONTINUE
      CHTIP=PERIM(4*NTIP)
      R4=4*NTIP
      XTIP=PERIM(N4-3)
      YTIP=PERIM(N4-2)
      ZTIP=PERIM(N4-1)
      IF(ROOT) GO TO 10
      C FOR NO ROOT SECTION.....
      XRLE=XTIP
      YRLE=YTIP
      ZRLE=ZTIP
      XRTE=PERIM(N4+1)
      YRTE=PERIM(N4+2)
      ZRTE=PERIM(N4+3)
      GO TO 20
      C FOR LIFTING PANEL WITH ROOT SECTION.....
10  NS4=4*NSPACE
      XRLE=PERIM(NS4+1)
      YRLE=PERIM(NS4+2)
      ZRLE=PERIM(NS4+3)
      N1=4*(NPERPT-NSPACE-1)
      XRTE=PERIM(N1+1)
      YRTE=PERIM(N1+2)

```

```

      ZRTE=PERIM(N1+3)
      C  NLROOT EQUALS NO. OF TR. VORTICES IN ROOT SECTION.
      NLROOT=NSPACE+1
20  XJLE=PERIM(1)
      YJLE=PERIM(2)
      ZJLE=PERIM(3)
      N4=4*NPTRPT
      XJTE=PERIM(N4-3)
      YJTE=PERIM(N4-2)
      ZJTE=PERIM(N4-1)
      IF(ROOT) GO TO 30
      C  FOR NO ROOT, DIVIDE JUNCTURE TR. VORTEX INTO BOUND VORTEX COORDINATES
      CX=XJTE-XJLE
      CY=YJTE-YJLE
      CZ=ZJTE-ZJLE
      XJ(1)=XJLE
      YJ(1)=YJLE
      ZJ(1)=ZJLE
      DO 25 I=2,NBVVP
      DPER= PXCV(I)-PXCV(I-1)
      XJ(I)=XJ(I-1)+CX*DPER
      YJ(I)=YJ(I-1)+CY*DPER
      ZJ(I)=ZJ(I-1)+CZ*DPER
25  N1=NBVVP+1
      XJ(N1)=XJTE
      YJ(N1)=YJTE
      ZJ(N1)=ZJTE
      GO TO 50
30  CONTINUE
      C
      C
      C  COMPUTE XJ,YJ,ZJ ARRAYS FOR ROOT-BODY JUNCTURE.
      XJ(1)=XJLE
      C  COMPUTE Y,Z ON TR. VORTEX FOR 20 X'S.
      NA=20
      DX=(XJTE-XJLE)/NA
      XA(1)=XJLE+XPO
      DO 45 I=1,NA
45  XA(I+1)=XA(I)+DX

```

```

NAP1=NA+1
CALL CODIM(XVT,YVT,N1,XA,YA,NAP1)
CALL CODIM(ZVT,N1,XA,ZA,NAP1)
WRITE(6,451) (XA(I),YA(I),ZA(I),I=1,NAP1)
451 FORMAT(10XA,YA,ZA/(1P6E20.5))
SAI=0.0
SA(1)=0.0
DO 46 I=1,NA
  DY= YA(I+1)-YA(I)
  DZ= ZA(I+1)-ZA(I)
  SAI=SAI+SQRT(DX**2+DY**2+DZ**2)
  SA(I+1)=SAI
46  SJ(1)=0.0
  DO 47 I=1,NBVVP
    SJ(I+1)=SJ(I)+(PXCVC(I+1)-FXCV(I))*SAI
  CALL CODIM(SA,XA,NAP1,SJ,XJ,NBP1)
  CALL CODIM(SA,YA,NAP1,SJ,YJ,NBP1)
  CALL CODIM(SA,ZA,NAP1,SJ,ZJ,NBP1)
  CONTINUE
50  DO 49 I=1,NBP1
    XJ(I)=XJ(I)-XPO
    YJ(I)=YJ(I)-YPO
    ZJ(I)=ZJ(I)-ZPO
  C COMPUTE XR,YR,ZR ARRAYS
    DX= XRTE-XRLE
    XR(1)=XRLE
    YR(1)=YRLE
    ZR(1)=ZRLE
    DO 55 I=1,NBVVP
      XR(I+1)=XRLE + DX*PXCVC(I+1)
      YR(I+1)=YRLE
      ZR(I+1)=ZRLE
55  WRITE(6,200)(I,XJ(I),YJ(I),ZJ(I),I=1,NBP1)
    WRITE(6,200)(I,XR(I),YR(I),ZR(I),I=1,NBP1)
  C
    CALL PANEL2
    RETURN
    END

```

1880  
 1890  
 1900  
 1910  
 1920  
 1930  
 1940  
 1950  
 1960  
 1970  
 1980  
 1990  
 2000  
 2010  
 2020  
 2030  
 2040  
 2050  
 2060  
 2070  
 2080  
 2090  
 2100  
 2110  
 2120  
 2130  
 2140  
 2150  
 2160  
 2170  
 2180  
 2190  
 2200  
 2210  
 2220  
 2230  
 2240  
 2250

```

SUBROUTINE PANDAT
COMMON/PANEL/ NPAN,IPSYN,INC,NBVVP,NTVVP,LNCFP,LNCFP,LNCFP,LNCFP,LNCFP,LNCFP
1  ,NPERPT,NSPACE,NATICH,NTRATT,NPRCLN,NPRCLT,NVCTXC,NVCTET,NTHXC
2  ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
3  ,MPI,NP2,NP3,NP4,NP5,NP6,NP7,NP8,NP9,NP10
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1  ,NCT,NB,NBODS,NPAN,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3  ,LNDIVR(7),LTDIVP(7),NBJP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON DA(5000)
EQUIVALENCE(DA(3420),PN), (DA(3426),PSYN), (DA(3429),NCI)
EQUIVALENCE(DA(3437),PNBVV), (DA(3438),PNTVV), (DA(3439),PLNCF)
EQUIVALENCE(DA(3440),PLTCF), (DA(3441),PLNCP), (DA(3442),PLTCP)
2  , (DA(3443),PTSPER), (DA(3444),ROOTSP), (DA(3445),ATTCH)
3  , (DA(3447),PCLN), (DA(3448),PCLT), (DA(3600),PANXC)
4  , (DA(3620),ETALF), (DA(4120),XCCHK), (DA(4160),ETATH)
5  , (DA(4680),CPLN), (DA(4720),CPLT)
6  EQUIVALENCE (DA(3430),VLNI), (DA(3431),VLTI)
1  EQUIVALENCE (DA(4600),PXCVC), (DA(4640),PETV)
EQUIVALENCE (DA(3423),SPAN)
COMMON/COMPRS/ BETAM
DIMENSION PXCVC(40), PETV(40)
DIMENSION CPLN(40), CPLT(40)
COMMON/PANATT/ NATT(30),XATT(200)
COMMON/CONTRV/ LNN(3,40),LTT(3,40)
DIMENSION NLOOK(50)
EQUIVALENCE(NLOOK,NVPTS)
WRITE(6,1) NLOOK
1  FORMAT(*OPANDAT*(1H ,10I10))
CALL LATAWR(DA)
SPAN=SPAN*BETAM
NPAN=PN
IPSYN=PSYN
INC=NCI
NBVVP=PNBVV
NTVVP=PNTVV
NPP= NPAN +NBODS
NVL(NPP) =NBVVP
NVT(NPP) =NTVVP

```

2650	ALL(NPP) = RBVVP+1	2
2660	ALT(NPP) = TVVP+1	2
2670	NPTS(NPP) = RBVVP*TVVP	2
2680	LNCFP=PLNCF	2
2690	LTCFP=PLTCF	2
2700	LFUNC(NPAN+1) = LNCFP	2
2710	TFUNC(NPAN+1) = LTCFP	2
2720	LNCPP=PLNCP	2
2730	LTCPP=PLTCP	2
2740	LTC(NPP) = LTCPP	2
2750	LNC(NPP) = LNCPP	2
2760	NPTS(NPP) = LNCPP*LTCPP	2
2770	NPERPT=PTSPR	2
2780	NSPACE=ROOTSP	2
2790	NAP=(NPAN-1)*3+2	2
2800	NATTCH=NATT(NAP)	2
2810	NTRATT=NATT(NAP+1)	2
2820	IF(NATTCH.EQ.0) GO TO 101	2
2830	WRITE(6,100) NPAN,NATTCH,NTRATT	2
2840	FORMAT(*OPANEL*,I3,* IS ATTACHED TO COMPONENT*,I3,*AT TRAILING VCR2	2
2850	1TEX*,I2)	2
2860	GO TO 103	2
2870	WRITE (6,102) NPAN,NATTCH	2
2880	FORMAT(*OPANEL*,I3,* IS NOT ATTACHED. NATTCH=*,I2)	2
2890	CONTINUE	2
2900	NPRCLN=PCLN	2
2910	NPRCLT=PCLT	2
2920	NACTXC=PARXC	2
2930	NACTET=ETALE	2
2940	NCTXC=XCTHCK	2
2950	NTHET=ETATH	2
2960	IF(CPLN(1).NE.0.0) GO TO 6	2
2970	DO 5 I=1,LNCP	2
2980	CPLN(I)=1	2
2990	IF(CPLT(1).NE.0.0) GO TO 3	2
3000	DO 7 I=1,LTCPP	2
3010	CPLI(I)=1	2
3020	CONTINUE	2
3030		2

C SAVE COEFFICIENT LOCATIONS FOR THE IN-VELOCITY CALCULATION.

```

15 DO 15 I=1,LNCP
   LNI(NPAX,I)=CPLA(I)
16 DO 15 I=1,LTCPP
   LTI(NPAY,I)=CPLT(I)
   LNI=VLNI
   IF(LNI) 10,20,25
   PXCV(1)=0.0
   D=1.0/NBVVP
   DO 11 I=1,NBVVP
   PXCV(I+1)=PXCV(I)+D
   GO TO 25
20 PXCV(1)=0.0
   D=180.0/NBVVP
   DO 21 I=1,NBVVP
   PXCV(I+1)=0.5*(1.0-COS(D*I))
25 CONTINUE
   NLI=NBVVP+1
   WRITE(6,26) (PXCV(I),I=1,NLI)
26 FORMAT(*3PXCV ARRAY*/(I6E15.6))
   LTI=VLTII
   IF(LTI) 40,30,55
   PETV(1)=0.0
   D=1.0/NTVVP
   DO 31 I=1,NTVVP
   IF(I-NTVVP) 311,312,311
311 PETV(I+1)=PETV(I)+D
   GO TO 31
312 PETV(I+1)=1.0
31 CONTINUE
   GO TO 55
40 PETV(1)=0.0
   D=180.0/NTVVP
   DO 41 I=1,NTVVP
   IF(I-NTVVP) 411,412,411
411 PETV(I+1)=0.5*(1.0-COS(D*I))
   GO TO 41
412 PETV(I+1)=1.0

```

```

41  CONTINUE
55  NT1=NTVVP+1
42  WRITE(6,42) (PEIV(I),I=1,NT1)
    FORMAT(*OPETV ARRAY*/(1P6E15.6))
    RETURN
    END

```

```

2  3430
2  3440
2  3450
2  3460
2  3470
2  3480

```

SUBROUTINE PANEL2

THIS SUBROUTINE (CALLED BY PANEL1) COMPUTES VORTEX PTS. AND  
CONTROL PTS. FOR ROOT AND OUTER PANEL.

COMMON DA(5000)

COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),NVO(10,20)  
1 ,YVO(10,20),ZVO(10,20),PVL(500),PLT(500),YSPR(100),CHORD(100)

2 ,XCVC( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)

3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)

4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)

5 ,SXRT(1000),SMT(1000),SZRT(1000),SY6(1000),DZ6(1000)

6 ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)

7 ,XVS(100),YVS(100),ZVS(100)

COMMON/PANEL/ NPAR,IPSYM,IVC,ASVVP,NTVVP,LNCFP,LTCFP,LTCPP,LTCPP

1 ,NPERPT,NSPACE,NATCH,NTRATT,NRACLN,PRCLT,NCTXC,NCTET,NTHXC

2 ,NTHET,NTIP,CHTIP,ROST,OUTER,NMATT

3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10

COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)

DIMENSION XCR(1000), YCR(1000), ZCR(1000)

EQUIVALENCE(XCR,XG), (YCR,YO), (ZCR,ZO)

COMMON/SCRAT/ XA(21),YA(21),ZA(21),SA(21),SJ(20)

1 ,DUMMY(1300),ETC(50),ETV(50)

COMMON/NUMBER/ NVPTS(7),NCPIS(7),NLN(7),NLT(7),LTC(7),LNC(7)

1 ,NCT,NP,NBODS,NPARS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,NTAPE,JTAPE

2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)

3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)

EQUIVALENCE(DA(3450),PLRIV), (DA(4600),PXCV), (DA(4650),CPLN)

1 , (DA(4720),CPLT), (DA(4640),PETV)

EQUIVALENCE(DA(3432),XPO), (DA(3433),YPO), (DA(3434),ZPO)

DIMENSION PERIM(50),PXCV(40),PETV(40),CPLN(40),CPLT(40)

DIMENSION DCX(20),DCY(20),DCZ(20)

DIMENSION BODYR(31000)

EQUIVALENCE(XVR(1,1),BODYR(1))

DIMENSION STAPAY(20),SLCF(50),SAVEC(6000)

EQUIVALENCE(DA(4840),UTARAY), (DA(4850),SLCF), (DA(3440),FNU)

1 , (DA(3433),FNU), (BODYR(15101),G/FU)

LOGICAL ROOT,OUTER,KLT,KLN

LOGICAL FIRST

2 3490  
2 3500  
2 3510  
2 3520  
2 3530  
2 3540  
2 3550  
2 3560  
2 3570  
2 3580  
2 3590  
2 3600  
2 3610  
2 3620  
2 3630  
2 3640  
2 3650  
2 3660  
2 3670  
2 3680  
2 3690  
2 3700  
2 3710  
2 3720  
2 3730  
2 3740  
2 3750  
2 3760  
2 3770  
2 3780  
2 3790  
2 3800  
2 3810  
2 3820  
2 3830  
2 3840  
2 3850  
2 3860  
2 3870



```

LOGICAL ROOTP,OUTERP
NSPP(NPAN)=NSPACE
ROOTP(NPAN)=ROOT
OUTERP(NPAN)=OUTER
NU=FNU
NW=FNW
LETV=0
LETC=0
NBPI=NBVVP+1
NLROOT=NSPACE + 1
IF(.NOT.ROOT) GO TO 65
COMPUTE ALL PANEL POINTS BOUNDED BY JLE,RLE,RTE,JTE.
N1=NSPACE-1
DO 58 J=1,NBPI
  XVR(NLROOT,J)=XR(J)
  YVR(NLROOT,J)=YR(J)
  ZVR(NLROOT,J)=ZR(J)
  XVR(1,J)=XJ(J)
  YVR(1,J)=YJ(J)
  ZVR(1,J)=ZJ(J)
DO 581 I=1,NLROOT
  XVR(I,1)=XLE(I)
  YVR(I,1)=YLE(I)
  ZVR(I,1)=ZLE(I)
  XVR(I,NBPI)=XTE(I)
  YVR(I,NBPI)=YTE(I)
  ZVR(I,NBPI)=ZTE(I)
DO 60 J=2,NBVP
  CSX=XR(J)-XJ(J)
  CSY=YR(J)-YJ(J)
  CSZ=ZR(J)-ZJ(J)
  CS2=CSX**2+CSY**2+CSZ**2
  PER= PXCV(J)
  DENOM=0.0
DO 59 I=1,NSPACE
  DXLE=XLE(I+1)-XLE(I)
  DYLE=YLE(I+1)-YLE(I)
  DZLE=ZLE(I+1)-ZLE(I)
  CALL NORM(DXLE,DYLE,DZLE)

```

2	3880
2	3890
2	3900
2	3910
2	3920
2	3930
2	3940
2	3950
2	3960
2	3970
2	3980
2	3990
2	4000
2	4010
2	4020
2	4030
2	4040
2	4050
2	4060
2	4070
2	4080
2	4090
2	4100
2	4110
2	4120
2	4130
2	4140
2	4150
2	4160
2	4170
2	4180
2	4190
2	4200
2	4210
2	4220
2	4230
2	4240
2	4250
2	4260

ORIGINAL PAGE IS  
OF POOR QUALITY

```

DXTE=XTE(I+1)-XTE(I)
DYTE=YTE(I+1)-YTE(I)
DZTE=ZTE(I+1)-ZTE(I)
CALL NORM(DXTE,DYTE,DZTE)
DCX(I)=DXLE+(DXTE-DXLE)*PER
DCY(I)=DYLE+(DYTE-DYLE)*PER
DCZ(I)=DZLE+(DZTE-DZLE)*PER
CALL NORM(DCX(I),DCY(I),DCZ(I))
DENOM=DENOM + DCI(DCX(I),DCY(I),DCZ(I),CSX,CSY,CSZ)
DS=CS2/DENOM
DO 60 I=1,N1
  XVR(I+1,J)=XVR(I,J)+DS*DCX(I)
  YVR(I+1,J)=YVR(I,J)+DS*DCY(I)
  FORMAT(1H,I3,1P3E16.6)
  ZVR(I+1,J)=ZVR(I,J)+DS*DCZ(I)
DO 618 I=2,NSPACE
  SAI=0.0
  SAI(1)=0.0
  DO 601 J=1,NBVVP
    DX= XVR(I,J+1)-XVR(I,J)
    DY= YVR(I,J+1)-YVR(I,J)
    DZ=ZVR(I,J+1)-ZVR(I,J)
    SAI= SAI + SQRT(DX**2 + DY**2 + DZ**2)
    SAI(J+1)=SAI
  DO 603 J=1,NBPI
    XA(J)=XVR(I,J)
    YA(J)=YVR(I,J)
    ZA(J)=ZVR(I,J)
    SJ(1)=0.0
  DO 604 J=1,NBVVP
    SJ(J+1)=SJ(J)+SAI*(PXC(J+1)-PXC(J))
    CALL CODI(SA,XA,NBPI,SJ,DCX,NBVVP)
    CALL CODI(SA,YA,NBPI,SJ,DCY,NBVVP)
    CALL CODI(SA,ZA,NBPI,SJ,DCZ,NBVVP)
  DO 618 J=2,NBVVP
    XVR(I,J)=DCX(J)
    YVR(I,J)=DCY(J)
    ZVR(I,J)=DCZ(J)
  618 ZVR(I,J)=DCZ(J)

```

65 CONTINUE  
 DO 70 I=1,NTVVP  
 IF(I.NE.1) GO TO 68  
 IF(ROOT) GO TO 66  
 CH = XJ(NBPI)-XJ(1)  
 GO TO 70  
 66 CH=0.0  
 DO 67 J=1,NBVVP  
 DX=XVR(I,J)-XVR(I,J+1)  
 DY=YVR(I,J)-YVR(I,J+1)  
 DZ=ZVR(I,J)-ZVR(I,J+1)  
 CH=CH+SQRT(DX\*\*2+DY\*\*2+DZ\*\*2)  
 GO TO 70  
 67 IF(ROOT.AND.I.LE.NLROOT) GO TO 66  
 CH=CHORD(I-1)+(PETV(I)-PETV(I-1))\*(CHTIP-CHORD(I-1))/  
 1 (1.0-PETV(I-1))  
 70 CHORD(I)=CH  
 CHORD(NTVVP+1)=CHTIP  
 NTV1=NTVVP+1  
 WRITE(6,200)(I,CHORD(I),PETV(I),CHTIP,I=1,NTV1)  
 NCO=0  
 NCOO=0  
 IF(NBODS.EQ.0.AND.(NPAN.EQ.1) NCT=0  
 NCP=NCT  
 IF(NPAN.GT.1) NCP=MPI  
 IF(.NOT.ROOT) GO TO 85  
 COMPUTE VORTEX AND CONTROL POINTS IN ROOT REGION.  
 LT=1  
 DO 80 I=1,NSPACE  
 KLT=.FALSE.  
 LTCP=CPLT(LT)  
 IF(I.NE.LTCP) GO TO 71  
 KLT=.TRUE.  
 LT=LT+1  
 71 CONTINUE  
 LN=1  
 DO 75 J=1,NBVVP  
 KLN=.FALSE.  
 LNCP=CPLN(LN)

```

5050 IF(J,NE,LNCP) GO TO 72
5060 KLN=.TRUE.
5070 LN=LN+1
5080 CONTINUE
5090 DX1=XVR(I,J+1)-XVR(I,J)
5100 DY1=YVR(I,J+1)-YVR(I,J)
5110 DZ1=ZVR(I,J+1)-ZVR(I,J)
5120 DX2=XVR(I+1,J+1)-XVR(I+1,J)
5130 DY2=YVR(I+1,J+1)-YVR(I+1,J)
5140 DZ2=ZVR(I+1,J+1)-ZVR(I+1,J)
5150 NCOO=NCOO+1
5160 PL1=SQRT(DX1**2+DY1**2+DZ1**2)
5170 PL2=SQRT(DX2**2+DY2**2+DZ2**2)
5180 PLL(NCOO)=0.5*(PL1+PL2)
5190 DX11=XVR(I+1,J)-XVR(I,J)
5200 DY11=YVR(I+1,J)-YVR(I,J)
5210 DZ11=ZVR(I+1,J)-ZVR(I,J)
5220 DX21=XVR(I+1,J+1)-XVR(I,J+1)
5230 DY21=YVR(I+1,J+1)-YVR(I,J+1)
5240 DZ21=ZVR(I+1,J+1)-ZVR(I,J+1)
5250 PL1=SQRT(DX11**2+DY11**2+DZ11**2)
5260 PL2=SQRT(DX21**2+DY21**2+DZ21**2)
5270 PLT(NCOO)=0.5*(PL1+PL2)
5280 T1=DX11/SQRT(DY11**2+DZ11**2)
5290 T2=DX21/SQRT(DY21**2+DZ21**2)
5300 FIRST=.FALSE.
5310 R1=0.25
5320 NCO=NCO+1
5330 XSS(NCO)=XPG+0.5*(XVR(I,J)+R1*DX1+XVR(I+1,J)+R1*DX2)
5340 YSS(NCO)=YPG+0.5*(YVR(I,J)+R1*DY1+YVR(I+1,J)+R1*DY2)
5350 ZSS(NCO)=ZPG+0.5*(ZVR(I,J)+R1*DZ1+ZVR(I+1,J)+R1*DZ2)
5360 TS(NCO)=T1+R1*(T2-T1)
5370 DYS(NCO)=DY1+R1*(DY21-DY11)
5380 DZS(NCO)=DZ1+R1*(DZ21-DZ11)
5390 IF(FIRST) GO TO 724
5400 FIRST=.TRUE.
5410 R1=0.75
5420 GO TO 722
5430 CONTINUE

```

IF(.NOT.KLT) GO TO 73	5440
IF(.NOT.KLN) GO TO 73	5450
NCP=NCP+1	5460
XCR(NCP)=0.5*(XVR(I,J)+0.75*DX1+XVR(I+1,J))+0.75*DX2) +XPO	5470
YCR(NCP)=0.5*(YVR(I,J)+0.75*DY1+YVR(I+1,J))+0.75*DY2) +YPO	5480
ZCR(NCP)=0.5*(ZVR(I,J)+0.75*DZ1+ZVR(I+1,J))+0.75*DZ2) +ZPO	5490
CONTINUE	5500
XVR(I,J)=XVR(I,J)+0.25*DX1 +XPO	5510
YVR(I,J)=YVR(I,J)+0.25*DY1 +YPO	5520
ZVR(I,J)=ZVR(I,J)+0.25*DZ1 +ZPO	5530
IF(I.NE.NSPACE) GO TO 75	5540
XVR(I+1,J)=XVR(I+1,J)+0.25*DX2 +XPO	5550
YVR(I+1,J)=YVR(I+1,J)+0.25*DY2 +YPO	5560
ZVR(I+1,J)=ZVR(I+1,J)+0.25*DZ2 +ZPO	5570
CONTINUE	5580
LETV=LETV+1	5590
ETV(LETV)=0.5*(PETV(I+1)+PETV(I))	5600
IF(.NOT.KLT) GO TO 80	5610
LETC=LETC+1	5620
ETC(LETC)=ETV(LETV)	5630
CONTINUE	5640
DO 82 I=1,NLROOT	5650
XVR(I,NBPI)=XVR(I,NBPI)+XPO	5660
YVR(I,NBPI)=YVR(I,NBPI)+YPO	5670
ZVR(I,NBPI)=ZVR(I,NBPI)+ZPO	5680
NCR=NCOO	5690
IF(.NOT.OUTER) GO TO 120	5700
DO 90 I=1,NBVVP	5710
XCVO(I)=0.75*PXCV(I)+0.25*PXCV(I+1)	5720
XCVO(NBVVP+1)=PXCV(NBVVP+1)	5730
DC 92 I=1,LNCP	5740
J=CPLN(I)	5750
XCCG(I)=0.25*PXCV(J)+(0.75*PXCV(J+1)	5760
CONTINUE	5770
NTVOUT=NTVVP-NSPACE	5780
COMPUTE VORTEX POINTS AND CONTROL POINTS FOR OUTER PAULL.	5790
C SPACE CPT ARRAY OUT OF FOOT PEGIC.	5800
	5810
	5820

```

96 DO 96 I=1,LTCPP
97 IC=I
    LTCP=CPLT(I)
    IF(LTCP.GT.NSPACE) GO TO 97
    CONTINUE
    CONTINUE
    DO 110 I=1,NTVOUT
    INS=I+NSPACE
    ET1=PEIV(INS)
    ET2=PEIV(INS+1)
    ET=0.5*(ET1+ET2)
    CH1=CHORD(INS)
    CH2=CHORD(INS+1)
    CHAVG=0.5*(CH1+CH2)
    DO 98 L=1,NTIP
    IF(ET.LE.ETLE(L+1).AND.ET.GT.ETLE(L)) GO TO 99
    CONTINUE
    DELET=ETLE(L+1)-ETLE(L)
    RATIO=(ET-ETLE(L))/DELET
    XLE1=XLE(L)+(XLE(L+1)-XLE(L))*RATIO
    YLE1=YLE(L)+(YLE(L+1)-YLE(L))*RATIO
    ZLE1=ZLE(L)+(ZLE(L+1)-ZLE(L))*RATIO
    R1=(ET1-ETLE(L))/DELET
    R2=(ET2-ETLE(L))/DELET
    DX=XLE(L+1)-XLE(L)
    DY=YLE(L+1)-YLE(L)
    DZ=ZLE(L+1)-ZLE(L)
    XL1=XLE(L)+DX*R1
    YL1=YLE(L)+DY*R1
    ZL1=ZLE(L)+DZ*R1
    XL2=XLE(L)+DX*R2
    YL2=YLE(L)+DY*R2
    ZL2=ZLE(L)+DZ*R2
    DX=XTE(L+1)-XTE(L)
    DY=YTE(L+1)-YTE(L)
    DZ=ZTE(L+1)-ZTE(L)
    XT1=XTE(L)+DX*R1
    YT1=YTE(L)+DY*R1
    ZT1=ZTE(L)+DZ*R1
98
99

```

XT2=XTE(L)+DX*R2	2	6220
YT2=YTE(L)+DY*R2	2	6230
ZT2=ZTE(L)+DZ*R2	2	6240
	2	6250
C		
C		
SAVE VORTEX POINTS ON TRAILING EDGE. (THOSE THAT LIE OUTSIDE ROOT.)	2	6260
XVS(I)=XT1+XP0	2	6270
YVS(I)=YT1+YP0	2	6280
ZVS(I)=ZT1+ZP0	2	6290
IF(I.NE.NTVOUT) GO TO 990	2	6300
XVS(I+1)=XT2+XP0	2	6310
YVS(I+1)=YT2+YP0	2	6320
ZVS(I+1)=ZT2+ZP0	2	6330
CONTINUE	2	6340
990	2	6350
DXL=XL2-XL1	2	6360
DYL=YL2-YL1	2	6370
DZL=ZL2-ZL1	2	6380
YSUBV(I)=0.5*SQRT(DYL**2+DZL**2)	2	6390
DXT=XT2-XT1	2	6400
DYT=YT2-YT1	2	6410
DZT=ZT2-ZT1	2	6420
TITLE=DXL/SQRT(DYL**2+DZL**2)	2	6430
TITE=DXT/SQRT(DYT**2+DZT**2)	2	6440
DO 100 J=1,NBVVP	2	6450
NCOO=NCOO+1	2	6460
PD=PXCV(J+1)-PXCV(J)	2	6470
PLL(NCOO)=0.5*PD*(CH1+CH2)	2	6480
FIRST=.FALSE.	2	6490
XCTERM=XCVO(J)	2	6500
XVO(I,J)=XP0+XLE1+XCTERM*CHAVG	2	6510
YVO(I,J)=YP0+YLE1	2	6520
ZVO(I,J)=ZP0+ZLE1	2	6530
NCO=NCO+1	2	6540
IF(FIRST) GO TO 992	2	6550
XSS(NCO)=XVO(I,J)	2	6560
YSS(NCO)=YVO(I,J)	2	6570
ZSS(NCO)=ZVO(I,J)	2	6580
GO TO 993	2	6590
991	2	6600
XSS(NCO)=XP0+XLE1+XCTERM*CHAVG	2	
YSS(NCO)=YP0+YLE1	2	
992	2	

```

993 ZSS(NCO)=ZP0+ZLE1
DYS(NCO)=DYL
DZS(NCO)=DZL
TS(NCO)=TITLE+(TITLE-TITLE)*XCTERM
IF(FIRST) GO TO 100
FIRST=.TRUE.
XCTERM=0.25*PXCV(J)+0.75*PXCV(J+1)
GO TO 991
100 CONTINUE
LETV=LETV+1
ETV(LETV)=ET
LTCP=CPLT(IC)
IF(I+NSPACE.NE.LTCP) GO TO 110
IC=IC+1
LETC=LETC+1
ETC(LETC)=ET
DO 105 I1=1,LNCP
NCP=NCP+1
XCR(NCP)=XP0+XLE1+XCCO(I1)*CHAVG
YCR(NCP)=YP0+YLE1
105 ZCR(NCP)=ZP0+ZLE1
110 CONTINUE
120 CONTINUE
MPI=NCP
C
C
CALL PANFNC(NCT,ETV,ETC)
NCT=NCP
C
IF(NU+NE.EQ.0) GO TO 1111
CALL SUBROUTINES REQUIRED TO SET UP PANEL CONSTRAINT MATRIX.
LOC1=(NPAN-1)*1000+1
LOC2=LOC1+400
LOC3=LOC2+400
C ARRAY XA IS USED FOR SCRATCH.
CALL TXCC(NU,XA,0,XA,0,NU,NEVVP,
1 SAVEC(LOC1),SAVEC(LOC2),ILPAC,XA,XCVC,XCCO)
CALL MATETA(SAVEC(LOC3),...ITV,PIA,Y,ETV,SUCF)
1111 CONTINUE

```



WRITE(18) BODYWR  
RETURN  
END

2 7000  
2 7010  
2 7020

```

SUBROUTINE PERM(PERIM,ARRAY)
  DIMENSION PERIM(1),ARRAY(1)
  COMMON/PANEL/ NPA,N,IPSYN,IWC,NBVVP,NTVVP,LICFP,LICPP,LICDP
  1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,NLCTXC,NRCTET,NTHAC
  2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
  3 ,NP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
  COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
  1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHPRN(100)
  2 ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
  3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
  4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XP( 20),YP( 20),ZR( 20)
  LOGICAL ROOT,OUTER
  COMMON DA(5000)
  EQUIVALENCE (DA(3432),XP0), (DA(3433),YP0), (DA(3434),ZP0)
  NP=NTIP
  N4=4*NTIP
  DO 10 I=1,N4
    ARRAY(I)=PERIM(I)
    N=2*NTIP-NPERPT
    DO 20 I=1,N
      NP=NP+1
      N1=4*(NTIP+I-1)+1
      N2=4*(NTIP-I)+1
      ARRAY(N1)=ARRAY(N2)+ARRAY(N2+3)
      ARRAY(N1+1)=ARRAY(N2+1)
      ARRAY(N1+2)=ARRAY(N2+2)
      ARRAY(N1+3)=0.0
    IF(ROOT) CALL ATTACH(XVT,YVT,ZVT,50,1)
    NNATT=NP3
    N=NTIP-N
    IF(N.EQ.0) GO TO 35
    DO 30 I=1,N
      NP=NP+1
      N1=N1+4
      N2=N4+(I-1)*4+1
      ARRAY(N1)=PERIM(N2)
      ARRAY(N1+1)=PERIM(N2+1)
      ARRAY(N1+2)=PERIM(N2+2)
      ARRAY(N1+3)=0.0
    
```

10

20

2 7030  
2 7040  
2 7050  
2 7060  
2 7070  
2 7080  
2 7090  
2 7100  
2 7110  
2 7120  
2 7130  
2 7140  
2 7150  
2 7160  
2 7170  
2 7180  
2 7190  
2 7200  
2 7210  
2 7220  
2 7230  
2 7240  
2 7250  
2 7260  
2 7270  
2 7280  
2 7290  
2 7300  
2 7310  
2 7320  
2 7330  
2 7340  
2 7350  
2 7360  
2 7370  
2 7380  
2 7390  
2 7400  
2 7410

```

IF(I,NE,N) GO TO 30
IF(.NOT.ROOT) GO TO 30
ARRAY(N1)=ARRAY(N1)+XPO
CALL CODIM(XVT,YVT,NNATT,ARRAY(N1),ARRAY(N1+1),1)
CALL CODIM(XVT,ZVT,NNATT,ARRAY(N1),ARRAY(N1+2),1)
ARRAY(N1)=ARRAY(N1)-XPO
ARRAY(N1+1)=ARRAY(N1+1)-YPO
ARRAY(N1+2)=ARRAY(N1+2)-ZPO
CONTINUE
30  NPERPT = NP
35  NP4=4*NP
    WRITE(6,100) NTIP
    WRITE(6,100) NSPACE
    WRITE(6,100) NPERPT,(I,ARRAY(I),I=1,NP4)
100 FORMAT(*OPANEL PERIMETER*/I4/(I4,1PE15.5))
    RETURN
    END

```

2	7420
2	7430
2	7440
2	7450
2	7460
2	7470
2	7480
2	7490
2	7500
2	7510
2	7520
2	7530
2	7540
2	7550
2	7560
2	7570
2	7580







```

SUBROUTINE PANFNC(KL,ETV,ETC)
C IWC=WING-CONTOUR INDICATOR
C =0 FOR LOCAL ANGLES OF ATTACK, ALPHA, GIVEN.
C =1 FOR DEFLECTIONS, Z/C GIVEN.
DIMENSION ETV(1),ETC(1)
COMMON DA(5000)
COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHCRD(100)
2 ,XCV( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
5 ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
6 ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
1 ,XN(1320),YN(1320),ZN(1320)
COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCP,LTCP
1 ,NPERPT,NSPACE,NATTCH,NTRATT,NPRCLN,NPRCLT,AWCTXC,NWCTET,NTHXC
2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
EQUIVALENCE(ALPHA,DA(3660)), (TWIST,DA(4100)), (XA,DA(3601))
1 , (ETAI,DA(3631)), (FNXA,DA(3600)), (FNETAI,DA(3630))
2 , (FNXC,DA(3437)), (FNEC,DA(3442))
C DUMS(1504) PRESERVES /SCRAT/ FROM PANEL2.
COMMON/SCRAT/ DUMS(1504),ALP(400),TW(20)
1 ,DYI(400),DZI(400),THK(820),XC(50),XCTHK(51)
2 ,DUMX(101),DUMY(2020),DUMZ(101),XP(40),ZP(40),CH(21),CHC(20)
DIMENSION ALPHA(40),TWIST(30),XA(29),ETAI(29)
EQUIVALENCE(DA(4600),PXC), (DA(4131),XTHICK), (DA(4130),FIXTHK)
1 , (DA(4161),ETATHK), (DA(4160),FETTHK), (DA(4190),THICK)
2 , (DA(4161),ETATHK), (DA(4160),FETTHK), (DA(4190),THICK)
DIMENSION PXC(40),XTHICK(29),ETATHK(29),THICK(410)
EQUIVALENCE(DA(4640),PETV)
COMMON /COMPRS/ BETAI
DIMENSION PETV(40)
COMMON/SLOPE/SIGMAP(500), DZDXT(500),DZDXC(500),TANP1(500)
EQUIVALENCE(MP5,LSIGMA,LS)
EQUIVALENCE(DA(7),XCG), (DA(8),YCG), (DA(9),ZCG)
1 , (DA(11),BETAI), (DA(12),PSTAR), (DA(13),QSTAR), (DA(14),RSTAR)
EQUIVALENCE(DA(4720),CPLT), (DA(4680),CPLN)

```

```

DIMENSION CPLT(40),CPLN(40)
EQUIVALENCE(B,DA(3423)),(REFA,DA(3421)),(CHRM,DA(3422))
1  ,(SPCF,DA(4880))
DIMENSION SPCF(40)
COMMON/NUMBER/ NLOCK(50)
IF(NPAN.EQ.1) MP5=0
C MP5 WILL BE TOTAL NO. OF SOURCE POINTS.
C
NXA=FNXA
NETAI=FNETAI
NXC=FNXC
NEC=FNEC
NXC2=2*NXC
NXTHK=FNXTHK
NETTHK=FETTHK
WRITE(6,100)(ETV(I),I=1,4)
WRITE(6,100)(ETC(I),I=1,4)
FORMAT(*OPANFNC*/(1P8E15.6))
100 INO=100
INI=INO+1
DO 1 I=1,INI
DUMX(I)=(I-1)*0.01
NT1=NTVVP+1
K=0
DX=0.01
DO 10 I=1,LNCP
K=K+1
XP(K)=XCCO(I)-DX
K=K+1
10 XP(K)=XCCO(I)+DX
IF(NXA.NE.0) GO TO 33
N1=LNCP*NEC
DO 31 I=1,N1
ALP(I)=0.0
31 DO 32 I=1,NEC
TW(I)=0.0
32 GO TO 34
33 CONTINUE
WRITE(6,100)(XP(I),I=1,20)

```

9060  
9070  
9080  
9090  
9100  
9110  
9120  
9130  
9140  
9150  
9160  
9170  
9180  
9190  
9200  
9210  
9220  
9230  
9240  
9250  
9260  
9270  
9280  
9290  
9300  
9310  
9320  
9330  
9340  
9350  
9360  
9370  
9380  
9390  
9400  
9410  
9420  
9430  
9440



```

DO 2 J=1,NETAI
J1=(J-1)*NXA+1
J2=(J-1)*INI+1
CALL CODIM(XA,ALPHA(J1),NXA,DUMX,DUMY(J2),INI)
IF(IWC.EQ.1) GO TO 2
CALL QTFG(DUMX,DUMY(J2),DUMY(J2),INI)
CONTINUE
WRITE(6,100)(DUMY(I),I=1,202)
CALL XLINE(PETV,CHORD,NT1,ETAI,CH,NETAI)
CALL XLINE(PETV,CHORD,NT1,ETC,CHC,LTCPP)
WRITE(6,100)(CHORD(J),J=1,NT1)
WRITE(6,100)(CH(J),J=1,NETAI)
WRITE(6,100)(CHC(J),J=1,LTCPP)
WRITE(6,100)(ETAI(J),J=1,NETAI)
WRITE(6,100)(DUMX(J),J=1,INI)
K1=0
L=2
DO 6 I=1,NEC
IF(ETC(I).GT.ETAI(L)) GO TO 5
LM1=L-1
D1=ETC(I)-ETAI(LM1)
D2=ETAI(L)-ETAI(LM1)
R=D1/D2
L1=(LM1-1)*INI
L2=L1+INI
K=0
DO 4 J=1,INI
L1=L1+1
L2=L2+1
ZZZ = CH(LM1)*DUMY(L1)+R*(CH(L)*DUMY(L2)-CH(LM1)*DUMY(L1))
K=K+1
DUMZ(K) = ZZZ/CHC(I)
LN2=2*LNCP
WRITE(6,100) CH(LM1),CH(L),R
CALL CODIM(DUMX,DUMZ,INI,XP,ZP,LN2)
WRITE(6,100)(DUMZ(J),J=1,INI)
WRITE(6,101) LN2
WRITE(6,100)(XP(J),ZP(J),J=1,LN2)
DO 20 J=1,LNCP

```

2	9450
2	9460
2	9470
2	9480
2	9490
2	9500
2	9510
2	9520
2	9530
2	9540
2	9550
2	9560
2	9570
2	9580
2	9590
2	9600
2	9610
2	9620
2	9630
2	9640
2	9650
2	9660
2	9670
2	9680
2	9690
2	9700
2	9710
2	9720
2	9730
2	9740
2	9750
2	9760
2	9770
2	9780
2	9790
2	9800
2	9810
2	9820
2	9830

```

20  K1=K1+1
    J2=2*J
    ALP(K1)=(ZP(J2)-ZP(J2-1))*0.5/DX
    WRITE(6,100)(ALP(J),J=1,K1)
    GO TO 6
5   L=L+1
    GO TO 3
6   CONTINUE
    WRITE(6,100) (ETAI(I),TWIST(I),I=1,NETAI)
    CALL CODIM(ETAI,TWIST,NETAI,ETC,TW,NEC)
34  CONTINUE
    L=0
    DO 9 J=1,NXC
        L=L+1
        XCTHK(L)=PXCV(J)
        XC(L)=0.75*PXCV(J)+0.25*PXCV(J+1)
        L=L+1
        XCTHK(L)=0.5*(PXCV(J)+PXCV(J+1))
        XC(L)=0.25*PXCV(J)+0.75*PXCV(J+1)
        XCTHK(L+1)=PXCV(NXC+1)
        CALL FINDF(XC,NXC2,ETV,NTVVP,DYS,XCCU,LACPP,ETC,NEC,DYI)
        WRITE(6,100)(DYI(I),I=1,80)
        CALL FINDF(XC,NXC2,ETV,NTVVP,DZS,XCCU,LACPP,ETC,NEC,DZI)
        WRITE(6,100)(DZI(I),I=1,80)
        CALL XLINE(PETV,CHORD,NT1,ETV,CHC,NTVVP)
        IF(NXTHK.NE.0) GO TO 8
        NT=NTVVP*NXC2
        DO 7 I=1,NT
            LSIGMA=LSIGMA+1
            SIGMA(LSIGMA)=0.0
7   GO TO 81
8   CONTINUE
        CALL XLINE(PETV,CHORD,NT1,ETATHK,CH,NETTHK)
        DO 102 J=1,NETTHK
            J1=(J-1)*NXTHK+1
            J2=(J-1)*IN1+1
102  CALL CODIM(XTHICK,THICK(J1),XTHK,DZS,DYI(DZ1),I,1)
        L=2
        DO 106 I=1,NTVVP

```

```

9840
9850
9860
9870
9880
9890
9900
9910
9920
9930
9940
9950
9960
9970
9980
9990
10000
10010
10020
10030
10040
10050
10060
10070
10080
10090
10100
10110
10120
10130
10140
10150
10160
10170
10180
10190
10200
10210
10220

```

103	IF(ETV(I),ST,ETATHK(L)) GO TO 105	2	10230
	LM1=L-1	2	10240
	D1=ETV(I)-ETATHK(LM1)	2	10250
	D2=ETATHK(L)-ETATHK(LM1)	2	10260
	R=D1/D2	2	10270
	L1=(LM1-1)*IN1	2	10280
	L2=L1+IN1	2	10290
	K=0	2	10300
	DO 104 J=1,IN1	2	10310
	L1=L1+1	2	10320
	L2=L2+1	2	10330
	TH = CH(LM1)*DUMY(L1)+R*(CH(L)*DUMY(L2)-CH(LM1)*DUMY(L1))	2	10340
	K=K+1	2	10350
104	DUMZ(K) = TH/CHC(I)	2	10360
	L3=(I-1)*(NXC2+1)+1	2	10370
	CALL CODIM(DUMX,DUMZ,IN1,XCTHK,THK(L3),NXC2+1)	2	10380
	GO TO 106	2	10390
105	L=L+1	2	10400
	GO TO 103	2	10410
106	CONTINUE	2	10420
	K1=0	2	10430
	K=KL	2	10440
	L2=1	2	10450
	DO 110 I=1,NEC	2	10460
	MT=(CPLT(I)-1.0)*(NXC2+1)+0.01	2	10470
	DO 110 J=1,LNCP	2	10480
	M1=CPLN(J)*2.0+0.01	2	10490
	M=MT+M1	2	10500
	ST=(THK(M+1)-THK(M))/(XCTHK(M+1)-XCTHK(M))	2	10510
	K=K+1	2	10520
	TANP1(K) = 1.0 +TS(L2)**2 *FETA**2	2	10530
	L2=L2+2	2	10540
	DZDXT(K)=ST	2	10550
	K1=K1+1	2	10560
	DZDXC(K)=ALP(K1)	2	10570
110	CONTINUE	2	10580
31	CONTINUE	2	10590
	WRITE(6,100)(XC(I),I=1,NXC2)	2	10600
	WRITE(6,100)(XCTHK(I),I=1,NXC2)	2	10610



```

        WRITE(6,6001) (SIGMA(I), I=1,230)
6001  FORMAT(*CSIGMA*/(1P10E13.4))
        WRITE(6,100)(SIGMAP(I), I=1,40)
        WRITE(6,6000) NLOOK
6000  FORMAT(*ONLOOK IN PANFAC*/(10I5))
        CALL XLINE(ETLE,YLE,NTIP,PETV,DUNY,NT1)
        CALL XLINE(ETLE,ZLE,NTIP,PETV,DUNZ,NT1)
        BS=0.0
        DO 200 I=1,NTVVP
            DUMX(I)=SQRT((DUMY(I+1)-DUNY(I))*2+(DUNZ(I+1)-DUNZ(I))*2)
            BS=BS+DUMX(I)
200   CONTINUE
        CALL XLINE(ETLE,YLE,NTIP,ETC,DUMY,LICPP)
        CALL XLINE(ETLE,ZLE,NTIP,ETC,DUNZ,LICPP)
        CALL XLINE(ETLE,XLE,NTIP,ETC,XP,LICPP)
        WRITE(12) 8,REFA,LICPP,(ETC(I), I=1,50),LNCPP,XCCO,DYI,DZI,Ta,CHC
1      ,CHRM,(XP(I),DUNY(I),DUNZ(I), I=1,LICPP),CPLT,SPCF,BS,DUNX
        RETURN
        END

```

```

11000
11010
11020
11030
11040
11050
11060
11070
11080
11090
11100
11110
11120
11130
11140
11150
11160
11170
11180

```

REAL FUNCTION P(T,TS)

```

1 S = SIN(.5*(TS-T))
SD= SIN(.5*(TS+T))
IF(S*SD) 4,3,4
3 T = T + 1.0E-6
GOTO 1
4 C = COS(.5*(TS+T))
CD= COS(.5*(TS-T))
IF(C*CD) 8, 3, 8
8 CTS = COS(TS)
CT = COS(T)
CTSPCT = CTS + CT
P = ((CTS-CT)*ALOG(ABS(S/SD)) + CTSPCT*CTSPCT*ALOG(ABS(C/CD)) +
1(4.0*TS*CTS-2.0*SIN(TS))*SIN(T)) / (6.2321 85307 17958*(1.0-CTS))
IF(LEGVAR(P)) 3,9, 3
9 RETURN
C BLAINE D. GAITHER 9/72
END

```

2 11190  
2 11200  
2 11210  
2 11220  
2 11230  
2 11240  
2 11250  
2 11260  
2 11270  
2 11280  
2 11290  
2 11300  
2 11310  
2 11320  
2 11330  
2 11340  
2 11350  
2 11360

```

      REAL FUNCTION R(TH,THS)
11370
11380
11390
11400
11410
11420
11430
11440
11450
11460
11470
11480
11490
11500
11510
11520
11530

      5 = SIN(THS-TH)*.5)
      SD= SIN(THS+TH)*.5)
      IF(S*SD) 3,2,3
      TH = TH + 1.0E-6
      GOTO 1
      C = COS((THS+TH)*.5)
      CD= COS((THS-TH)*.5)
      IF(C*CD) 4,2,4
      CTHS = COS(THS)
      CTH = COS(TH)
      M= 3.1830 98861 83790E-1*((CTHS -CTH)*ALOG(ABS(S/SD)))+(CTHS+CTH)*2
      1ALOG(ABS(C/CD))+2.0*THS*SIN(TH))
      IF(LEGVAR(M)) 2,5,2
      5 RETURN
      BLAINE D. GAITHER 9/72
      END

```

000

2

2 11570

2 11580  
2 11590

2 11590  
2 11600

2 11610

2 11620

2 11630  
2 11640

2 11640  
2 11650

2 11730

2 11750

2 11760

2 11770  
2 11780

11780  
11790

11800

11810

11820  
11830

11850  
11840

058T

11860  
11870

11870  
11880

11890

11900

11910  
11920

07677



```

C      SCALED MATRIX A AND APPROPRIATE TOLERANCE
C      INTERPRETED THAT MATRIX A HAS THE NAME N.
C      GIVEN IN CASE N=1.
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C      NONE
C
C      METHOD
C      SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION BY
C      COMPLETE PIVOTING.
C
C      .....
C
C      DIMENSION A(1),R(1)
C      IF(M)23,23,1
C
C      SEARCH FOR GREATEST ELEMENT IN MATRIX A
C      1 IER=0
C      PIV=0.
C      MM=N*M
C      NM=N*M
C      DO 3 L=1,MM
C      TB=ABS(A(L))
C      IF(TB-PIV)3,3,2
C      2 PIV=TB
C      I=L
C      3 CONTINUE
C      TOL=EPS*PIV
C      A(I) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).
C
C      START ELIMINATION LOOP
C      LST=1
C      DO 17 K=1,M
C
C      TEST ON SINGULARITY
C      IF(PIV)23,23,4

```

ORIGINAL PAGE IS  
OF POOR QUALITY

4	IF(IER)7,5,7	2	12320
5	IF(PIV-TOL)6,6,7	2	12330
6	IER=K-1	2	12340
7	PIVI=1./A(I)	2	12350
	J=(I-1)/M	2	12360
	I=I-J*M-K	2	12370
	J=J+1-K	2	12380
	I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT	2	12390
	PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R	2	12400
	DO 8 L=K,NM,M	2	12410
	LL=L+1	2	12420
	TB=PIVI*R(LL)	2	12430
	R(LL)=R(L)	2	12440
	R(LL)=TB	2	12450
8	R(L)=TB	2	12460
	IS ELIMINATION TERMINATED	2	12470
	IF(K-M)9,18,18	2	12480
	COLUMN INTERCHANGE IN MATRIX A	2	12490
9	LEND=LST+M-K	2	12500
	IF(J)12,12,10	2	12510
10	II=J*M	2	12520
	DO 11 L=LST,LEND	2	12530
	TB=A(L)	2	12540
	LL=L+II	2	12550
	A(L)=A(LL)	2	12560
	A(LL)=TB	2	12570
11	A(LL)=TB	2	12580
	ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A	2	12590
12	DO 13 L=LST,MN,M	2	12600
	LL=L+I	2	12610
	TB=PIVI*A(LL)	2	12620
	A(LL)=A(L)	2	12630
13	A(L)=TB	2	12640
	SAVE COLUMN INTERCHANGE INFORMATION	2	12650
	A(LST)=J	2	12660
		2	12670
		2	12680
		2	12690
		2	12700

C ELEMENT REDUCTION AND NEXT PIVOT SEARCH

```

PIV=0.
LST=LST+1
J=0
DO 16 II=LST,LEND
  PIVI=-A(II)
  IST=II+M
  J=J+1
DO 15 L=IST,MM,M
  LL=L-J
  A(L)=A(L)+PIVI*A(LL)
  TB=ABS(A(L))
  IF(TB-PIV)15,15,14
14 PIV=TB
  I=L
15 CONTINUE
DO 16 L=K,NM,M
  LL=L+J
16 R(LL)=R(LL)+PIVI*R(L)
17 LST=LST+M
END OF ELIMINATION LOOP

BACK SUBSTITUTION AND BACK INTERCHANGE
18 IF(N-1)23,22,19
19 IST=NM+M
  LST=M+1
DO 21 I=2,M
  II=LST-I
  IST=IST-LST
  L=IST-M
  L=A(L)+.5
DO 21 J=II,NM,M
  TB=R(J)
  LL=J
DO 20 K=IST,MM,M
  LL=LL+1
20 TB=TB-A(K)*R(LL)
  K=J+L

```

R(J)=R(K)

21 R(K)=TB

22 RETURN

C

C

C

ERROR RETURN

23 IER=-1

RETURN

END

2 13100

2 13110

2 13120

2 13130

2 13140

2 13150

2 13160

2 13170

2 13180

SUBROUTINE QTFG(X,Y,Z,NDIM)

SUBROUTINE QTFG

PURPOSE

TO COMPUTE THE VECTOR OF INTEGRAL VALUES FOR A GIVEN  
GENERAL TABLE OF ARGUMENT AND FUNCTION VALUES.

USAGE

CALL QTFG (X,Y,Z,NDIM)

DESCRIPTION OF PARAMETERS

X - THE INPUT VECTOR OF ARGUMENT VALUES.  
Y - THE INPUT VECTOR OF FUNCTION VALUES.  
Z - THE RESULTING VECTOR OF INTEGRAL VALUES. Z MAY BE  
IDENTICAL WITH X OR Y.  
NDIM - THE DIMENSION OF VECTORS X,Y,Z.

REMARKS

NO ACTION IN CASE NDIM LESS THAN 1.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED

NONE

METHOD

BEGINNING WITH Z(1)=C, EVALUATION OF VECTOR Z IS DONE BY  
MEANS OF TRAPEZOIDAL RULE (SECOND ORDER FORMULA).  
FOR REFERENCE, SEE  
F.B.HILDGARD, INTRODUCTION TO NUMERICAL ANALYSIS,  
MCGRAW-HILL, NEW YORK/TORONTO/LONDON, 1956, PP.75.

DIMENSION X(1),Y(1),Z(1)

	SUM2=0.	2	13580
	IF (NDIM-1)4,3,1	2	13590
C		2	13600
C	INTEGRATION LOOP	2	13610
	1 DO 2 I=2,NDIM	2	13620
	SUM1=SUM2	2	13630
	SUM2=SUM2+.5*(X(I)-X(I-1))*(Y(I)+Y(I-1))	2	13640
	2 Z(I-1)=SUM1	2	13650
	3 Z(NDIM)=SUM2	2	13660
	4 RETURN	2	13670
	END	2	13680

SUBROUTINE EFIL( XCJ, XCI, NI, E)

C

FILL E MATRIX

C

XCI = .25 PANEL  
XCJ = .75 PANEL

DIMENSION E(NI,NI), XCI(1), XCJ(1)  
NJJ=NI-1

DO 10 J=1, NJJ  
DO 10 I=1, NI

10 E(J,I) = 1.0/(XCJ(J)-XCI(I))

DO 20 I=1, NI  
20 E(NI,I) = 1.0

RETURN

BLAINE D. GAITHER 9/72

END

2 13690  
2 13700  
2 13710  
2 13720  
2 13730  
2 13740  
2 13750  
2 13760  
2 13770  
2 13780  
2 13790  
2 13800  
2 13810  
2 13820  
2 13830  
2 13840  
2 13850

SUBROUTINE FILNFK(W , NF , NJ , THETA , FLAG , THET)  
 SUBROUTINE TO FILL THE NF OR NK SECTION OF THE W MATRIX

W - NF MATRIX  
 NF - NUMBER OF COL IN W  
 NJ - NUMBER OF ROWS IN NF  
 THETA - ARRAY OF UNIQUE THETA FSOR KS  
 FLAG - LOGICAL VARIABLE T(WORK ON NF) F(JORK ON NK)  
 THET - ARRAY OF THETAS FROM FILLNU  
 DIMENSION W(NJ,NF), THETA(1), THET(1)  
 LOGICAL FLAG

DATA PI/ 3.1415 92653 58979/

IF(NF.LE.0) RETURN

I = 1

NJJ = NJ - 1

DO 400 J=1, NF

TH = THETA(M)

THMPI = TH - PI

DO 300 J=1, NJJ

IF((FLAG.AND.(THET (J).GE.TH)).OR.((NOT.FLAG).AND.(THET (J).GT.TH2  
 1))) GOTO 221

W(J,I) = THMPI

GOTO 300

221 W(J,I) = TH

300 CONTINUE

W(NJ,I) = .5 \* SIN(TH)

400 I=I+1

RETURN

BLAINE D. GAITHER 9/72

END

2 13860  
 2 13870  
 2 13880  
 2 13890  
 2 13900  
 2 13910  
 2 13920  
 2 13930  
 2 13940  
 2 13950  
 2 13960  
 2 13970  
 2 13980  
 2 13990  
 2 14000  
 2 14010  
 2 14020  
 2 14030  
 2 14040  
 2 14050  
 2 14060  
 2 14070  
 2 14080  
 2 14090  
 2 14100  
 2 14110  
 2 14120  
 2 14130  
 2 14140  
 2 14150



[illegible]

```

FUNCTION TANDEL(ETA, XOC)
COMMON STATEMENT THAT PUTS FLP(1,1) ON THE FLAPS
THIS FUNCTION RETURNS TAN(DELTA) IF A CP FALLS ON A FLAP,
OTHERWISE ZERO
DIMENSION FLP(8,1)
NF = 1
TANDEL = 0.0
7 IF(FLP(1,NF))2,1,2
2 IF(FLP(3,NF)-ETA) 3,3,4
3 IF(FLP(4,NF)-ETA) 4,5,5
5 T = FLP(5,NF) + (FLP(6,NF)-FLP(5,NF))*(ETA - FLP(3,NF)) /
1 (FLP(4,NF) - FLP(3,NF))
IF((XOC - T)* FLP(1,NF)) 6, 6, 4
4 NF = NF + 1
GOTO 7
6 TANDEL = TAN( FLP(2,NF) * 0.0174 53292 51994 32957)
1 RETURN
BLAINE D. GAITHER OCT 72
END

```

```

2 14490
2 14500
2 14510
2 14520
2 14530
2 14540
2 14550
2 14560
2 14570
2 14580
2 14590
2 14600
2 14610
2 14620
2 14630
2 14640
2 14650
2 14660
2 14670

```

4880

```

C C SUBROUTINE FINDF(X,NX,E,NE,F,XC,XC,EC,NEC,FC) 14680
C C 2 14690
C C PROGRAM TO INTERPOLATE FOR A FUNCTION AT A GIVEN (X/C,ETA) POINT. 14700
C C GRID UPON WHICH INTERPOLATION IS MADE IS GIVEN BY 14710
C C X = GIVEN VALUES OF X/C. NX=NUMBER OF X VALUES. 14720
C C E = GIVEN VALUES OF ETA. NE=NUMBER OF ETA VALUES. 14730
C C THE INPUT GRID IS DESCRIBED BY ONE SET OF X/C VALUES 14740
C C AND ONE SET OF ETA VALUES. 14750
C C F = GIVEN VALUES OF A FUNCTION OF (X,E), GIVEN FIRST LONGITUDINALLY. 14760
C C AND THEN SPANWISE. (EX. IF NX=10,NE=5, F(42) IS AT X(2) AND E(4).) 14770
C C (NUMBER OF F VALUES TO BE AVAILABLE MUST EQUAL NX*NE.) 14780
C C 2 14790
C C XC = XC ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND. 14800
C C NXC = NUMBER OF XC GIVEN, TO MAXIMUM OF 20. (DIMENSION OF F1,F2 = 20) 14810
C C EC = ETA ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND. 14820
C C NEC = NUMBER OF EC GIVEN. 14830
C C FC = INTERPOLATED FUNCTION VALUES, GIVEN CHORDWISE THEN SPANWISE. 14840
C C NUMBER OF OUTPUT VALUES OF FC = NXC*NEC 14850
C C 2 14860
C C INTERPOLATION IN THE X DIRECTION IS MADE BY SUBROUTINE CODIN. 14870
C C INTERPOLATION IN THE ETA DIRECTION IS LINEAR. 14880
C C 2 14890
C C DIMENSION X(1),E(1),XC(1),EC(1),F(1),FC(1) 14900
C C DIMENSION F1(20),F2(20) 14910
C C NE1=NE-1 14920
C C L=0 14930
C C DO 50 J=1,NEC 14940
C C E1=EC(J) 14950
C C DO 5 IE=1,NE1 14960
C C IEL=IE 14970
C C IF(E1.GT.E(IE+1)) GO TO 5 14980
C C ETA1=E(IE) 14990
C C ETA2=E(IE+1) 15000
C C GO TO 6 15010
C C CONTINUE 15020
C C CONTINUE 15030
C C DELET=ETA2-ETA1 15040
C C D = C1-ETA1 15050
C C I1=(IEL-1)*NX+1 15060

```

ORIGINAL PAGE IS  
OF POOR QUALITY

```

I2=I1+NX
CALL CODIM(X,F(I2),NX,XC,F2,NXC)
CALL CODIM(X,F(I1),NX,XC,F1,NXC)
DO 50 I=1,NXC
L=L+1
FC(L)=F1(I)+D*(F2(I)-F1(I))/DELET
CONTINUE
RETURN
END

```

50

```

2 15070
2 15080
2 15090
2 15100
2 15110
2 15120
2 15130
2 15140
2 15150

```

```

C
C
C
C
C
C
SUBROUTINE XLINE(X,Y,N,X1,Y1,N1)
THIS ROUTINE DOES LINEAR INTERPOLATION AT N1 (X1,Y1) POINTS
ON SEGMENTS BETWEEN N (X,Y) POINTS (GIVEN).
ALL X1 MUST BE WITHIN RANGE OF VALUES OF X.
VALUES OF X MUST BE ALGEBRAICALLY ASCENDING.
C
    DIMENSION X(1),Y(1),X1(1),Y1(1)
    L2=2
    DO 10 I=1,N1
    IF(X1(I).GT.X(L2)) GO TO 5
    L1=L2-1
    DX=X(L2)-X(L1)
    D1=X1(I)-X(L1)
    DY=Y(L2)-Y(L1)
    Y1(I)=D1*DY/DX+Y(L1)
    GO TO 10
    IF(L2.GE.N) GO TO 3
    L2=L2+1
    GO TO 2
    CONTINUE
    RETURN
    END

```

2	15160
2	15170
2	15180
2	15190
2	15200
2	15210
2	15220
2	15230
2	15240
2	15250
2	15260
2	15270
2	15280
2	15290
2	15300
2	15310
2	15320
2	15330
2	15340
2	15350
2	15360
2	15370



```

C
C      NPANS=NPANS
C      TEMPORARY VALUES FOR BREF,CBAR
C      BREF=1.0
C      CBAR=10.0
C      IF(NBODS.EQ.0) GO TO 1111
C      READ 2 RECORDS ON 18 TO SPACE OVER ALNGTH,ELNGTH ARRAYS
C      READ(18) B
C      READ(18) B
C      1111 CONTINUE
C
C      NBP=NBODS+NPANS
C      M=0
C      DO 10 KK=1,NBP
C      LTPTS=LTC(KK)
C      LNPTS=LNC(KK)
C      DO 10 JJ=1,LTPTS
C      DO 10 II=1,LNPTS
C      M=M+1
C      XX=XQ(M)-XCG
C      YY = YQ(M) -YCG *BETAM
C      ZZ = ZQ(M) -ZCG *BETAM
C      VY   = -BETA*BETAM-2.0*(PSTAR*ZZ-RSTAR*XX)/BREF
C      VZ   = ALPHA*BETAM+2.0*(PSTAR*YY/3REF+QSTAR*XX/CBAR)
C      VX=1.0-2.0*(OSTAR*ZZ/CBAR-RSTAR*YY/3REF)
C      BOUND(M)=-XN(M)*VX-YI(M)*VY-ZI(M)*VZ
C      NB=0
C      NCTV=0
C      KCON=0
C      IF(NBODS.EQ.0) GO TO 99
C      DO 50 I=1,NBODS
C      NB=NB+1
C      SYML=SYML(I)
C      NBVV=IVL(I)
C      NTVV=IVT(I)
C      N3=NBVV+1
C      N2=NL(I)
C      NSEG=LSEG(I)+TSEG(I)
C      KCON=KCON+1,SEG
10

```

LNDIVB(I)=1	3	0790
LNDIV=1	3	0800
LTDIV=LTDIVB(I)	3	0810
READ(18) 5B	3	0820
CALL INFL(NSEG,NBP)	3	0830
REWIND 19	3	0840
REWIND 20	3	0850
CONTINUE	3	0860
50		
CONTINUE	3	0870
C 'A' MATRIX FOR INFLUENCE OF BODY VORTICES IS ON UNIT 21.		
C AX,AY,AZ MATRICES FOR BODY INFLUENCES IS ON UNIT 20.	3	0880
C UNIT TAPE (20 OR 19) IS AVAILABLE FOR ANOTHER USE.	3	0890
CALL MATA(KCON,NCIV,HTAPE,0)	3	0900
IF(KCON.NE.0).REWIND 23	3	0910
CONTINUE	3	0920
IF(NPANS.EQ.0) GO TO 101	3	0930
	3	0940
NCOLP=0	3	0950
DO 100 I=1,NPANS	3	0960
SYN=PANSYM(I)	3	0970
NB = NB+1	3	0980
NBVVP=NVL(NB)	3	0990
NTVVP=NVL(NB)		
ROOT=ROOTP(I)	3	1000
OUTER=OUTERP(I)	3	1010
WRITE(6,40) I,ROOT,OUTER	3	1020
WRITE(6,40) I,(ROOTP(K),OUTERP(K),K=1,NPANS)	3	1030
FORMAT(*O1=*,I4/10L5)	3	1040
NSPACE=NSPP(I)	3	1050
READ(18) BODYRE	3	1060
IF(.NOT.ROOT) GO TO 55	3	1070
NLROOT=NSPACE+1	3	1080
N3=NBVVP+1	3	1090
IF(.NOT.OUTER) GO TO 60	3	1100
NVTOUT=NTVVP-NSPACE	3	1110
CONTINUE	3	1120
IP=1	3	1130
CALL PANMAT(XG,YO,ZO,NCI,NCOLP,IP)	3	1140
CONTINUE	3	1150
100	3	1160
C XP2 = NCNIT FOR STORAGE OF X,Y,AZ FOR PANEL VORTICES.	3	1170



NZERO = 0  
CALL MATA(NZERO,NCOLP,MP2,1)

REWIND 21

CONTINUE

REWIND 11

END

3 1180

3 1190

3 1200

3 1210

3 1220

3 1230

```

SUBROUTINE PANMAT(XO,YO,ZO,NCP,NCCLP,IP)
COMMON DA(5000)
COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1 ,YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSURV(100),CHORD(100)
2 ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)
3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)
4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)
5 ,SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)
6 ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
7 ,XVS(100),YVS(100),ZVS(100)
COMMON/CONPTS/ DUMQ(3960)
1 ,XN(1320),YN(1320),ZN(1320)
COMMON/SCRAT/XSOL(5000),BOUND(5000),AXR(5000),AYR(5000),AZR(5000)
EQUIVALENCE(AX,AXB),(AY,AYB),(AZ,AZB),(X3,XSOL(1)),(Y3,XSOL(101))
1 , (Z3,XSOL(201)),(XT,XSOL(301)),(YT,XSOL(401))
2 , (ZT,XSOL(501)),(SUM,XSOL(601))
DIMENSION AX(5000),AY(5000),AZ(5000),X3(100),Y3(100),Z3(100)
1 ,XT(100),YT(100),ZT(100),SUM(100)
DIMENSION B(31000)
EQUIVALENCE (B,XVR(1,1))
FREE TRAILING VORTEX VARIABLES.....
C XN,YN,ZN, USE, SAME SPACE AS UTV,VTV,MTV TO BE COMPUTED IN TRAIL.
C
C
DIMENSION XTV(1000),YTV(1000),ZTV(1000),XTVI( 500),YTVI(500)
1 ,ZTVI(500),UTV(1000),VTV(1000),MTV(1000),V1(100),V2(100)
2 ,V3(100),NIP(100),NTVE(100)
EQUIVALENCE(B(15001),UTV), (B(16001),VTV), (R(17001),WTV)
1 , (B(18001),XTV), (B(19001),YTV), (B(20001),ZTV)
2 , (B(21001),XTVI), (R(21501),YTVI), (R(22001),ZTVI)
3 , (B(22501),V1), (B(22601),V2), (B(22701),V3)
4 , (B(22801),NIP), (B(22901),NTVE)
5 , (B(23001),NV)
C MP9=NTR=NO. OF FREE TRAILING VORTICES.
C MP10 =NTRV=TOTAL NO. OF INITIAL PTS.
EQUIVALENCE (MP9,NTR),(MP10,NTRV)
COMMON/PANEL/ MPAN,IPSYM,INC,BVVP,ITVVP,LNCFP,LICFP,LNCPD,LICPP
1 ,NPERPT,NSPACE,MAATCH,MPATT,MPRCL,MPRCLT,LNCTXC,NHCTET,NHXC
2 ,NTHET,NTHP,CHTIP,ROOT,OUTER,NNAIT
3 ,MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10

```

```

COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1  ,NCT,NB,NBODS,WPANS,NVL(7),NVT(7),VTAPE,NTAPE,NCTV,ITAPE,JTAPE 3 1630
2  ,LSEG(7),LSEG(7),LFUNC(7),TFUNC(7) 3 1640
3  ,LNDIVr(7),LNDIVr(7),NSPP(7),ROJTP(7),OUTERP(7),SYN(7) 3 1650
COMMON/PANINF/ PANSYN(10) 3 1660
EQUIVALENCE(DA(3432),XPO), (DA(3433),YPO), (DA(3434),ZPO) 3 1670
DIMENSION XQ(1),YQ(1),ZQ(1) 3 1680
DIMENSION SAVEC(6000), ASSP(1000),AS(1000) 3 1690
EQUIVALENCE(R(15001),SAVEC), (R(21001),ASSP), (R(22001),AS) 3 1700
1  , (DA(3439),FNU), (DA(3440),FNA) 3 1710
DIMENSION YPD(200),ZPD(200) 3 1720
EQUIVALENCE(YPD(1),S(30001)), (ZPD(1),S(30201)) 3 1730
REAL LLEGX,LLEGY,LLEGZ 3 1740
LOGICAL FLAG 3 1750
LOGICAL ROOT,OUTER 3 1760
DATA LSIG70/ 3 1770
DATA LDD70/ 3 1780
IF(IP.GT.1) GO TO 999 3 1790
NTR=0 3 1800
NTRV=0 3 1810
NU=FNU 3 1820
NW=FNW 3 1830
999 CONTINUE 3 1840
NVL1=NBVVP 3 1850
PI=3.141592654 3 1860
IC=0 3 1870
NBPI=NBVVP+1 3 1880
IF(.NOT.ROOT) GO TO 2 3 1890
C 3 1900
XVR1=XVR(1,NBPI) 3 1910
IF(XVR1.GE.XVT(NNATT)) GO TO 2 3 1920
DO 1 I=1,NNATT 3 1930
IF(XVT(I).GT.XVR1) IC=IC+1 3 1940
CONTINUE 3 1950
IF(IC.EQ.1) IC=2 3 1960
C LET IC BE NUMBER OF LONGITUDINAL PANELS IN TRAILING VORTEX SECTION. 3 1970
N1=NSPACE+1 3 1980
DO 3 I=1,N1 3 1990
X3(I)=XVR(I,NBPI) 3 2000
3 2010

```

```

3      Y3(I)=YVR(I,NSP1)
      Z3(I)=ZVR(I,NSP1)
      WRITE(6,101)(X3(I),Y3(I),Z3(I),I=1,N1)
      FORMAT(1H0/('H',IP3E20.6))
      CALL TVGH1(XVI,YVI,ZVI,NNATT,IC,X3,Y3,Z3,N1,XI,YI,ZI,NT)
      WRITE(6,100) NNATT,IC,N1,NT
      FORMAT(' 7H PANMAT/1015')
      WRITE(6,101)(XI(I),YI(I),ZI(I),I=1,NT)
      IF(IX.EQ.1) WRITE(6,502) NCP
502    FORMAT(*ONCP=*,I4)
      NIX=NT/N1
      DO 503 I=1,N1
      NTR=NTR+1
503    NIP(NTR)=NIX
      K=0
      DO 504 KK=1,N1
      DO 504 KK1=1,NTX
      K=K+1
      NTRV=NTRV+1
      XTVI(NTRV)=XT(K)
      YTVI(NTRV)=YT(K)
      ZTVI(NTRV)=ZT(K)
      IF(NTX.NE.KK1) GO TO 504
      LDD=LDD+1
      YPD(LDD)=YTVI(NTRV)
      ZPD(LDD)=ZTVI(NTRV)
      CONTINUE
504    CONTINUE
      NIT=NTVVP+1-NSPACE
      DO 505 K=1,NIT
      IF(NSPACE.NE.0.AND.K.EQ.1) GO TO 505
      NTR=NTR+1
      NIP(NTR)=1
      NTRV=NTRV+1
      XTVI(NTRV)=XVS(K)
      YTVI(NTRV)=YVS(K)
      ZTVI(NTRV)=ZVS(K)
      LDD=LDD+1
      YPD(LDD)=YTVI(NTRV)

```

```

ZPD(LCD)=ZIVI(NTRV)
WRITE(6,7002) (I,YPD(I),ZPD(I),I=1,LCD)
7002 FORMAT(*PANEL DRAG COORDINATES*/(I5,2F15.5))
505 CONTINUE
WRITE(6,800)(XTVI(I),YIVI(I),ZIVI(I),I=1,NTRV)
800 FORMAT( 26HCXTVI,YIVI,ZIVI, IN PANHAT/(I5,20.6))
C SET NV=NO. OF FREE TRAILING VORTICES.
NV=NTR
WRITE(6,100) NSPACE,NVL1
DO 1000 IX=1,NCP
XC=XQ(IX)
YC=YQ(IX)
ZC=ZQ(IX)
MA=0
MS=0
IF(.NOT.ROOT) GO TO 105
DO 104 I=1,NSPACE
DO 104 J=1,NVL1
DO 5 K1=1,3
SUM(K1)=0.0
DO 50 K=1,4
IF(J.EQ.NVL1.AND.K.EQ.4) GO TO 41
IF(K.GT.1) GO TO 20
X1=XVR(I,J+1)
Y1=YVR(I,J+1)
Z1=ZVR(I,J+1)
X2=XVR(I,J)
Y2=YVR(I,J)
Z2=ZVR(I,J)
GO TO 40
20 X1=X2
Y1=Y2
Z1=Z2
IF(K-3) 25,30,35
X2=XVR(I+1,J)
Y2=YVR(I+1,J)
Z2=ZVR(I+1,J)
GO TO 40
30 X2=XVR(I+1,J+1)

```

3	2410
3	2420
3	2430
3	2440
3	2450
3	2460
3	2470
3	2480
3	2490
3	2500
3	2510
3	2520
3	2530
3	2540
3	2550
3	2560
3	2570
3	2580
3	2590
3	2600
3	2610
3	2620
3	2630
3	2640
3	2650
3	2660
3	2670
3	2680
3	2690
3	2700
3	2710
3	2720
3	2730
3	2740
3	2750
3	2760
3	2770
3	2780
3	2790

	Y2=YVR(I+1,J+1)	3	2900
	Z2=ZVR(I+1,J+1)	3	2810
	GO TO 40	3	2820
35	X2=XVR(I,J+1)	3	2830
	Y2=YVR(I,J+1)	3	2840
	Z2=ZVR(I,J+1)	3	2850
40	CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)	3	2860
	GO TO 50	3	2870
41	CONTINUE	3	2880
	IF(IC.EQ.0) GO TO 480	3	2890
	SUM1=SUM(1)	3	2900
	SUM2=SUM(2)	3	2910
	SUM3=SUM(3)	3	2920
	IF(I.EQ.1) GO TO 42	3	2930
	RLEGX1=RLEGX	3	2940
	RLEGY1=RLEGY	3	2950
	RLEGZ1=RLEGZ	3	2960
42	DO 43 K1=1,3	3	2970
43	SUM(K1)=0.0	3	2980
	DO 46 K1=1,IC	3	2990
	IF(K1.EQ.1) GO TO 44	3	3000
	X1=X2	3	3010
	Y1=Y2	3	3020
	Z1=Z2	3	3030
	GO TO 45	3	3040
44	I1=I*(IC+1)+1	3	3050
	X1=XT(I1)	3	3060
	Y1=YT(I1)	3	3070
	Z1=ZT(I1)	3	3080
45	I2=I*(IC+1)+1+K1	3	3090
	X2=XT(I2)	3	3100
	Y2=YT(I2)	3	3110
	Z2=ZT(I2)	3	3120
46	CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)	3	3130
	RLEGX=SUM(1)	3	3140
	RLEGY=SUM(2)	3	3150
	RLEGZ=SUM(3)	3	3160
	Y2H=Y2	3	3170
	SYMPAS=-1.0	3	3180

409	CONTINUE	3	3190
C	NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR RIGHT LEG.	3	3200
	T1=SQRT((Y2-YC)**2+(Z2-ZC)**2)	3	3210
	IF(T1-0.00001) 412,411,411	3	3220
411	CONTINUE	3	3230
	T2=(X2-XC)/SQRT((X2-XC)**2+(Y2-YC)**2+(Z2-ZC)**2)	3	3240
	QT=0.25*(1.0-T2)/(PI*T1)	3	3250
	RLEGY=RLEGY-SYMPAS*QT*(Z2-ZC)/T1	3	3260
	RLEGZ=RLEGZ+SYMPAS*QT*(Y2-YC)/T1	3	3270
412	CONTINUE	3	3280
	IF(SYMPAS.EQ.-1.0.AND.PANSYM(IP).EQ.0) GO TO 471	3	3290
	Y2=Y2H	3	3300
	GO TO 472	3	3310
471	SYMPAS=1.0	3	3320
	Y2=-Y2	3	3330
	GO TO 409	3	3340
472	CONTINUE	3	3350
	IF(I.EQ.1) GO TO 421	3	3360
	LLEGX=-RLEGX1	3	3370
	LLEGY=-RLEGY1	3	3380
	LLEGZ=-RLEGZ1	3	3390
	GO TO 4721	3	3400
421	DO 431 K1=1,3	3	3410
431	SUM(K1)=0.0	3	3420
	DO 461 K1=1,IC	3	3430
	IF(K1.EQ.1) GO TO 441	3	3440
	X2=X1	3	3450
	Y2=Y1	3	3460
	Z2=Z1	3	3470
	GO TO 451	3	3480
441	X2=XT(1)	3	3490
	Y2=YT(1)	3	3500
	Z2=ZT(1)	3	3510
451	I1=K1+1	3	3520
	X1=XT(I1)	3	3530
	Y1=YT(I1)	3	3540
	Z1=ZT(I1)	3	3550
461	CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)	3	3560
	LLEGX=SUM(1)	3	3570

```

LLEGY=SUM(2)
LLEGZ=SUM(3)
YIH=YI
SYMPAS=-1.0
4091 CONTINUE
C NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR LEFT LEG.
T1=SQRT((Y1-YC)**2+(Z1-ZC)**2)
IF(T1-0.00001) 470,415,415
415 CONTINUE
T2=(X1-XC)/SQRT((X1-XC)**2+(Y1-YC)**2+(Z1-ZC)**2)
QT=0.25*(1.0-T2)/(PI*T1)
LLEGY=LLEGY+SYMPAS*QT*(Z1-ZC)/T1
LLEGZ=LLEGZ-SYMPAS*QT*(Y1-YC)/T1
470 CONTINUE
IF(SYMPAS.EQ.-1.0.AND.PANSYV(IP).EQ.0) GO TO 4711
YI=YIH
GO TO 4721
4711 SYMPAS=1.0
YI=-YI
GO TO 4091
4721 CONTINUE
SUM(1)=SUM1 + RLEGX+LLEGX
SUM(2)=SUM2 + RLEGY+LLEGY
SUM(3)=SUM3 + RLEGZ+LLEGZ
480 CONTINUE
C PROVIDE HERE FOR ST. LINE TR. VORTICES (HERE PANEL TR. EDGE IS
C EITHER AT END OF BODY OR BEHIND IT. (IC=0 CASE)
50 CONTINUE
MA=MA+1
AX(MA)=SUM(1)
AY(MA)=SUM(2)
AZ(MA)=SUM(3)
DO 90 IS=1,2
MS=MS+1
IV=0
JV=0
T=TS(MS)
DZ=DZS(MS)
DY=DYS(MS)

```

3	3530
3	3540
3	3550
3	3610
3	3620
3	3630
3	3640
3	3650
3	3660
3	3670
3	3680
3	3690
3	3700
3	3710
3	3720
3	3730
3	3740
3	3750
3	3760
3	3770
3	3780
3	3790
3	3800
3	3810
3	3820
3	3830
3	3840
3	3850
3	3860
3	3870
3	3880
3	3890
3	3900
3	3910
3	3920
3	3930
3	3940
3	3950
3	3960



```

          KSOL=1
          CALL PVSK(T,DY,DZ,XC,YC,ZC,MS,MA,IA,JV,KSOL)
          CONTINUE
104      CONTINUE
105      CONTINUE
C 200  OUTER PANEL INFLUENCE EQUATIONS AND SOURCE INFLUENCE EQUATIONS.
          NVTOUT=NTVVP-NSPACE
          DO 200 I=1,NVTOUT
          DO 200 J=1,NBVVP
              MS=NS+1
              MA=MA+1
              KSOL=2
              T=TS(MS)
              DZ=DZS(MS)
              DY=DYS(MS)
              CALL PVSK(T,DY,DZ,XC,YC,ZC,MS,MA,I,J,KSOL)
              IF(KSOL.EQ.1) GO TO 115
              KSOL=1
              MS=NS+1
              GO TO 110
115      CONTINUE
200      CONTINUE
          IF(DA(5000).LT.0) GO TO 351
          WRITE(6,300) IX(I),AX(I),AY(I),AZ(I),I=1,MA)
          FORMAT(//'/OINFLUENCE COEFFICIENTS FOR CONTROL PT.,I3//
              (I3,1P3E18.6))
          WRITE(6,350) IX(I),SYNT(I),SZAT(I),I=1,NS)
          FORMAT(//'/OOSOURCE COEFFICIENTS FOR CONTROL PT.,I3//
              (I3,1P3E18.6))
351      CONTINUE
C  IN SUPROUTINE PANMAT, WRITE THE SYNT,CYNT,SZAT MATRICES.
C  THERE WILL BE NCT ROWS OF SYNT FOR EACH PANEL.
C  NCT = TOTAL NUMBER OF CONTROL POINTS.
          WRITE(11)(SYNT(I),CYNT(I),SZAT(I),I=1,NCT)
          XN1=XN(IX)
          YN1=YN(IX)
          ZN1=ZN(IX)
          SN=0.0
          PAN=0.0

```

```

3970
3980
3990
4000
4010
4020
4030
4040
4050
4060
4070
4080
4090
4100
4110
4120
4130
4140
4150
4160
4170
4180
4190
4200
4210
4220
4230
4240
4250
4260
4270
4280
4290
4300
4310
4320
4330
4340
4350

```

ORIGINAL PAGE IS  
OF POOR QUALITY

BNY=0.0	3	4360
J=LSIG	3	4370
DO 355 I=1,NS	3	4380
J=J+1	3	4390
BNX=SIGNA(J)*SXMT(I)+BNX	3	4400
BNY=SIGNA(J)*SYMT(I)+BNY	3	4410
355 BN=SIGMA(J)*(XNI*SXMT(I) + YNI*SYMT(I) + 2*1*SZMT(I)) + 9N	3	4420
IF(NBODS.EQ.0) GO TO 358	3	4430
IF(IX.LE.NCPTS(1)) GO TO 359	3	4440
BOUND(IX)=BOUND(IX)-BN	3	4450
CONTINUE	3	4460
IF(NU+NW.EQ.0) GO TO 3060	3	4470
CALL CONFUN FOR CONSTRAINT MATRIX MULTIPLY.	3	4480
LOC1=(IP-1)*1000+1	3	4490
LOC2=LOC1+400	3	4500
LOC3=LOC2+400	3	4510
FLAG=.FALSE.	3	4520
IF(NW.EQ.0)FLAG=.TRUE.	3	4530
CALL CONFUN(AX,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE	3	4540
1 ,ASSP,NTVVP,NBVVP,AS,NU,FLAG,NU)	3	4550
IF(FLAG) GO TO 1050	3	4560
MA=NU*NW	3	4570
DO 1005 I=1,MA	3	4580
1005 AX(I)=ASSP(I)	3	4590
GO TO 1060	3	4600
1050 MA=NU*NTVVP	3	4610
DO 1055 I=1,MA	3	4620
1055 AX(I)=AS(I)	3	4630
1060 CONTINUE	3	4640
CALL CONFUN(AY,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE	3	4650
1 ,ASSP,NTVVP,NBVVP,AS,NU,FLAG,NU)	3	4660
IF(FLAG) GO TO 2050	3	4670
DO 2005 I=1,MA	3	4680
2005 AY(I)=ASSP(I)	3	4690
GO TO 2060	3	4700
2050 DO 2055 I=1,MA	3	4710
2055 AY(I)=AS(I)	3	4720
2060 CONTINUE	3	4730
CALL CONFUN(AZ,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE	3	4740



501	IF (IX.EQ.1) WRITE(6,501) IP,MUNPR,MUNIT,NCPRE,NA	5130
	FORMAT(*OIP,MUNPR,MUNIT,NCPRE,NA=' ',5I5)	5140
	READ(MUNPR)(AX(I),AY(I),AZ(I),I=1,NCPRE)	5150
	IF (IX.EQ.1) NCOLP=NCPRE + NA	5160
	WRITE(MUNIT)(AX(I),AY(I),AZ(I),I=1,NCOLP)	5170
900	CONTINUE	5180
1000	CONTINUE	5190
	LSIG=LSIGH	5200
	NCPRE=NCOLP	5210
	MUNPR=MUNIT	5220
	MP2=MUNIT	5230
	MP3=NCOLP	5240
	REWIND MUNPR	5250
	REWIND MTAPE	5260
	REWIND 24	5270
	RETURN	5280
	END	5290

```

SUBROUTINE INFL(KCN,NBP)
COMPUTE AX,AY,AZ MATRICES AND BOUNDARY CONDITIONS.
COMMON DA(5000)
1 ,NX,NXTH,LNAYOR,LTIVOR,NTIV,L3VV,NTV,NXTHV,NBV,NTH(49)
2 ,LNDIV,LTDIV,LNPTS,LTPTS
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LIC(7),LAC(7)
1 ,NCT,NS,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,ITAPE
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 ,LNDIVS(7),ETDIVS(7),NSPP(7),ROTP(7),QUIERP(7),SYMM(7)
COMMON/BODY/ X(151,31),Y(151,31),Z(151,31)
1 ,TX(1320),THY(1320),TNZ(1320)
2 ,TTX(1320),TTY(1320),TTZ(1320)
COMMON/COMPTS/ XQ(1320),YQ(1320),ZQ(1320)
1 ,XN(1320),YN(1320),ZN(1320)
COMMON/SCRAT/ XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
COMMON/INDEX/ X,Y,Z,I1,I12,I1F,I1F2
DIMENSION SUM(3)
EQUIVALENCE (DA(7),XCG), (DA(8),YCG), (DA(9),ZCG), (DA(10),ALPHA)
1 , (DA(11),DETA), (DA(12),PSTAR), (DA(13),QSTAR), (DA(14),RSTAR)
EQUIVALENCE(DA(19),SYMB)
DIMENSION XQQ(1320),YQQ(1320),ZQQ(1320)
DIMENSION BB(31000)
DIMENSION B (5000)
EQUIVALENCE(B,XV1(1,1))
EQUIVALENCE(B,XV1(1,1))
EQUIVALENCE (XQG,XQ), (YQG,YQ), (ZQG,ZQ)
EQUIVALENCE(N,NCTV), (N,NCT)
EQUIVALENCE(ALNGTH,XSOL)
DIMENSION ALNGTH(5000)
C IF KCN DOES NOT = 0, ALNGTH IS NEEDED FOR XYZRL.
IF(KCN.EQ.0) GO TO 5
C REWIND 18 AND THEN READ ALNGTH ARRAY.
REWIND 18
READ(18) ALNGTH
C NOW SKIP 2 RECORDS SO THAT 18 IS POSITIONED TO READ PANEL DATA.
READ(18) B
READ(18) BB
CONTINUE
PI4=12.5663704

```

NSTART=NCTV	5690
NCT=0	5700
DO 65 KK=1,NBP	5710
LTPTS=LTC(KK)	5720
LNPTS=LNC(KK)	5730
DO 65 JJ=1,LTPTS	5740
DO 65 II=1,LNPTS	5750
NCT=NCT+1	5760
L=M	5770
X=XQ(M)	5780
Y=YQ(M)	5790
Z>ZQ(M)	5800
N=NSTART	5810
DO 60 J=1,NTVV	5820
DO 60 I=1,NBVV	5830
N=N+1	5840
TOT1=0.0	5850
TOT2=0.0	5860
TOT3=0.0	5870
DO 55 K=1,4	5880
DO 8 KS=1,3	5890
SUM(KS)=0.0	5900
GO TO (10,20,30,40),K	5910
CONTINUE	5920
12 I11=LNDIV*(I-1)+1	5930
IF1=I11	5940
I12=(J-1)*LTDIV	5950
DO 14 L=1,LTDIV	5960
I12=I12+1	5970
IF2=I12+1	5980
CALL VORTEX(SUM)	5990
GO TO 50	6000
I12=1+J*LTDIV	6010
IF2=I12	6020
I11=(I-1)*LNDIV	6030
DO 24 L=1,LNDIV	6040
I11=I11+1	6050
IF1=I11+1	6060
CALL VORTEX(SUM)	6070

```

30  GO TO 50
    I11=IF1
    I12=I12+1
    DO 34 L=1,LTDIV
    I12=I12-1
    IF2=I12-1
    IF(I.NE.NBVV) GO TO 33
    C ADD TRAILING VORTEX CONTRIBUTION.
    C RIGHT SIDE
    SYMLOO=1.0
    301 CONTINUE
    DX=XV1(I11,I12)-X
    DY=SYMLOO*YV(I11,I12)-Y
    DZ=ZV(I11,I12)-Z
    TYZ=DY**2+DZ**2
    T1=SQRT(TYZ)
    T2=DX/SQRT(DX**2+TYZ)
    QT=(1.0-T2)/T1
    QTT=SYMLOO*QT
    SUM(2)=SUM(2)+QTT*DZ/T1
    SUM(3)=SUM(3)-QTT*DY/T1
    C LEFT SIDE
    DX=XV1(IF1,IF2)-X
    DY=SYMLOO*YV(IF1,IF2)-Y
    DZ=ZV(IF1,IF2)-Z
    TYZ=DY**2+DZ**2
    T1=SQRT(TYZ)
    T2=DX/SQRT(DX**2+TYZ)
    QT=(1.0-T2)/T1
    QTT=SYMLOO*QT
    SUM(2)=SUM(2)-QTT*DZ/T1
    SUM(3)=SUM(3)+QTT*DY/T1
    C
    IF(SYM3.NE.0.0) GO TO 24
    IF(SYMLOO.EQ.-1.0) GO TO 34
    SYMLOO=-1.0
    GO TO 301
    30 CALL VORTEX(SUM)

```

```

5080
5090
5100
5110
5120
5130
5140
5150
5160
5170
5180
5190
5200
5210
5220
5230
5240
5250
5260
5270
5280
5290
5300
5310
5320
5330
5340
5350
5360
5370
5380
5390
5400
5410
5420
5430
5440
5450
5460

```

34	CONTINUE	3	6470
	GO TO 50	3	6480
40	CONTINUE	3	6490
42	II2=IF2	3	6500
	III=III+1	3	6510
	DO 44 L=1,LNDIV	3	6520
	III=III-1	3	6530
	IF1=III-1	3	6540
44	CALL VORTEX(SUM)	3	6550
50	TOT1=TOT1+SUM(1)	3	6560
	TOT2=TOT2+SUM(2)	3	6570
	TOT3=TOT3+SUM(3)	3	6580
55	CONTINUE	3	6590
	AXB(N)=TOT1/PI4	3	6600
	AYB(N)=TOT2/PI4	3	6610
	AZB(N)=TOT3/PI4	3	6620
	IF(I.NE.1) GO TO 60	3	6630
	AXB(N)=0.0	3	6640
	AYB(N)=0.0	3	6650
	AZB(N)=0.0	3	6660
60	CONTINUE	3	6670
C	IF BODY HAS CONSTRAINT FUNCTIONS, CONVERT AX TO AXRL. SAME FOR AY,AZ	3	6680
	IF(KCN.EQ.0) GO TO 601	3	6690
	NS1=NSTART+1	3	6700
	CALL XYZRL(NS1,ALNGTH,AXB,AYB,AZB)	3	6710
601	CONTINUE	3	6720
	N1=1	3	6730
	IF(NB.EQ.1) GO TO 62	3	6740
	NB1=NB-1	3	6750
	N2=C	3	6760
	DO 61 J=1,NB1	3	6770
61	N2=N2+NVL(J)*NVT(J)	3	6780
	READ(NTAPE)(AXB(I),AYB(I),AZB(I),I=N1,N2)	3	6790
62	WRITE(MTAPE)(AXB(I),AYB(I),AZB(I),I=N1,NCTV)	3	6800
65	CONTINUE	3	6810
	MTAPE=39-MTAPE	3	6820
	NTAPE=39-NTAPE	3	6830
	RETURN	3	6840
	END	3	6850



```

SUBROUTINE CONFUN(A, SCRCH, TETA, NROOT, ASSP, NETA, NXOC,
1 AS, NW, FLAG, NCOL)
C
C   A - A MATRIX (ONE ROW FROM FRED)
C   SCRCH - T(XOC)
C   SCRCH - T(XOC) FOR USE IN ROOT SECTION
C   TETA - T(ETA)
C   NROOT NUMBER OF ETAS IN ROOT SECTION OF PANEL
C   ASSP - OUTPUT A//
C   NETA - NUMBER OF ETAS
C   NXOC - NUMBER OF XOCs
C   AS - CHORDWISE TRANSFORMED
C   NW - NUMBER OF LATERAL CONSTRAINT FUNCTIONS
C   FLAG - IF FALSE SOLVE FOR COMPLETELY CONSTRAINED
C   NCOL - NU + NF + NK (NUMBER OF COLS OF AS)
C   DIMENSION A(NXOC, NETA), SCRCH(NXOC, NCOL), SCRCH(NXOC, NCOL),
1 TETA(NETA, NW), AS(NCOL, NETA), ASSP(NCOL, NW)
C   LOGICAL FLAG
C   IF(NROOT) 41, 41, 1
1 DO 40 IETA=1, NROOT
DO 40 IC = 1, NCOL
TEMP = 0.0
DO 30 K=1, NXOC
30 TEMP = TEMP + SCRCH(K, IC) * A(K, IETA)
40 AS(IC, IETA) = TEMP
41 IS = NROOT + 1
IF(IS.GT.NETA) RETURN
DO 60 IETA=IS, NETA
DO 60 IC=1, NCOL
TEMP = 0.0
DO 50 K=1, NXOC
50 TEMP = TEMP + SCRCH(K, IC) * A(K, IETA)
60 AS(IC, IETA) = TEMP
IF(FLAG) RETURN
SUBROUTINE TO SOLVE COMPLETELY CONSTRAINED MATRIX
DO 70 INW=1, NW
DO 70 IUKF=1, NCOL
TEMP = 0.0
DO 20 IETA=1, NETA

```

```

20 TEMP = TEMP + TETA(IETA,INW)*AS(INWF,ISTM)
   ASSP(IURF,INW) = TEMP
70 CONTINUE
   RETURN
   BLAINE D. CAITHER 9/72
   END

```

3	7250
3	7260
3	7270
3	7280
3	7290
3	7300

```

SUBROUTINE TVG71(X,Y,Z,N,NS,X1,Y1,Z1,N1,XT,YT,ZT,NT)
CALCULATES FIRST ITERATION TRAILING VORTEX GEOMETRY.
C X,Y,Z = JUNCTURE TRAILING VORTEX. N=NO.OF POINTS.
C X1,Y1,Z1 = PANEL TRAILING EDGE. N1=NO.OF POINTS.
C XT,YT,ZT = COMPUTED TRAILING VORTEX POINTS. NT=NO.OF POINTS.
C
    DIMENSION X(1),Y(1),Z(1),X1(1),Y1(1),Z1(1),XT(1),YT(1),ZT(1)
    DIMENSION XS(21),YS(21),ZS(21),SA(21),SJ(21)
    DIMENSION DCX(20),DCY(20),DCZ(20),XTT(21),YTT(21),ZTT(21)
    WRITE(6,100)(I,X(I),Y(I),Z(I),I=1,N)
    WRITE(6,100)(I,X1(I),Y1(I),Z1(I),I=1,N1)
    FORMAT(1H6/('3,1P3E18.5'))
    NT=(NS+1)*N1
    DX=(X(N)-X1(1))/NS
    XS(1)=X1(1)
    DO 1 I=1,NS
        XS(I+1)=XS(I)+DX
        NS1=NS+1
        CALL CODIM(X,Y,N,XS,YS,NS1)
        CALL CODIM(X,Z,N,XS,ZS,NS1)
        SAI=0.0
        SAI=0.0
        SAI=0.0
        SD=SAI/NS
        SJ(1)=0.0
        DO 10 I=1,NS
            SJ(I+1)=SJ(I)+SD
        CALL CODIM(SA,XS,NS1,SJ,XT,NS1)
        CALL CODIM(SA,YS,NS1,SJ,YT,NS1)
        CALL CODIM(SA,ZS,NS1,SJ,ZT,NS1)
C
C DIVIDE LAST BOUND VORTEX INTO EQUAL SPACES.
N11=N1-1
XF=X(N)
YF=Y1(N1)

```

```

ZF=Z1(N1)
DX=0.0
DY=(YF-Y(N1)/N1)
DZ=(ZF-Z(N1)/N1)
DO 15 I=2,N1
  IT1=(I-1)*NS1
  C (IT1+1) PT. IS ON FIRST BOUND VORTEX.
  N3=IT1+1
  XT(N3)=X1(I)
  YT(N3)=Y1(I)
  ZT(N3)=Z1(I)
  IT2= I*NS1
  XT(IT2)=XT(IT1)
  YT(IT2)=YT(IT1)+DY
  ZT(IT2)=ZT(IT1)+DZ
15 CALL NORM(DX,DY,DZ)
  N2=N1-1
  NN=NS1*N1+1
  C
  COMPUTE DX ALONG LAST TRAILING VORTEX.
  DXL=(XF-X1(N1))/NS
  XT(NN)=X1(N1)
  DO 16 I=1,NS
    NN=NN+1
    XT(NN)=XT(NN-1)+DXL
    YT(NN)=Y1(N1)
    ZT(NN)=Z1(N1)
16 NP=N1*NS1
  N4=N2-1
  DO 20 J=2,NS
    CSX=XT(NP+J)-XT(J)
    CSY=YT(NP+J)-YT(J)
    CSZ=ZT(NP+J)-ZT(J)
    CS2=CSX**2+CSY**2+CSZ**2
    PER=(J-1.0)/NS
    DENOM=0.0
    DO 18 I=1,N2
      DXLE=X1(I+1)-X1(I)
      DYLE=Y1(I+1)-Y1(I)

```

	DZLE=Z1(I+1)-Z1(I)	3	8090
	CALL NORM(DXLE,DYLE,DZLE)	3	8100
	DIRX=DXLE+(DX-DXLE)*PER	3	8110
	DIRY=DYLE+(DY-DYLE)*PER	3	8120
	DIRZ=DZLE+(DZ-DZLE)*PER	3	8130
	CALL NORM(DIRX,DIRY,DIRZ)	3	8140
	DCX(I)=DIRX	3	8150
	DCY(I)=DIRY	3	8160
	DCZ(I)=DIRZ	3	8170
18	DENOM=DENOM + DOT(DIRX,DIRY,DIRZ,CSX,CSY,CSZ)	3	8180
	DS=CS2/DENOM	3	8190
	NN=NS1+J	3	8200
	DO 20 I=1,N4	3	8210
	N3=NN-NS1	3	8220
	XT(NN)=XT(N3)+DS*DCX(I)	3	8230
	YT(NN)=YT(N3)+DS*DCY(I)	3	8240
	ZT(NN)=ZT(N3)+DS*DCZ(I)	3	8250
	NN=NN+NS1	3	8260
20	DO 30 I=2,N11	3	8270
	NN=(I-1)*NS1	3	8280
	SAI=0.0	3	8290
	SA(I)=0.0	3	8300
	DO 22 J=1,NS	3	8310
	N2=NN+J	3	8320
	N4=N2+1	3	8330
	DX= XT(N2)-XT(N4)	3	8340
	DY= YT(N2)-YT(N4)	3	8350
	DZ= ZT(N2)-ZT(N4)	3	8360
	SAI= SAI+ SQRT(DX**2 +DY**2 +DZ**2)	3	8370
22	SA(J+1)=SAI	3	8380
	SD=SAI/NS	3	8390
	DO 23 J=1,NS1	3	8400
	N2=NN+J	3	8410
	XTT(J)=XT(N2)	3	8420
	YTT(J)=YT(N2)	3	8430
23	ZTT(J)=ZT(N2)	3	8440
	SJ(I)=0.0	3	8450
	DO 24 J=1,NS	3	8460
24	SJ(J+1)=SJ(J)+SD	3	8470

```

CALL CODIM(SA,XT,NSI,SJ,DCX,NS)
CALL CODIM(SA,YT,NSI,SJ,DCY,NS)
CALL CODIM(SA,ZT,NSI,SJ,DCZ,NS)
DO 30 J=2,NS
  N2=NN+J
  XT(N2)=DCX(J)
  YT(N2)=DCY(J)
  ZT(N2)=DCZ(J)
RETURN
END

```

30

```

3 8480
3 8490
3 8500
3 8510
3 8520
3 8530
3 8540
3 8550
3 8560
3 8570

```

SUBROUTINE MATCON

C

PERFORM PHI,THEIA MATRIX MULTIPLY ON AXB,AYB,AZB MATRICES.

NO REPLACEMENT IS MADE FOR BODIES HAVING NO CONSTRAINT FUNCTIONS.

COMMON DA(5000)

1 ,NX,NXTH,LNVOR,LTVOR,NIVV,NBVV,NTV,NXTHV,NBV,NTH(49)

2 ,LNDIV,LTDIV,LNPTS,LTPIS

COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)

1 ,NCT,NB,NBODS,APANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE

2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)

3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)

COMMON/BODY/ ARRAY(31000)

DIMENSION BX(5000),RY(5000),RZ(5000),PHTH(5000)

EQUIVALENCE(BX,ARRAY(1)),(BY,ARRAY(5001)),(BZ,ARRAY(10001))

1 ,((PHTH,ARRAY(15001))

COMMON/SCRAT/XSCL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)

DO 1000 I=1,NCT

READ(NTAPE)(AXB(K),AYB(K),AZB(K),K=1,NCTV)

M=0

IV=0

DO 100 NB=1,NBODS

NVP=NVL(NB)\*NVT(NB)

IF(LSEG(NB).EQ.0.0.AND.TSEG(NB).EQ.0.0) GO TO 50

KODE=1

NC=ITAPE

GO TO 70

NC=NVP

KODE=0

DO 60 I1=1,NC

M=M+1

IV=IV+1

BX(M)=AXB(IV)

BY(M)=AYB(IV)

BZ(M)=AZB(IV)

GO TO 100

CONTINUE

IS=IV

DO 80 I2=1,NC

SX=0.0

SY=0.0	3	8970
SZ=0.0	3	8980
READ(231)(PHTH(KC1),KC=1,NVP)	3	8990
M=M+1	3	9000
DO 75 I1=1,NVP	3	9010
IS11=IS+I1	3	9020
SX=SY+PHTH(I1)*AXS(IS11)	3	9030
SY=SY+PHTH(I1)*AYB(IS11)	3	9040
SZ=SZ+PHTH(I1)*AZB(IS11)	3	9050
IV=IV+1	3	9060
BX(N)=SX	3	9070
BY(M)=SY	3	9080
BZ(M)=SZ	3	9090
CONTINUE	3	9100
IF(KODE.EQ.1) REWIND 23	3	9110
1000 WRITE(MTAPE)(BX(K),BY(K),BZ(K),K=1,M)	3	9120
C CHANGE MEANING OF NCTV FROM TCT. NO. OF VORTICES ON BODIES	3	9130
C TO NUMBER OF COLUMNS OF 'A' MATRIX. (FOR DISCRETE CASE, NO CHANGE.)	3	9140
NCTV=M	3	9150
NTAPE=39-NTAPE	3	9160
MTAPE=39-MTAPE	3	9170
REWIND 19	3	9180
REWIND 20	3	9190
RETURN	3	9200
END	3	9210



```

SUBROUTINE MATA(KCON,NCOLS,IUNIT,IBP)
C KCON = 0 FOR PANEL SIDE. KCON = 0 OR NSEG FOR BODY SIDE (SEE INFLM.)
C NCOLS = NO. OF AXB,AYB,AZB. (NO. OF COLS OF BODY OR PANEL SIDE.)

COMMON DA(5000)
1 ,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NRV,NTH(49)
2 ,LNDIV,LTDIV,LNPTS,LTPTS
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1 ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,NTAPE,JTAPE
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYNN(7)
COMMON/CONPTS/XQ(1320),YQ(1320),ZQ(1320)
1 ,XN(1320),YN(1320),ZN(1320)
COMMON/SCRAT/ XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
DIMENSION A(5000)
EQUIVALENCE(A,AXB)
EQUIVALENCE(DA(2),PANS)
NPANS=PANS
IWR=21
IF(IBP.EQ.1) IWR=10
NBP=NBODS + NPANS
IF(KCON.EQ.0) GO TO 100
CALL MATCON
100 L=0
DO 300 I=1,NBP
LTPTS=LTC(I)
LNPTS=LNC(I)
DO 300 JJ=1,LTPTS
DO 300 II=1,LNPTS
L=L+1
XNN=XN(L)
YNN=YN(L)
ZNN=ZN(L)
READ(IUNIT)(AXB(K),AYB(K),AZB(K),K=1,NCOLS)
DO 200 K=1,NCOLS
200 A(K)=XNN*AXB(K)+YNN*AYB(K)+ZNN*Azb(K)
300 WRITE(IWR)(A(K),K=1,NCOLS)
REWIND IWR
REWIND NTAPE
JTAPE=21

```

RETURN  
END

3 9880  
3 9890

SUBROUTINE XYZRL(KSTART,ALNGTH,AXR,AYR,AZR)			
DIMENSION ALNGTH(1)			9220
DIMENSION AXB(1),AYB(1),AZB(1)			9230
COMMON DA(5000)			9240
1	,NX,NXTH,LNVOR,LTVOR,NTVV,NBVV,NTV,NXTHV,NBV,NTH(49)	3	9250
2	,LNDIV,LTDIV,LNPTS,LTPTS	3	9260
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)		3	9270
1	,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE	3	9280
2	,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)	3	9290
3	,LNDIVB(7),LTDIVB(7),NSPP(7),RCOTP(7),OUTERP(7),SYN(7)	3	9300
DO 100 J1=1,NTVV		3	9310
I2=J1*NBVV		3	9320
I3=KSTART-1+I2		3	9330
ADDX=0.0		3	9340
ADDY=0.0		3	9350
ADDZ=0.0		3	9360
DO 100 I1=1,NBVV		3	9370
ADDX=AXB(I3)+ADDX		3	9380
AXB(I3)=ADDX*ALNGTH(I3)		3	9390
ADDY=AYB(I3)+ADDY		3	9400
AYB(I3)=ADDY*ALNGTH(I3)		3	9410
ADDZ=AZB(I3)+ADDZ		3	9420
AZB(I3)=ADDZ*ALNGTH(I3)		3	9430
I3=I3-1		3	9440
CONTINUE		3	9450
RETURN		3	9460
END		3	9470
		3	9480

SUBROUTINE PVSK(T,DYY,DZZ,XC,YC,ZC,MS,MA,IV,JV,KSOL)	3	9900
COMMON DA(5000)	3	9910
EQUIVALENCE(DA(3426),SYM)	3	9920
COMMON/PANINF/ PANSYM(10)	3	9930
COMMON/SCRAT/XSOL(5000),BOUND(5000),AX(5000),AY(5000),AZ(5000)	3	9940
COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)	3	9950
1 YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)	3	9960
2 XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)	3	9970
3 XIE( 20),YTE( 20),ZIE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)	3	9980
4 ETE( 20),XVT( 50),YVT( 50),ZVT( 50),XRT( 20),YRT( 20),ZRT( 20)	3	9990
5 SXMT(1000),SYM(1000),SZMT(1000),DYS(1000),DZS(1000)	3	10000
6 IS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGNA(1000)	3	10010
COMMON/PANEL/ NPAN,IPSYM,INC,NBVVP,NTVVP,LNCFP,LTCFP,LNCP,LTICPP	3	10020
1 ,NPERPT,NSPACE,NATTCH,NIRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC	3	10030
2 ,NTHET,NTIP,CHIIP,ROOT,OUTER,NNATT	3	10040
3 ,MPI,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10	3	10050
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)	3	10060
1 ,NCT,NB,NBODS,NPAN5,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE	3	10070
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)	3	10080
3 ,LNDIVB(7),LNDIVB(7),NSPP(7),ROOIP(7),OUTERP(7),SYMM(7)	3	10090
	3	10100
C KSOL=1 FOR SOURCE PTS. ONLY	3	10110
C KSOL=2 FOR BOTH SOURCE PTS. AND VORTEX POINTS.	3	10120
C MS=SUBSCRIPT OF SOURCE PT.	3	10130
C XC,YC,ZC --- CONTROL PT.	3	10140
C IV = LATERAL VORTEX SUBSCRIPT.	3	10150
C JV = LONGITUDINAL VORTEX SUBSCRIPT.	3	10160
C INSERT SPECIFICATION STATEMENTS HERE.	3	10170
REAL I1,I2,I3,I4	3	10180
YVV=0.5*SQR(DYY**2+DZZ**2)	3	10190
YV2=2.0*YVV	3	10200
PI=3.141592654	3	10210
YV=YVV	3	10220
GO TO (10,20),KSOL	3	10230
10 YK=YSS(MS)	3	10240
ZK=ZSS(MS)	3	10250
XK=XSS(MS)	3	10260
GO TO 25	3	10270
25 IF(MS-2*(MS/2),EO.0) GO TO 10	3	10280

YK=YVO(IV,JV)	3	10290
ZK=ZVO(IV,JV)	3	10300
XK=XVO(IV,JV)	3	10310
25 CONTINUE	3	10320
SUMX=0.0	3	10330
SUMY=0.0	3	10340
SUMZ=0.0	3	10350
UTSUM=0.0	3	10360
VTSUM=0.0	3	10370
WTSUM=0.0	3	10380
SIGN=1.0	3	10390
DZ=ZC-ZK	3	10400
X=XC-XK	3	10410
CONTINUE	3	10420
DY=YC-SIGN*YK	3	10430
RY=DYY/YV2	3	10440
RZ=DZZ/YV2	3	10450
Y=RY*DY+RZ*DZ	3	10460
Z=-RZ*DY+RY*DZ	3	10470
R12=(Y+YV)**2+Z**2	3	10480
R22=(X-T*Y)**2+Z**2*(1.0+T*T)	3	10490
R32=(Y-YV)**2+Z**2	3	10500
R4= SQRT((X-T*YV)**2+(Y-YV)**2+Z**2)	3	10510
R5= SQRT((X+T*YV)**2+(Y+YV)**2+Z**2)	3	10520
I1=(X+T*YV)/R5	3	10530
I2=(Y+T*X+YV*(1.0+T**2))/R5	3	10540
I3=- (Y+T*X-YV*(1.0+T**2))/R4	3	10550
I4=(X-T*YV)/R4	3	10560
IF (ABS(Z).GT.YV2) GO TO 42	3	10570
Z=0.0	3	10580
R6D=(X+T*YV)**2+(Y+YV)**2	3	10590
R7D=(X-T*YV)**2+(Y-YV)**2	3	10600
RXTY=(X-T*Y)**2	3	10610
R6=RXTY/R6D	3	10620
R7=RXTY/R7D	3	10630
IF (R6.GE.0.0075968656) GO TO 41	3	10640
IF (R7.GE.0.0075968656) GO TO 41	3	10650
IF (ABS(Y).LE.YV) GO TO 41	3	10660
TERM1=ABS(1.0/R7D-1.0/R6D)*0.5/P	3	10670

```

41 GO TO 43
   IF(ABS(Y).GT.YV) GO TO 42
   IF(ABS(X-T*Y).GE.C.25*ABS(PLL(MA))) GO TO 42
   TERM1=0.0
   GO TO 43
42 TERM1=(I2+I3)/R22
43 CONTINUE
   TERM2=(I1+I.0)/R12
   TERM3=(I4+I.0)/R32
   TERM4=1.0/R4-1.0/R5
   P=SQRT(1.0+T*T)
   EUS=(T*TERM4+(X-T*Y)*TERM1)/P
   EVS=(TERM4-T*(X-T*Y)*TERM1)/P
   EWS=P*Z*TERM1
   US=0.25*EUS/PI
   VS=0.25*EVS/PI
   WS=0.25*EWS/PI
   UT=US
   VT=VS*RY-WS*RZ
   WT=VS*RZ+WS*RY
   UTSUM=UT+UTSUM
   VTSUM=VT+VTSUM
   WTSUM=WT+WTSUM
   IF(KSOL.EQ.1) GO TO 45
   EU=Z*TERM1
   EV=Z*(-T*TERM1+TERM2-TERM3)
   EW=- (X-T*Y)*TERM1-(Y+YV)*TERM2+(Y-YV)*TERM3
   UV=0.25*EU/PI
   VV=0.25*EV/PI
   WV=0.25*EW/PI
   UI=UV
   VI=RY*VV-RZ*WV
   WI=RZ*VV+RY*WV
   SUNX=UI+SUNX
   SUMY=VI+SUMY
   SUMZ=WI+SUMZ
45 CONTINUE
   C FOR SYMMETRY, GET IMAGE CONTRIBUTION.
   IF(SYM.NE.0.0) GO TO 60

```

```

IF(SIGN.LT.0.0) GO TO 60
SIGN=-1.0
DZZ=-DZZ
T=-T
GO TO 50
CONTINUE
SXMT(MS)=UTSUM
SYMT(MS)=VTSUM
SZMT(MS)=WTSUM
IF(KSOL.EQ.1) RETURN
AX(MA)=SUMX
AY(MA)=SUMY
AZ(MA)=SUMZ
RETURN
END

```

3	11070
3	11080
3	11090
3	11100
3	11110
3	11120
3	11130
3	11140
3	11150
3	11160
3	11170
3	11180
3	11190
3	11200
3	11210

SUBROUTINE VORRAN(SUM,XI,YI,ZI,XF,YF,ZF,X,Y,Z)

COMMON DA(5000)

EQUIVALENCE(DA(3426),SYM)

DIMENSION SUM(1)

R4PI=0.07957747

YIH=YI

YFH=YF

XFQ=XF-X

ZFQ=ZF-Z

XIQ=XI-X

ZIQ=ZI-Z

SYML00=1.0

YFQ=YF-Y

YIQ=YI-Y

DELX=XF-XI

DELY=YF-YI

DELZ=ZF-ZI

RXS1=YFQ\*DELZ-ZFQ\*DELY

RXS2=ZFQ\*DELX-XFQ\*DELZ

RXS3=XFQ\*DELY-YFQ\*DELX

RXS =SQRT(RXS1\*\*2+RXS2\*\*2+RXS3\*\*2)

TERM1=SQRT(DELX\*\*2+DELY\*\*2+DELZ\*\*2)

TERM2=SQRT(XFQ\*\*2+YFQ\*\*2+ZFQ\*\*2)

TERM3=SQRT(XIQ\*\*2+YIQ\*\*2+ZIQ\*\*2)

TERM4= XFQ\*DELX+YFQ\*DELY+ZFQ\*DELZ

RATIO = TERM4/TERM1\*\*2

COSA=(DELX\*XIQ+DELY\*YIQ+DELZ\*ZIQ)/(TERM1\*TERM3)

COSB = TERM4/(TERM1\*TERM2)

CC=COSE-COSA

HX=XFQ-RATIO\*DELX

HY=YFQ-RATIO\*DELY

HZ=ZFQ-RATIO\*DELZ

H=SQRT(HX\*HX+HY\*HY+HZ\*HZ)

IF(H=0.00001) 11,12,12

COEF=0.0

GO TO 13

CONTINUE

HRXS=H\*RXS

COEF=R4PI\*SYML00\*CC/HRXS



13	CONTINUE		
	SUM(1)=COEF*RXS1 + SUM(1)	3	11610
	SUM(2)=COEF*RXS2 + SUM(2)	3	11620
	SUM(3)=COEF*RXS3 + SUM(3)	3	11630
	YI=YIH	3	11640
	YF=YFH	3	11650
	IF (SYMLOO.EQ.-1.0.OR.SYM.NE.0.0) RETURN	3	11660
	SYMLOO=-1.0	3	11670
	YI=-YI	3	11680
	YF=-YF	3	11690
	GO TO 10	3	11700
	END	3	11710
		3	11720

SUBROUTINE VORTEX(SUM)	3	11730
COMMON/BODY/ XV(151,31),YV(151,31),ZV(151,31)	3	11740
COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)	3	11750
COMMON DA(5000)	3	11760
EQUIVALENCE (DA(19),SYM)	3	11770
COMMON/INDEX/ X,Y,Z,I1,I2,IF1,IF2	3	11780
DIMENSION SUM(1)	3	11790
XI=XV(I1,I2)	3	11800
YI=YV(I1,I2)	3	11810
ZI=ZV(I1,I2)	3	11820
XF=XV(IF1,IF2)	3	11830
YF=YV(IF1,IF2)	3	11840
ZF=ZV(IF1,IF2)	3	11850
XFQ=XF-X	3	11860
ZFQ=ZF-Z	3	11870
XIQ=XI-X	3	11880
ZIQ=ZI-Z	3	11890
SYMLOO=1.0	3	11900
YFQ=YF-Y	3	11910
YIQ=YI-Y	3	11920
DELX=XF-XI	3	11930
DELY=YF-YI	3	11940
DELZ=ZF-ZI	3	11950
RXS1=YFQ*DELZ-ZFQ*DELY	3	11960
RXS2=ZFQ*DELX-XFQ*DELZ	3	11970
RXS3=XFQ*DELY-YFQ*DELX	3	11980
RXS =SQRT(RXS1**2+RXS2**2+RXS3**2)	3	11990
TERM1=SQRT(DELX**2+DELY**2+DELZ**2)	3	12000
TERM2=SQRT(XFQ**2+YFQ**2+ZFQ**2)	3	12010
TERM3=SQRT(XIQ**2+YIQ**2+ZIQ**2)	3	12020
TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ	3	12030
RATIO = TERM4/TERM1**2	3	12040
COSA=(DELX*XIQ+DELY*YIQ+DELZ*ZIQ)/(TERM1*TERM3)	3	12050
COSB = TERM4/(TERM1*TERM2)	3	12060
CC=COSB-COSA	3	12070
HX=XFQ-RATIO*DELX	3	12080
HY=YFQ-RATIO*DELY	3	12090
HZ=ZFQ-RATIO*DELZ	3	12100
H=SQRT(HX*HX+HY*HY+HZ*HZ)	3	12110

IF(H=0.00001) 11,12,12

11 COEF=0.0  
GO TO 13

3 12120  
3 12130  
3 12140

12 CONTINUE  
HRXS=H\*RXS  
COEF=SYMLOO\*CC/HRXS

3 12150  
3 12160  
3 12170

13 CONTINUE  
SUM(1)=COEF\*RXS1 + SUM(1)  
SUM(2)=COEF\*RXS2 + SUM(2)  
SUM(3)=COEF\*RXS3 + SUM(3)  
IF(SYMLOO.EQ.-1.0.OR.SYM.NE.0.0) RETURN  
SYMLOO=-1.0

3 12180  
3 12190  
3 12200

YI=-YI  
YF=-YF  
GO TO 10  
END

3 12210  
3 12220  
3 12230  
3 12240  
3 12250  
3 12260  
3 12270

```

PROGRAM SOL
COMMON/SCRAT/XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
COMMON/PANEL/ NPAN,IPSYM,IWC,NBVVP,NTVVP,LNCFP,LTCFP,LNCP,LTICPP
1  ,NPERPT,NSPACE,NAATCH,NTRATT,NPRCLN,NPRCLT,NWCTXC,NWCTET,NTHXC
2  ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT
3  ,MPI,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
COMMON/BODY/ AA(31000)
DIMENSION PHTH(5000),ALNGTH(5000),GA(5000)
EQUIVALENCE(PHTH,AA(1)),(ALNGTH,AA(5001)),(GA,AA(10001))
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,NTAPE,NCTV,ITAPE,JTAPE
2  ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
DIMENSION BI(1000),PI(5000),SAVEC(6000),DUM(2000)
DIMENSION SAVEC1(6000),ATET(1000)
EQUIVALENCE (BI,AXB(1)),(PI,AXB(1001)),(SAVEC,AYB(1001))
EQUIVALENCE (DUM,AZB(2001)),(ATET,AZB(4001)),(DA(2),PANS)
EQUIVALENCE (SAVEC1,AA(15001))
COMMON DA(5100)
DIMENSION LT(10),LN(10),NP1(10)
COMMON/PANINF/ PANSYM(10),ASOL(600)
NPANS = PANS
IF(NPANS.EQ.0) GO TO 10
DO 8 I=1,6000
8 SAVEC(I) = SAVEC1(I)
10 CONTINUE
REWIND 23
REWIND 18 IN ORDER TO READ ALNGTH ARRAY.
REWIND 18
WRITE(6,1) NCTV,MP3,NCT
FORMAT(*OSURROUTINE SOL*/315)
WRITE(6,3) (BOUND(I),I=1,NCT)
3 FORMAT(20H0BOUNDARY CONDITIONS/(1P10E13.5))
2 FORMAT(*OSOLUTION*/(1P10E13.5))
NCOLS=NCTV+MP3
DO 20 I=1,NCOLS
IF(BOUND(I).NE.0.0) GO TO 22
20 CONTINUE
DO 21 I=1,NCOLS

```

```

21  XSOL(I)=0.0
    DO 211 I=1,600
211  ASOL(I) = 0.0
    GO TO 200
22  CONTINUE
    CALL MSOLX(NCTV,MP3,NCT,BOUND,XSOL,DUM)
    DO 2305 I=1,600
2305  ASOL(I)=XSOL(I)
23  CONTINUE
    NR=NCOLS
    KCN=LSEG(1)+TSEG(1)
    IF(KCN.EQ.0) GO TO 111
    READ(18) ALNGTH
    REWIND 18
    NTVV=NVT(1)
    NBVV=NVL(1)
    NR=NTVV*NBVV
    NC=ITAPE
    WRITE(6,231)(XSOL(I),I=1,NCOLS)
231  FORMAT(50H0SOLUTION FOR COEFFICIENTS OF CONSTRAINT EQUATIONS/
      1 (1P10E13,5))
C
CONVERT FIRST NC XSOL TERMS INTO NR K'S.
C  NOW COMPUTE THE PRODUCT OF PTH*A = GA, A COLUMN MATRIX
    DO 101 I=1,NR
101  GA(I)=0.0
    DO 102 J=1,NC
    READ(23)(PTH(I),I=1,NR)
    DO 102 I=1,NR
102  GA(I)=GA(I)+PTH(I)*XSOL(J)
C
C  NOW DO R*L MATRIX PRE-MULTIPLY OF GA VECTOR... RESULT IS K VECTOR.
    K=0
    DO 110 J=1,NTVV
    TL=0.0
    DO 110 I=1,NBVV
    K=K+1
    AL1=ALNGTH(K-1)
    AL2=ALNGTH(K)

```

```

AL3=ALNGTH(K+1)
IF(I.EQ.1) AL1=0.0
IF(I.EQ.NBVV) AL3=0.0
AL=0.5*(0.75*AL1+AL2+0.25*AL3)
XSOL(K)=AL*GA(K)+TL
TL=XSOL(K)
CONTINUE
CONTINUE
NB1=NVT(1)*NVL(1)
IF(KCN.NE.0) NB1 = ITAPE
IF(NBODS.EQ.0) NB1 = 0
IF(NBODS.EQ.0) GO TO 301
DO 300 I=1,NB1
300 B1(I) = XSOL(I)
301 IF(NPANS.EQ.0) GO TO 700
DO 305 I=1,NPANS
NBPI = NBODS+I
LT(I) = NVT(NBPI)
LN(I) = NVL(NBPI)
IF(LFUNC(I+1).NE.0) LN(I) = LFUNC(I+1)
IF(TFUNC(I+1).NE.0) LT(I) = TFUNC(I+1)
6000 FORMAT(6I5,3X,2F15.5)
305 NP1(I) = LT(I) * LN(I)
IPI = NB1
DO 310 I=1,NPANS
IT = (I-1) *1000
M = NP1(I)
WRITE(6,6000) I,NP1(I),IT,M,IPI
DO 308 K=1,M
PL(IT+K) = XSOL(IPI+K)
308 CONTINUE
310 IPI = IPI +M
C NOW DO ANY CONVERSION NECESSARY FOR PANEL A'S TO K'S
DO 500 IP=1,NPANS
I=IP
M = NP1(I)
IT = (I-1) *1000
LOC1 = (IP-1) *1000
LOC2 = LOC1 +400

```

LOC3 = LOC2 +400	4	1180
NBP1 = NBODS + IP	4	1190
IF(LN(I) .EQ. NVL(NBP1) .AND. LT(I) .EQ. NVT(NBP1)) GO TO 499	4	1200
DO 315 K=1,M	4	1210
315 DUM(K) = PL(IT+K)	4	1220
NTVVP = NVT(NBP1)	4	1230
NU = LN(IP)	4	1240
NW=LT(IP)	4	1250
NBVVP = NVL(NBP1)	4	1260
IF(LT(I) .NE. NVT(NBP1)) GO TO 330	4	1270
L = 0	4	1280
DO 320 J=1,NTVVP	4	1290
DO 320 I=1,NBVVP	4	1300
L = L + 1	4	1310
IF(J.GT. NSPP(IP)) GO TO 317	4	1320
SUM = 0.0	4	1330
DO 316 K=1,NU	4	1340
316 SUM = SUM + SAVEC(LOC2+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)	4	1350
GO TO 319	4	1360
317 SUM = 0.0	4	1370
DO 318 K=1,NU	4	1380
6001 FORMAT(I5,IPE20.6)	4	1390
318 SUM = SUM + SAVEC(LOC1+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)	4	1400
319 XSOL(L) = SUM	4	1410
320 CONTINUE	4	1420
GO TO 361	4	1430
330 CONTINUE	4	1440
L = 0	4	1450
DO 355 J=1,NTVVP	4	1460
DO 355 I=1,NU	4	1470
L = L + 1	4	1480
SUM = 0.0	4	1490
DO 346 K=1,NW	4	1500
346 SUM = SUM +DUM((K-1)*NU+I) *SAVEC((K-1)*NTVVP+J+LOC3)	4	1510
ATET(L) = SUM	4	1520
WRITE(6,6001) L,ATET(L)	4	1530
355 CONTINUE	4	1540
L = 0	4	1550
DO 360 J=1,NTVVP	4	1560

```

DO 360 I=1,NBVVP
  L = L + 1
  IF(J.GT. NSPP(IP)) GO TO 357
  SUM = 0.0
  DO 356 K=1,NU
    356 SUM = SUM +SAVEC(LOC2+(K-1)*NBVVP+I) *ATET((J-1)*NU+K)
  GO TO 359
  357 SUM = 0.0
  DO 358 K=1,NU
    358 SUM = SUM + SAVEC(LOC1+(K-1)*NBVVP+I) *ATET((J-1)*NU+K)
  359 XSOL(L) = SUM
  WRITE(6,6001) L,XSOL(L)
  360 CONTINUE
  361 CONTINUE
  IT=(IP-1)*1000
  WRITE(6,6001) IT
  DO 370 I=1,L
    370 P1(IT+I) = XSOL(I)
  499 CONTINUE
  500 CONTINUE
  L = 0
  IF(NBODS.EQ. 0) GO TO 600
  NB1=NVT(1)*NVL(1)
  DO 550 I=1,NB1
    L = L + 1
    550 XSOL(L) = B1(I)
  600 IF(NPANS.EQ. 0) GO TO 700
  DO 650 I=1,NPANS
    IT = (I-1)*1000
    M=NVT(NBODS+I)*NVL(NBODS+I)
    DO 625 K=1,M
      L = L + 1
      625 XSOL(L) = P1(IT+K)
    650 CONTINUE
    WRITE(6,50)
  50 FORMAT(47H0CONVERT SOLUTION FOR BODY CONSTRAINT FUNCTIONS)
  150 FORMAT(16H0SOLUTION MATRIX/(1P10E13.5))
  150 WRITE(6,150) (XSOL(I),I=1,L)
  150 FORMAT(16H0SOLUTION MATRIX/(1P10E13.5))

```

4 1570  
 4 1580  
 4 1590  
 4 1600  
 4 1610  
 4 1620  
 4 1630  
 4 1640  
 4 1650  
 4 1660  
 4 1670  
 4 1680  
 4 1690  
 4 1700  
 4 1710  
 4 1720  
 4 1730  
 4 1740  
 4 1750  
 4 1760  
 4 1770  
 4 1780  
 4 1790  
 4 1800  
 4 1810  
 4 1820  
 4 1830  
 4 1840  
 4 1850  
 4 1860  
 4 1870  
 4 1880  
 4 1890  
 4 1900  
 4 1910  
 4 1920  
 4 1930  
 4 1940  
 4 1950



200  
700

CONTINUE  
CONTINUE

WRITE(6,2)(XSOL(I),I=1,NCOLS)  
END

4 1960  
4 1970  
4 1980  
4 1990

```

SUBROUTINE MSOLX(NB,NP,NQT,B,XSOL,IL)
  C  NKTB = NO. OF COLUMNS IN BODY INFLUENCE MATRIX.
  C  NKT = NO. OF COLUMNS IN PANEL INFLUENCE MATRIX.
  DIMENSION B(1),XSOL(1),IL(1)
  COMMON BODY/ A(28000),AR(500),IXC(500)
  COMMON DA(5000)
  EQUIVALENCE (FUNCL,DA(34)), (FNB, DA(30)), (FNT,DA(31))
  NBV = FNB
  NTV = FNT
  WRITE(6,1000)
  1000 FORMAT(*1A MATRIX*)
  NKT=NB+NP
  IF(NB.EQ.0.OR.FUNCL.NE.0.0) GO TO 5
  NKT=NKT-NTV
  CONTINUE
  NKT=NKT+1
  N=(NKT*(NKT+3))/2
  DO 10 I=1,N
    10 A(I)=0.0
    NKT2=NKT+2
    DO 60 K=1,NQT
      IF(NB.EQ.0) GO TO 1
      READ(21)(AR(L),L=1,NB)
      IF(FUNCL.NE.0.0) GO TO 23
      L2 = 1
      L1 = 1
      NBM1 = NBV -1
      DO 22 J=1, NTV
        DO 21 I=1, NBM1
          AR(L1) = AR(L1+L2)
          21 L1 = L1 + 1
          22 L2 = L2 + 1
        23 CONTINUE
      IF(K.GT.20) GO TO 20
      1001 FORMAT('OROW',I3, ' BODY ON BODY',(1P10E13.4))
      20 CONTINUE
      1 IF(NP.EQ.0) GO TO 2
      NN=NB+1
      IF(NB.EQ.0.OR.FUNCL.NE.0.0)GOTO 24

```

```

NN = NN - NTVV
24 CONTINUE
  READ(10) (AR(L), L=NN, NKI)
  IF (K.GT.20) GO TO 30
1003 FORMAT('ROW', I3, ' WING ON BODY', (1P10E13.4))
30 CONTINUE
2 CONTINUE
  AR(NKT+1)=B(K)
  IXI=1
  DO 50 I=1, NKT
    R=SQRT(A(IXI)**2+AR(I)**2)
    IF (R.EQ.0.) GO TO 50
    C=A(IXI)/R
    S=AR(I)/R
    IXJ=IXI
    DO 40 J=1, NKTP
      T2=C*A(IXJ)+S*AR(J)
      AR(J)=-S*A(IXJ)+C*AR(J)
      A(IXJ)=T2
40 IXJ=IXJ+1
50 IXI=IXI+NKI2-1
60 CONTINUE
  REWIND 21
  REWIND 10
  II=1
  IXI=1
  DO 80 I=1, NKI
    IXC(I)=IXI
    XSOL(I)=0.
    IL(I)=0
    IF (A(IXI).LE.0.0000001) GO TO 80
    IL(I)=II
    II=II+1
80 IXI=IXI+NKT2-1
    II=NKT
    DO 210 J=1, NKT
      IF (IL(II).LE.0) GO TO 210
      JI=IL(II)
      JS=IXC(JI)-JI

```

```

JXI=JS+II
JXN=JS+NKTP
IF(II-NKT) 170,200,220
170 IK=II+1
JXK=JXI
DO 180 K=IK,NKT
JXK=JXK+1
180 XSOL(II)=XSOL(II)-A(JXK)*XSOL(K)
200 XSOL(II)=(XSOL(II)+A(JXN))/A(JXI)
210 II=II-1
220 CONTINUE
IF(NB.EQ.0.OR.FUNCL.NE.0.0) GOTO 310
L2 = 1
L1 = 1
DO 301 J=1, NTVV
DO 300 I=1, NBVV
IF(I.NE.1) GOTO 299
AR(L1) = 0.0
GOTO 300
299 AR(L1) = XSOL(L1-L2)
300 L1 = L1 + 1
301 L2 = L2 + 1
IF(NP.EQ.0) GOTO 304
DO 303 I=1, NP
AR(L1) = XSOL(L1-NTVV)
303 L1 = L1 + 1
304 L1 = L1 - 1
DO 305 I=1, L1
305 XSOL(I) = AR(I)
310 CONTINUE
RETURN
END

```

2780  
 2790  
 2800  
 2810  
 2820  
 2830  
 2840  
 2850  
 2860  
 2870  
 2880  
 2890  
 2900  
 2910  
 2920  
 2930  
 2940  
 2950  
 2960  
 2970  
 2980  
 2990  
 3000  
 3010  
 3020  
 3030  
 3040  
 3050  
 3060  
 3070  
 3080  
 3090

C	SUBROUTINE SFSOL (M,N)	4	3100
C	COEFFICIENT MATRIX ON UNIT 23 BY ROWS	4	3110
C	M ROWS	4	3120
C	N COLUMNS	4	3130
C	B VECTOR (RIGHT SIDE) IN BOUND	4	3140
	COMMON /BODY/S(150,150),X(1000), BWORK(150),XWORK(150),COL(150),	4	3150
	ITOPCOL(150),WORK(150),IWORK(150),ATA(1000), ATB(1000), A(1000),	4	3160
	2PAD(600)	4	3170
	COMMON /SCRAT/XSOL(5000),BOUND(5000)	4	3180
	INTEGER UNIT	4	3190
	REWIND 10	4	3200
C	GENERATE A TRANSPOSE A ON UNIT	4	3210
	CALL ATRAN (A,M,N,BOUND,ATB,ATA,UNIT)	4	3220
C	NOW SOLVE N X N SYSTEM	4	3230
C	DETERMINE NROW, MAXIMUM PARTITION IS 150 ROWS	4	3240
C	THERE MUST BE AT LEAST TWO PARTITION ROWS	4	3250
	IF (N - 150) 5,5,7	4	3260
	5 NROW = 2	4	3270
	GO TO 9	4	3280
	7 NROW = N / 150	4	3290
	IF (N - (NROW*150)) 8,9,8	4	3300
	8 NROW = NROW + 1	4	3310
C	DETERMINE SIZES OF SUBMATRICES	4	3320
C	INTERIOR SUBMATRICES	4	3330
C	UPPER LEFT SUBMATRIX	4	3340
	NP1 = N - NPO * NROW + NPO	4	3350
C	STORE SUBMATRICES IN UNIT 10	4	3360
	DO 15 KROW = 1,NROW	4	3370
C	SUBMATRIX SIZE FOR KROW, NE. 1	4	3380
C	NP = NPO	4	3390
C	SUBMATRIX SIZE FOR KROW = 1	4	3400
	IF (KROW.EQ. 1) NP = NP1	4	3410
C	ROW NUMBER OF TOP ELEMENT IN SUBMATRIX	4	3420
	KTOP = N - NP - (NROW-KROW)*NPO + 1	4	3430
C	ROW NUMBER OF BOTTOM ELEMENT IN SUBMATRIX	4	3440
	KBOT = KTOP + NP - 1	4	3450
C	STORE SUBMATRIX COLUMNS	4	3460
C	READ ONE COLUMN AT A TIME FROM UNIT, N ELEMENTS	4	3470
		4	3480

C	LOAD JCOL OF COEFFICIENT MATRIX FROM KTOP TO KBOT	4	3490
	DO 10 JCOL = 1,N	4	3500
	READ (UNIT) (A(I), I=1,N)	4	3510
C	10 WRITE (10) (A(I), I = KTOP,KBOT)	4	3520
	LOAD CONSTANT VECTOR FROM KTOP TO KBOT	4	3530
	WRITE (10) (ATB(I), I = KTOP,KBOT)	4	3540
	15 REWIND UNIT	4	3550
	REWIND 10	4	3560
C	PUT SOLUTION IN X	4	3570
	18 CALL XSOLVE (1000,150,N, NPO, NP1, NROW, S, XSOL, IWORK, WORK,	4	3580
	IBWORK, XWORK, COL, TOPCOL)	4	3590
	RETURN	4	3600
	END	4	3610

```

SUBROUTINE XSOLVE(MX,NPX,N,NPO,NP1,NROW,Z,X,IWORK,WORK,
C ***** SUBROUTINE FOR SOLUTION BY PARTITIONING USING FILE STORAGE *****
C ***** WRITTEN BY D.G. ELLIOTT USING SCHEME SUGGESTED BY C.O.L. LAWSON *****
C ***** JET PROPULSION LABORATORY, PASADENA, NOVEMBER 1, 1972 *****
C *
C * BWORK,XWORK,COL,TOCOL)
C * INTEGER IWORK(NPX),UNILL,UNIT2,UNIT3
C REAL WORK(NPX)
C REAL Z(NPX,NPX),X(MX),BWORK(NPX),XWORK(NPX),
C * COL(NPX),TOCOL(NPX)
C ***** FORTRAN UNITS 10, 11, AND 12 USED FOR STORAGE *****
C ***** TRIANGULARIZATION *****
C 6 NCXS = 0
C NCX1 = N+1-NP1
C DO 90 KROW=1,NROW
C SET UP UNITS
C IF (KROW - 1) 2,2,3
C 2 UNIT1 = 10
C UNIT2 = 21
C UNIT3 = 22
C GO TO 4
C 3 JU = UNIT3
C UNIT3 = UNIT2
C UNIT2 = UNIT1
C UNIT1 = JU
C CONTAINS OLD LOWER-RIGHT MATRIX
C TO RECEIVE TRANSFORMED TOP ROW
C TO RECEIVE NEW LOWER-RIGHT MATRIX
C SIZE OF INTERIOR SUBMATRICES
C SIZE OF UPPER-LEFT SUBMATRIX
C IF (KROW .EQ. 1) NP = NP1
C

```

```

C      NCX = N+1-NP1-(KROW-1) *NP0
C      DO 10 J=1,NP
C          READ (UNIT1) (Z(K,J),K=1,NP)
C          TRIANGULARIZE FIRST MATRIX
C          CALL DECOMP(NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
C          TRANSFORM REST OF ROW, INCLUDING RT-SIDE VCT
C      DO 20 J=1,NCX
C          NEXT COLUMN IN ROW
C          READ (UNIT1) (BWORK(K),K=1,NP)
C          PUT TRANSFORMED COLUMN IN XWORK
C          CALL SOLVE (NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
C          TRIANGULARIZATION COMPLETE
C      IF(KROW.EQ.NROW) GO TO 100
C          STORE TRANSFORMED COLUMN OF KROW
C      20  WRITE (UNIT2) (XWORK(K),K=1,NP)
C          NO PREVIOUS ROW TO STACK IN 2
C      IF(KROW.EQ.1) GO TO 40
C      NNP = NPO
C          STACK TRANSFORMED ROWS IN REVERSE ORDER
C      DO 30 J=1,NCXS
C          IF((NCXS-J).LT.NCX1) NNP = NP1
C          READ PRVIOUSLY TRANSFORMED ROWS
C          READ (UNIT3) (COL(K),K=1,NNP)
C          AND WRITE AFTER NEW ONE
C      30  WRITE (UNIT2) (COL(K),K=1,NNP)
C          FOR WRITING NEW LOWER-RT MATRIX IN NEXT OPER
C      REWIND UNIT3
C      40  KROWP1 = KROW +1
C          SUBTRACT MULTIPLES OF TOP ROW FROM EACH ROW
C      DO 80 KKROW=KROWP1,NROW
C          DO 50 J=1,NP
C          READ (UNIT1) (Z(K,J),K=1,NP0)
C          REPOSITION AT START OF TOP ROW
C      50

```





```

C      IF(KROW.EQ.1) NP = NP1
C      KSTART = N-NP-NMKROW *NP0
C      DO 130 KCOL=1,NMKROW
C      KCOLM1 = KCOL -1
C      DO 120 J=1,NP0
C      NEXT MATRIX IN KROW
C      READ (UNIT3) (Z(K,J),K=1,NP)
C      FORM PRODUCT OF MATRICES AND SUBVECTORS
C      DO 130 K=1,NP
C      DO 130 JJ=1,NP0
C      X(KSTART+K) = X(KSTART+K) -Z(K,JJ) *X(KSTART+NP+KCOLM1)
C      * *NP0+JJ)
C      READ (UNIT3) (BWORK(K),K=1,NP)
C      ADD RIGHT-SIDE VECTOR TO COMPLETE X FOR KROW4
C      DO 140 K=1,NP
C      X(KSTART+K) = X(KSTART+K) +BWORK(K)
C      RETURN
C      END

```

4780  
 4790  
 4800  
 4810  
 4820  
 4830  
 4840  
 4850  
 4860  
 4870  
 4880  
 4890  
 4900  
 4910  
 4920  
 4930  
 4940  
 4950  
 4960  
 4970

```

SUBROUTINE ATRAN (A,M,N,B,ATB,ATA, UNIT)
  M
  A TRANSPOSE TIMES A (ATRA)
  A STORED BY ROWS ON UNIT 23
  M ROWS, N COLUMNS, M IS GREATER THAN OR EQUAL N
  A TRANSPOSE A STORED BY COLUMNS (ROWS DUE TO SYMMETRY)
  UNIT IS AUXILIARY UNIT CONTAINING A TRANSPOSE A

  DIMENSION A(N), ATA(N), B(N), ATB(N)
  INTEGER UNIT, UNITN, UNITN1
  PREPARE AUXILIARY STORAGE

  1 UNITN = 21
  UNITN1 = 22
  IFLAG = -1
  REWIND 21
  REWIND 22
  REWIND 23
  DO 5 I = 1,N
  DO 4 J = 1,N
  4 ATA(J) = 0.0
  5 WRITE (21) ATA
  REWIND 21

  INITIALIZE STORAGE FOR A TRANSPOSE B VECTOR

  DO 6 I = 1,N
  6 ATB(I) = 0.0
  READ A ROW OF A

  DO 40 I = 1,M
  READ (23) A
  READ PARTIAL VALUE OF ELEMENTS IN A COLUMN OF ATA

  DO 30 J = 1,N
  READ (UNITN) ATA
  GENERATE NEXT TERM FOR ELEMENTS IN COLUMN O ATA

  DO 20 L = 1,N
  20 ATA(L) = ATA(L) + A(J) * A(L)
  WRITE (UNITN1) ATA
  30 CONTINUE
  GENERATE NEXT TERM FOR ELEMENTS OF ATB

  DO 25 K = 1,N
  25 ATB(K) = ATB(K) + A(K) * B(I)
  IFLAG = IFLAG * (-1)
  UNITN = UNITN + IFLAG

```

UNITN1=UNITN1 - IFLAG  
REWIND UNITN  
REWIND UNITN1  
40 CONTINUE  
UNIT = UNITN  
RETURN  
END

4 5370  
4 5380  
4 5390  
4 5400  
4 5410  
4 5420  
4 5430

```

SUBROUTINE DECOMP(MX,N,UL,B,X,SCALES,IPS)
C
C ***** SUBROUTINE FOR SIMULTANEOUS EQUATION SOLVING *****
C ***** REF. G. FORSYTHE AND C. MOLER, 'COMPUTER SOLUTION OF LINEAR
C          ALGEBRAIC SYSTEMS', PRENTICE-HALL, 1967 *****
C
C ***** NOMENCLATURE' UL=COEFF MATRIX, B=RIGHT-SIDE VCTR, X=UNKNOWN VCTR
C ***** CALL TO DECOMP TRIANGULARIZES THE COEFFICIENT MATRIX *****
C
      REAL    UL(MX,MX),B(MX),X(MX),PIVOT,EM,SUM
      DIMENSION SCALES(MX),IPS(MX)
C
C INITIALIZE IPS AND SCALES
      DO 5 I = 1,N
      IPS(I) = 1
      ROWNRM = 0.0
      DO 2 J = 1,N
        ROWNRM = AMAX1( ROWNRM, ABS(UL(I,J)) )
      IF (ROWNRM) 3,4,3
      SCALES(I) = 1.0/ROWNRM
      GO TO 5
      PRINT 20
      SCALES(I) = 0.0
      CONTINUE
C
C GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
      NML=N-1
      DO 17 K=1,NML
        BIG = 0.0
        DO 11 I = K,N
          IP = IPS(I)
          SIZE = ABS(UL(IP,K))*SCALES(IP)
          IF (SIZE-BIG) 11,11,10
          BIG = SIZE
          IDXPIV = I
          CONTINUE
        IF (BIG) 13,12,13
        PRINT 25
        GO TO 17
      10
      11
      12

```

```

13 IF (IDXPIV-K) 14,15,14
14 J = IPS(K)
   IPS(K) = IPS(IDXPIV)
   IPS(IDXPIV) = J
15 KP = IPS(K)
   PIVOT = UL(KP,K)
   KP1 = K+1
      DO 16 I = KP1,N
      IP = IPS(I)
      EM = -UL(IP,K)/PIVOT
      UL(IP,K) = -EM
      DO 161 J1 = KP1,N
      UL(IP,J1) = UL(IP,J1) + EM*UL(KP,J1)
161 CONTINUE
16 CONTINUE
17 CONTINUE
      KP = IPS(N)
      IF (UL(KP,N)) 19,18,19
18 PRINT 20
19 GO TO 500
20 FORMAT (* MAIRIX WITH ZERO ROW IN DECOMP*)
25 FORMAT (* SINGULAR MATRIX IN DECOMP*)
C
C ***** THIS ENTRY BACK-SUBSTITUTES THE RIGHT SIDE *****
C
ENTRY SOLVE
NPI = N + 1
C
IP = IPS(1)
X(1) = B(IP)
DO 30 I = 2,N
IP = IPS(I)
SUM = 0.0
IM1 = I-1
DO 27 J = 1,IM1
SUM = SUM + UL(IP,J)*X(J)
27 X(I) = B(IP) - SUM
30
C
IP = IPS(N)

```

```

4 5830
4 5840
4 5850
4 5860
4 5870
4 5880
4 5890
4 5900
4 5910
4 5920
4 5930
4 5940
4 5950
4 5960
4 5970
4 5980
4 5990
4 6000
4 6010
4 6020
4 6030
4 6040
4 6050
4 6060
4 6070
4 6080
4 6090
4 6100
4 6110
4 6120
4 6130
4 6140
4 6150
4 6160
4 6170
4 6180
4 6190
4 6200
4 6210

```

6220  
6230  
6240  
6250  
6260  
6270  
6280  
6290  
6300  
6310  
6320  
6330

4  
4  
4  
4  
4  
4  
4  
4  
4  
4  
4

```

X(N) = X(N)/UL(IP,N)
DO 40 IBACK = 2,N
  I = NP1-IBACK
  C I GOES (N-1),...,1
  IP = IPS(I)
  IP1 = I+1
  SUM = 0.0
  DO 35 J = IP1,N
    SUM = SUM + UL(IP,J)*X(J)
  X(I) = (X(I)-SUM)/UL(IP,I)
35
40
500 RETURN
END

```

```

PROGRAM VEL
REWIND 12
CALL VELOC
END
SUBROUTINE VELOC
COMMON/SLOPE/SIGMAP(500), DZDXT(500),DZDXC(500),TANP1(500)
EQUIVALENCE (B(11871),SIGMA)
DIMENSION SIGMA(1000)
COMMON/SCRAT/ AK(5000),XG(11,151),GANT(11,151),GAWR(1500)
1 DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPE(1500)
2 XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
3 CP(1320)
EQUIVALENCE(XCCO,B(2421))
DIMENSION XCCO(20),XOC(20)
COMMON/BODY/ B(31000)
EQUIVALENCE(B(14044),TMX), (B(15364),TMY), (B(16584),TMZ)
1 , (B(18004),TTX), (B(19324),TTY), (B(20644),TTZ)
EQUIVALENCE(DA(3),FMACH)
DIMENSION TMX(1320),TMY(1320),TMZ(1320)
1 TTX(1320),TTY(1320),TTZ(1320)
COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
1 XN(1320),YN(1320),ZN(1320)
COMMON DA(5000)
COMMON /COMPRS/ BETAM
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1 NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
2 LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYNCR(7)
C
C WHEN SOURCE MATRIX IS READ, GAMMA ARRAYS WILL BE DESTROYED.
EQUIVALENCE (SMX,XG),(SMY,GANT),(SMZ,GAMB)
DIMENSION SMX(800),SMY(800),SMZ(800)
C
DIMENSION VX(1320),VY(1320),VZ(1320)
EQUIVALENCE(VX,B(22001)),(VY,B(23321)),(VZ,B(24641))
C
C VX,VY,VZ EQUIVALENCING SHOULD NOT DESTROY SIGMA ARRAY.
C
EQUIVALENCE (DA(7), XCG),(DA(8),YCG),(DA(9),ZCG),(DA(10),ALPHA)
1 ,(DA(11),BETA),(DA(12),PSTAR),(DA(13),QSTAR),(DA(14),RSTAR)

```



COMMON/PANEL/ NPAR,IPSYN,IAC,NEVVP,NTVVP,LNCFP,LUCFP,LTCPP	5	0400
1 ,NPERPT,NSPACE,NATICH,NTRATT,NPRCLN,NPRCLT,NACTXC,NACTET,NTHXC	5	0410
2 ,NTHET,NTIP,CHIIP,ROOT,CUTER,NMAT	5	0420
3 ,MPI,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10	5	0430
DIMENSION AX(4681),AY(4681),AZ(4681)	5	0440
EQUIVALENCE(B(1),AX),(B(4682),AY),(B(9363),AZ)	5	0450
EQUIVALENCE (B,BB,BP)	5	0460
EQUIVALENCE(AKIB,B(24001)),(AKIP,B(24501)),(DA(2),PANS)	5	0470
DIMENSION AKIB(500),AKIP(500)	5	0480
DIMENSION BD(22000),BP(13000)	5	0490
C	5	0500
C BB IS READ INTO CORE TO GET TMX - ITZ ARRAYS INTO CORE.	5	0510
C BB IS NOT LONG ENOUGH TO DESTROY SIGMA ARRAY.	5	0520
C	5	0530
C MP2 IS UNIT FOR PANEL INFLUENCE MATRIX	5	0540
C MP3 IS NUMBER OF AX,AY,AZ TERMS IN EACH ROW OF THE MATRIX ON MP2.	5	0550
C NCB = NO. OF CONTROL POINTS ON BODY.	5	0560
C	5	0570
COMMON/PANINF/ PANSYM(10),ASOL(600)	5	0580
DO 1 I=1,20	5	0590
1 XOC(I)=XCCO(I)	5	0600
C	5	0610
C TEMPORARY BREF,CBAR	5	0620
BREF=1.0	5	0630
CBAR=10.0	5	0640
C	5	0650
GAM14=1.4	5	0660
IF(NBODS.NE.0) REWIND NTAPE	5	0670
IF(PANS.NE.0.0) REWIND MP2	5	0680
CALL GAMMA FOR CAPITAL GAMMA ARRAYS.	5	0690
CALL GAMMA	5	0700
C	5	0710
CALL DELV FOR DELVM AND DELVI ARRAYS.	5	0720
CALL DELV	5	0730
WRITE(6,5000) (DELVT(I),I=1,200)	5	0740
5000 FORMAT( 6HDELVT/(1H,10F12.6))	5	0750
WRITE(6,5001) (DELVM(I),I=1,200)	5	0760
5001 FORMAT( 6HDELVM/(1H,10F12.6))	5	0770
IF(NBODS.EQ.0) GO TO 4	5	0780

C

READ(18)

READ(18)

READ(18) B3

CONTINUE

C

C

NCB=NCPTS(1)

IF(NBODS.EQ.0) NCB=0

NPANS=NPANS

DO 5 I=1,NCT

XX=XQ(I)-XCG

YY=YQ(I)-YCG

ZZ=ZQ(I)-ZCG

VX(I)=1.0-2.0\*(QSTAR\*ZZ/CBAR-RSTAR\*YY/BREF)

VY(I)=-BETA-2.0\*(PSTAR\*ZZ-RSTAR\*XX)/BREF

VZ(I)=ALPHA+2.0\*(PSTAR\*YY/BREF+QSTAR\*XX/CBAR)

CONTINUE

NPWT=(NCT-NCB)\*2+NCB

DO 6 I=1,NPWT

VM(I)=0.0

VT(I)=0.0

IF(NPANS.EQ.0) GO TO 2005

ISIG=0

C READ PANEL DATA TO GET SIGMA

DO 7 I=1,NPANS

READ(18) BP

REWIND 18

WRITE(6,8000) (ASOL(I),I=1,220)

FORMAT(1H,1P10E13.4)

WRITE(6,8000) (SIGMA(I),I=1,280)

WRITE(6,8000) (SIGMAP(I),I=1,280)

WRITE(6,8000) (TANPI(I),I=1,140)

WRITE(6,8000) (DZDXT(I),I=1,140)

WRITE(6,8000) (DZDXC(I),I=1,140)

DO 2000 I=1,NPANS

NSIG=NVPTS(NBODS+1)\*2

C

COMPUTE TERMS INVOLVING SOURCE MATRICES.

```

DO 1000 IC=1,NCT
T1=C.0
T2=0.0
T3=0.0
T4=0.0
T5=0.0
T6=0.0
READ(11)(SMX(J),SMY(J),SMZ(J),J=1,NSIG)
DO 50 J=1,NSIG
IS=ISIG+J
T1=T1+(SMX(J)*TMX(IC)+SMY(J)*TMY(IC)+SMZ(J)*TMZ(IC))*SIGMA(IS)
T2=T2+(SMX(J)*TTX(IC)+SMY(J)*TTY(IC)+SMZ(J)*TTZ(IC))*SIGMA(IS)
T3=T3+SMX(J)*SIGMA(IS)
SIGDIF=SIGMA(IS)-SIGMAP(IS)
T4=T4+SMX(J)*SIGDIF
TERM=ZN(IC)*SMY(J)-YN(IC)*SMZ(J)
T5=T5+TERM*SIGMA(IS)
T6=T6+TERM*SIGDIF
50 CONTINUE
C NOW A4D MORE TERMS TO THOSE ABOVE.
IF(I.NE.NPANS) GO TO 1000
IF(9C.GT.NCB) GO TO 500
VM(IC)=T1
VT(IC)=T2
WRITE(6,8003) IC,VM(IC),VT(IC)
8003 FORMAT(I4,1P6E20.5)
GO TO 600
C
500 IX=2*IC-NCB-1
COMPUTE SUBSCRIPT FOR TOP CONTROL POINT ON PANEL.
C TOP WILL HAVE VELOCITY(IX)
C BOTTOM WILL HAVE VELOCITY(IX+1).
ICP=IC-NCB
VM(IX)=T3+DELVM(IC)*(1.0+SQRT(TANPL(ICP))*T4)
VM(IX+1)=T3-DELVM(IC)*(1.0+SQRT(TANPL(ICP))*T4)
TERM = T5+ZN(IC)*VY(IC)*BETAM -YN(IC)*VZ(IC)*BETAM
TERM2=DELVT(IC)*(1.0+SQRT(TANPL(ICP))*T6)
VT(IX)=TERM+TERM2
VT(IX+1)=TERM-TERM2

```

1180  
1190  
1200  
1210  
1220  
1230  
1240  
1250  
1260  
1270  
1280  
1290  
1300  
1310  
1320  
1330  
1340  
1350  
1360  
1370  
1380  
1390  
1400  
1410  
1420  
1430  
1440  
1450  
1460  
1470  
1480  
1490  
1500  
1510  
1520  
1530  
1540  
1550  
1560

WRITE(6,8003) IC,YN(IC),ZN(IC),VY(IC),VZ(IC)  
WRITE(6,8003) IX,VM(IX),VT(IX),T3,T4,T5,T6  
IXX=IX+1

5 1570  
5 1580  
5 1590

WRITE(6,8003) IXXX,VM(IXXX),VT(IXXX),DELM(IC),DELVT(IC)

600 CONTINUE  
1000 CONTINUE

ISIG=IS

2000 CONTINUE

2005 NBI=NBODS+1

DO 3000 IC=1,NCT

FLAG=1.0

IF(NPANS.EQ.0) GO TO 2014

IF(IC-NCB-NCPTS(NBI)-GT.0) NBI=NBI+1

LPTS=LNC(NBI)

ICR=IC-((IC-1)/LPTS)\*LPTS

2014 CONTINUE

IX=2\*IC-NCB-1

IF(NBODS.EQ.0) GO TO 2050

READ(NTAPE)(AX(I),AY(I),AZ(I),I=1,NCTV)

IF(IC-GT.NCB) GO TO 2020

TERM=0.0

DO 2015 I=1,NCTV

2015 TERM=TERM+(TMX(IC)\*AX(I)+TMY(IC)\*AY(I)+TMZ(IC)\*AZ(I))\*ASOL(I)

VM(IC)=VM(IC)+TERM

TERM=0.0

DO 2016 I=1,NCTV

2016 TERM=TERM+(ITX(IC)\*AX(I)+ITY(IC)\*AY(I)+ITZ(IC)\*AZ(I))\*ASOL(I)

VT(IC)=VT(IC)+TERM

VM(IC) = VM(IC)+BETAM\*\*2\*TMX(IC)\*VX(IC)+BETAM\*TMY(IC)\*VY(IC)+

1BETAM\*TMZ(IC)\*VZ(IC) +DELVT(IC)

VT(IC) = VT(IC)+BETAM\*\*2\*ITX(IC)\*VX(IC)+BETAM\*ITY(IC)\*VY(IC)+

1BETAM\*ITZ(IC)\*VZ(IC) +DELVT(IC)

DBETA=SQRT(BETAM\*\*2\*(TMX(IC)\*\*2+TMY(IC)\*\*2+TMZ(IC)\*\*2))

VM(IC) = VM(IC) /BETAM/DBETA

DBETA=SQRT(BETAM\*\*2\*(ITX(IC)\*\*2+ITY(IC)\*\*2+ITZ(IC)\*\*2))

VT(IC) = VT(IC) /BETAM/DBETA

GO TO 2050

2020 TERM=0.0

DO 2025 I=1,NCTV

2025	TERM=TERM+AX(I)*ASOL(I)	5	1960
	WRITE(6,8001) IC,TERM	5	1970
3001	FORMAT(1H0,I5,1PE15.5)	5	1980
	VM(IX)=VM(IX)+TERM	5	1990
	VM(IX+1)=VM(IX+1)+TERM	5	2000
	TERM=0.0	5	2010
	DO 2026 I=1,NCTV	5	2020
2026	TERM=TERM+(ZN(IC)*AY(I)-YI(IC)*AZ(I))*ASOL(I)	5	2030
	WRITE(6,8001) IC,TERM	5	2040
	VT(IX)=VT(IX)+TERM	5	2050
	VT(IX+1)=VT(IX+1)+TERM	5	2060
2050	CONTINUE	5	2070
	IF(NPANS.EQ.0) GO TO 2900	5	2080
	REA4(MP2)(AX(I),AY(I),AZ(I),I=1,MP3)	5	2090
	IF(IC.GT.NCB) GO TO 2070	5	2100
	TERM=0.0	5	2110
	DO 2065 I=1,MP3	5	2120
	IK=NCTV+1	5	2130
2065	TERM=TERM+(TMX(IC)*AX(I)+TMY(IC)*AY(I)+TMZ(IC)*AZ(I))*ASOL(IK)	5	2140
	VM(IX)=VM(IX)+TERM	5	2150
	TERM=0.0	5	2160
	DO 2066 I=1,MP3	5	2170
	IK=NCTV+1	5	2180
2066	TERM=TERM+(TTX(IC)*AX(I)+TTY(IC)*AY(I)+TTZ(IC)*AZ(I))*ASOL(IK)	5	2190
	VT(IX)=VT(IX)+TERM	5	2200
	GO TO 2900	5	2210
2070	TERM=0.0	5	2220
	DO 2075 I=1,MP3	5	2230
	IK=I+NCTV	5	2240
2075	TERM=TERM+AX(I)*ASOL(IK)	5	2250
	WRITE(6,8001) IC,TERM	5	2260
	VM(IX)=VM(IX)+TERM	5	2270
	VM(IX+1)=VM(IX+1)+TERM	5	2280
	TERM=0.0	5	2290
	DO 2076 I=1,MP3	5	2300
	IK=I+NCTV	5	2310
2076	TERM=TERM+(ZN(IC)*AY(I)-YI(IC)*AZ(I))*ASOL(IK)	5	2320
	WRITE(6,8001) IC,TERM	5	2330
	VT(IX)=VT(IX)+TERM	5	2340

VT(IX+1)=VT(IX+1)+TERM	5	2350
IF(IC.LE.NCB) GO TO 2900	5	2360
ICP=IC-NCB	5	2370
TOP=1.0/SQRT(1.0+TANP1(ICP)*(DZDXT(ICP)+DZDXC(ICP))*2)	5	2380
BOTTOM=1.0/SQRT(1.0+TANP1(ICP)*(DZDXT(ICP)-DZDXC(ICP))*2)	5	2390
VM(9X) = TOP*(VM(IX)/BETAM**2 + VX(IC))	5	2400
VM(IX+1) = BOTTOM*(VM(IX+1)/BETAM**2 + VX(IC))	5	2410
VT(IX) = TOP* VT(IX)/BETAM	5	2420
VT(9X+1) = BOTTOM*VT(IX+1)/BETAM	5	2430
WRITE(6,8002) IX,VM(IX),VM(IX+1),VT(IX),VT(IX+1)	5	2440
8002 FORMAT(14,1P4E20.5//)	5	2450
2900 IF(IC-NCB) 2901,2901,2902	5	2460
2901 ICX=IC	5	2470
GO TO 2903	5	2480
2902 ICX=IX	5	2490
2903 A=1.0-VT(ICX)**2-VM(ICX)**2	5	2500
AM2=A*FMACH**2	5	2510
IF(6MACH-0.1) 2905,2905,2904	5	2520
2904 CP(9CX)=(2.0/(GAM14*FMACH**2))*((1.0+(GAM14-1.0)*0.5*AM2)	5	2530
1 ** (GAM14/(GAM14-1.0))-1.0)	5	2540
GO TO 2906	5	2550
2905 CP(ICX)=A*(1.0+0.25*A**2*(1.0-0.1*AM2))	5	2560
2906 CONTINUE	5	2570
IF(FLAG.EQ.-1.0) GO TO 2905	5	2580
IF(IC-NCB) 2908,2908,2907	5	2590
2907 FLAG=-1.0	5	2600
ICX=ICX+1	5	2610
GO TO 2903	5	2620
2908 CONTINUE	5	2630
3000 CONTINUE	5	2640
IXX=IX+1	5	2650
IF(NPANS.EQ.0) IXX=IC-1	5	2660
REWIND 11	5	2670
IF(NBODS.NE.0) REWIND NIAPE	5	2680
IF(PANS.NE.0.0) REWIND MP2	5	2690
C	5	2700
C SET TRAILING EDGE X VALUES.	5	2710
L=0	5	2720
IV=0	5	2730

```

IF (NBODS.EQ.0) GO TO 310
NT=NVT(1)
NL=NVL(1)
IV=NT*NL
DO 305 I=1,NT
J=I*NL
LTD=LTDIV(1)
DO 305 II=1,LTD
L=L+1
305 AKTB(L)=AK(J)
WRITE(6,7000) (I,AKTP(I),I=1,L)
7000 FORMAT(*AKTB*/(15,1PE20.6))
310 IF (PANS.EQ.0.0) GO TO 400
L=0
DO 350 K=1,NPANS
NT=NVT(NBODS+K)
NL=NVL(NBODS+K)
DO 340 I=1,NT
J=I*NL+IV
L=L+1
340 AKTP(L)=AK(J)
WRITE(6,7001) (I,AKTP(I),I=1,L)
7001 FORMAT(*AKTP*/(15,1PE20.6))
350 IV=IV+NT*NL
400 CONTINUE
RETURN
END

```

5	2740
5	2750
5	2760
5	2770
5	2780
5	2790
5	2800
5	2810
5	2820
5	2830
5	2840
5	2850
5	2860
5	2870
5	2880
5	2890
5	2900
5	2910
5	2920
5	2930
5	2940
5	2950
5	2960
5	2970
5	2980
5	2990
5	3000

ORIGINAL PAGE IS  
OF POOR QUALITY

```

SUBROUTINE DELV
COMMON DA(5000)
COMMON/NUMBER/ NVPTS(7),VCPTS(7),NLN(7),NLI(7),LTC(7),LNC(7)
1  ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
2  ,LSEG(7),TSFG(7),LFUNC(7),TFUNC(7)
3  ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON/SCRAT/XSOL(5000),XG(11,151),GAMT(11,151),GAMR(1500)
1  ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
2  ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
3  ,CP(1320),XGW(50,20),GAMTW(50,20)
EQUIVALENCE(DA(2),PANS)
EQUIVALENCE(DA(2800),FLNC), (DA(3100),FLTC)
DIMENSION FLNC(150),FLTC(40)
COMMON/BODY/ B(31000)
EQUIVALENCE(ALNGTH,B(20000)),(BLNGTH,B(25000))
DIMENSION ALNGTH(5000),BLNGTH(5000)
COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
1  ,XN(1320),YN(1320),ZN(1320)
COMMON/CONTRV/ LNN(3,40),LTT(3,40)
COMMON/AF/ ATTFB(10),ATTFW(50)
IF(NBODS.EQ.0) GO TO 5

REWIND 18
NNV=NVL(1)*NVT(1)
READ(18) (ALNGTH(I),I=1,NNV)
READ(18) (BLNGTH(I),I=1,NNV)
REWIND 18
CONTINUE

5
NPANS=PANS
KC=0
WRITE(6,4998)(GAMB(I),I=1,100)
4998 FORMAT(5H0GAMB/(1P10E12.4))
IF(NBODS.EQ.0) GO TO 100

C
COMPUTE DELVM,DELVT AT BODY CONTROL POINTS.
LTC3=LTC(1)
LNCC=LNC(1)
DO 50 J=1,LTC

```



C	IT1, IT2 ARE TRAILING VORTICES ON WHICH INTERPOLATED GAMT ARE FOUND.	5	3400
C	IT1=FLTC(J)+0.01	5	3410
	IT2=IT1+1	5	3420
	DO 50 I=1, LNCC	5	3430
	KC=KC+1	5	3440
C	IB1, IB2 ARE BOUND VORTICES.	5	3450
	IB1=FLNC(I)+0.01	5	3460
	IB2=IB1+1	5	3470
	L2=(IT1-1)*NVL(1)+IB1-1	5	3480
	IF (IB1.NE.1) GO TO 10	5	3490
	ALM2=0.0	5	3500
	GO TO 12	5	3510
10	ALM2=ALNGTH(L2)	5	3520
12	L5=L2+1	5	3530
	ALM5=ALNGTH(L5)	5	3540
	IF (IB1.NE.NVL(1)) GO TO 14	5	3550
	ALM8=0.0	5	3560
13	ALM11=0.0	5	3570
	GO TO 20	5	3580
14	L8=L5+1	5	3590
	ALM8=ALNGTH(L8)	5	3600
	IF (IB2.GE.NVL(1)) GO TO 13	5	3610
	ALM11=ALNGTH(L8+1)	5	3620
20	CONTINUE	5	3630
	NGAM1=NVL(1)	5	3640
	NGAM2=NGAM1	5	3650
	IF (ATTFB(IT1).EQ.99.0) NGAM1=20	5	3660
25	CONTINUE	5	3670
	IF (ATTFB(IT2).EQ.99.0) NGAM2=20	5	3680
26	CONTINUE	5	3690
	DO 261 M=1, NGAM1	5	3700
	XD(M)=XG(IT1, M)	5	3710
261	GD(M)=GAMT(IT1, M)	5	3720
	IF (XG(KC).GT.XD(NGAM1)) XD(NGAM1) = XG(KC)	5	3730
	CALL CODIM(XD, GD, NGAM1, XG(<C), GAM1, 1)	5	3740
	DO 262 M=1, NGAM2	5	3750
	XD(M)=XG(IT2, M)	5	3760
262	GD(M)=GAMT(IT2, 1)	5	3770
		5	3780

	IF(XQ(KC).GT.XD(NGAM2)) XD(NGAM2) = XQ(KC)	
	CALL CODIM(XD,GD,NGAM2,XQ(KC),GAM2,1)	
	L2=(IT1-1)*NVL(1)+IB1	5 3790
	L1=L2-NVL(1)	5 3800
	L3=L2+NVL(1)	5 3810
	BL2=BLNGTH(L2)	5 3820
	IF(IT1.NE.1) GO TO 35	5 3830
	BL1=BL2	5 3840
	BL3=BLNGTH(L3)	5 3850
	GO TO 40	5 3860
	IF(IT1.NE.NVT(1)) GO TO 36	5 3870
35	BL1=BLNGTH(L1)	5 3880
	BL3=BLNGTH(L3)	5 3890
	GO TO 40	5 3900
	BL1=BLNGTH(L1)	5 3910
	BL3=BLNGTH(L3)	5 3920
36	CONTINUE	5 3930
40	D = 0.75*ALM5+0.25*ALM8	5 3940
	IF(IB1-NVL(1)) 42,41,42	5 3950
41	GAMB2=0.0	5 3960
	GO TO 43	5 3970
	GAMB2=GAMB(L2+1)	5 3980
42	CONTINUE	5 3990
43	FORMAT(8I5)	5 4000
6002	DELVM(KC)=GAMB(L2)/(0.75*ALM2+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D	5 4010
6001	+GAMB2/(0.75*ALM5+ALM8+0.25*ALM11)*0.5*ALM5/D	5 4020
1	DELVT(KC)=-0.5*(GAM1/(BL1+BL2) +GAM2/(BL2+BL3))	5 4030
	CONTINUE	5 4040
50	CONTINUE	5 4050
100	IF(NPANS.EQ.0) GO TO 300	5 4060
	KTRV=0	5 4070
	IF(NBODS) 101,101,102	5 4080
101	NBVORT=0	5 4090
	GO TO 103	5 4100
102	NBVORT=NVL(1)*NVT(1)	5 4110
103	CONTINUE	5 4120
	DO 250 II=1,NPANS	5 4130
	NBP=N90DS+II	5 4140
		5 4150
		5 4160
		5 4170

COMPUTE D5LVM,DELVT ON PANELS.  
C

```

LTCC=LTC(NBP)
LNCC=LNC (NBP)
DO 200 J=1,LTCC
  IT1=LT(I,I,J)+0.01+KTRV
  IT2=IT1+1
DO 200 I=1,LNCC
  KC=KC+1
  IB1=LN(I,I)+0.01
  IB2=IB1+1
  IT1L=IT1-KTRV
  IT2L=IT1L+1
  L2=(IT1L-1)*NVL(NBP)+IB1-1+NRVORT
  L5=L2+1
  L8=L5+1
  L11=L8+1
  ALM2=(ALNGTH(L2))
  ALM5=(ALNGTH(L5))
  ALM8=(ALNGTH(L8))
  ALM11=(ALNGTH(L11))
  IF (IB1.EQ.1) ALM2=ALM5
  IF (IB1.EQ.NVL(NBP)) ALM8=ALM5
  IF (IB2.EQ.NVL(NBP)) ALM11=ALM8
  NGAM1=NVL(NBP)
  NGAM2=NGAM1
  IF (ATTFW(IT1).EQ.99.0) NGAM1=20
125 CONTINUE
  IF (ATTFW(IT2).EQ.99.0) NGAM2=20
126 CONTINUE
DO 361 M=1,NGAM1
  XD(M)=XGW(IT1,M)
  GD(M)=GAMTW(IT1,M)
361 CALL CODIM(XD,GD,NGAM1,XQ(KC),GAM1,1)
DO 362 M=1,NGAM2
  XD(M)=XGW(IT2,M)
  GD(M)=GAMTW(IT2,M)
362 CALL CODIM(XD,GD,NGAM2,XQ(KC),GAM2,1)
  L2=(IT1L-1)*NVL(NBP)+IB1+NRVORT

```

L1=L2-NVL(NBP)	5	4570
L3=L2+NVL(NBP)	5	4580
BL2=BLNGTH(L2)	5	4590
IF(IT1L.NE.1) GO TO 135		
BL1=BL2	5	4600
BL3=BLNGTH(L3)	5	4610
GO TO 140	5	4620
	5	4630
	5	4640
	5	4650
135 IF(IT1L.NE.NVT(NBP)) GO TO 136		
BL1=BLNGTH(L1)	5	4660
BL3=BL2	5	4670
GO TO 140	5	4680
136 BL1=BLNGTH(L1)	5	4690
BL3=BLNGTH(L3)	5	4700
140 CONTINUE	5	4710
D=0.75*ALM5+0.25*ALM8		
IF(IB1-NVL(NBP)) 142,141,142	5	4720
141 GAMB2=0.0	5	4730
GO TO 143	5	4740
142 GAMB2=GAMB(L2+1)	5	4750
143 CONTINUE	5	4760
DELVM(KC)=GAMB(L2)/(0.75*ALM2+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D	5	4770
1 +GAMB2/(0.75*ALM5+ALM8+0.25*ALM11)*0.5*ALM5/D		
TERMPP=-DELVM(KC)*SLOPEC(L2)	5	4780
	5	4790
	5	4800
C ADD TERMPP ONLY FOR PANEL.	5	4810
DELVT(KC)=-0.5*(GAMB1/(BL1+BL2)+GAMB2/(BL2+BL3)) + TERMPP	5	4820
200 CONTINUE	5	4830
KTRV=KTRV+NVT(NBP)+1		
NBVORT=NVPTS(NBP)+NBVORT	5	4840
250 CONTINUE	5	4850
300 CONTINUE	5	4860
RETURN	5	4870
END	5	4880
	5	4890

```

SUBROUTINE GAMMA
C
C THIS PROGRAM COMPUTES GAMMA'S FOR BOUND AND TRAILING VORTICES.
C
C *****
C INSERT NATT
C COMMON/PANATT/ NATT(30),XATT(200)
C *****
C COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
C *****
1 ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON/SCRAT/ AK(5000),XG(11,151),GAMT(11,151),GAMB(1500)
1 ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
2 ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
3 ,CP(1320),XGW(50,20),GAMTW(50,20)
COMMON DA(5000)
COMMON/AF/ ATTFB(10),ATTFW(50)
EQUIVALENCE (DA(2),PANS)
DATA L/O/,KBODS/O/,NTBODS/O/
REWIND 18
IF(NBODS.EQ.0) GO TO 1111
C SKIP ALNGTH AND BLNGTH ARRAYS
READ(18)
READ(18)
1111 CONTINUE
NPANS=PANS
IF(NBODS.EQ.0) GO TO 100
DO 10 I=1,NBODS
IF(I-1) 1,1,2
1 N=1
GO TO 3
2 M=NCPTS(I-1)+1
3 CONTINUE
CALL BOLGAM(I,AK(N),NVL(I),NVT(I),L)
KBODS=KBODS+NCPTS(I)
CONTINUE
10 CONTINUE
100 IF(NPANS.EQ.0) GO TO 205
KPI=KBODS+1

```

DO 200 I=1,NPANS	5	5290
KP=KPI	5	5300
INB=I+NBODS	5	5310
CALL PANGAM(I,AK,NVL(INB),NVT(INB),KP,L,NIBODS)	5	5320
WRITE(6,7007) (AK(IX),IX=1,100)	5	5330
7007 FORMAT(*OAK AFTER PANGAM*/(IP1OE13.4))	5	5340
KPI=KPI+NCPTS(INB)	5	5350
200 CONTINUE	5	5360
205 CONTINUE	5	5370
REW9ND 18	5	5380
IF(NPANS.EQ.0) RETURN	5	5390
NTRVP=0	5	5400
N1PRE=0	5	5410
DO 300 I=1,NPANS	5	5420
NA=NATT((I-1)*3+2)	5	5430
NA=COMPONENT (BODY OR PANEL) TO WHICH PANEL I IS ATTACHED.	5	5440
IF(NA.EQ.0) GO TO 260	5	5450
	5	5460
DETERMINE WHICH TRAILING VORTICES NEED TO HAVE MODIFIED GAMT'S.	5	5470
THESE WILL BE THE VORTICES NUMBERED N1 AND N2.	5	5480
N1=0	5	5490
IF(NA.EQ.1) GO TO 190	5	5500
N11=NA-1	5	5510
DO 180 K=1,N11	5	5520
180 N1=NVT(K)+N1+1	5	5530
190 N1=N1+NATT(3*I)	5	5540
N2=NTRVP+1	5	5550
N1=TRAILING VORTEX LINE TO WHICH PANEL I IS ATTACHED.	5	5560
N2=INBOARD TRAILING VORTEX LINE OF ATTACHED PANEL I.	5	5570
DETERMINE XMIN AND XMAX OF TR. VORTICES N1,N2.	5	5580
IF(XG(N1,1)-XGW(N2,1)) 202,202,203	5	5590
202 XMIN=XG(N1,1)	5	5600
GO TO 204	5	5610
203 XMIN=XGW(N2,1)	5	5620
204 NVOR1=NVL(NA)	5	5630
NVOR2=NVL(NBODS+1)	5	5640
IF(N1.EQ.N1PRE) NVOR1=20	5	5650
IF(XG(N1,NVOR1)-XGW(N2,NVOR2)) 206,206,207	5	5660
206 XMAX=XGW(N2,NVOR2)	5	5670

207	GO TO 208	5	5680
208	XMAX=XG(N1,NVOR1)	5	5690
	CONTINUE	5	5700
	N20=19	5	5710
	XINT=XMAX-XMIN	5	5720
	DX=XINT/N20	5	5730
	N21=N20+1	5	5740
	DO 210 M=1,N21	5	5750
210	XAA(M)=XMIN+(M-1)*DX	5	5760
	DO 211 M=1,NVOR1	5	5770
	XD(M)=XG(N1,M)	5	5780
211	GD(M)=GAMT(N1,M)	5	5790
	CALL CODIM(XD,GD,NVOR1,XAA,GAMMA1,N21)	5	5800
	DO 230 M=1,N21	5	5810
	IF(XAA(M)-XG(N1,1)) 222,224,224	5	5820
222	GAMMA1(M)=0.0	5	5830
	GO TO 230	5	5840
224	IF(XAA(M)-XG(N1,NVOR1)) 228,228,226	5	5850
226	GAMMA1(M)=GLAST	5	5860
	GO TO 230	5	5870
228	GLAST=GAMMA1(M)	5	5880
230	CONTINUE	5	5890
	DO 213 M=1,NVOR2	5	5900
	XD(M)=XGW(N2,M)	5	5910
213	GD(M)=GAMTW(N2,M)	5	5920
	CALL CODIM(XD,GD,NVOR2,XAA,GAMMA2,N21)	5	5930
	DO 240 M=1,N21	5	5940
	IF(XAA(M)-XGW(N2,1)) 232,234,234	5	5950
232	GAMMA2(M)=0.0	5	5960
	GO TO 240	5	5970
234	IF(XAA(M)-XGW(N2,NVOR2)) 238,238,236	5	5980
236	GAMMA2(M)=GLAST	5	5990
	GO TO 240	5	6000
238	GLAST=GAMMA2(M)	5	6010
240	CONTINUE	5	6020
	DO 250 M=1,N21	5	6030
	XG(N1,M)=XAA(M)	5	6040
	XGW(N2,M)=XAA(M)	5	6050
	WRITE(6,261) N1,N2,N1PRE	5	6060

261	FORMAT(*0N1,N2,N1PRE* 315)	5	6070
	WRITE(6,262) M,GAMMA1(M),GAMMA2(M)	5	6080
262	FORMAT(*0M,GAMMA1,GAMMA2 = *I3,1P2E20.5)	5	6090
	IF(N1.NE.N1PRE) GO TO 249		
	GAMT(N1,M)= GAMMA2(M)+GAMT(N1,M)	5	6100
	GO TO 250	5	6110
		5	6120
249	GAMT(N1,M)=GAMMA1(M)+GAMMA2(M)	5	6130
250	GAMTW(N2,M)=GAMT(N1,M)	5	6140
		5	6150
	ATTFB(N1)=99.0	5	6160
	ATTFW(N2)=99.0	5	6170
	WRITE(6,5998)(XG(N1,M),M=1,20)	5	6180
5998	FORMAT(*0X-COORDINATES FOR ATTACHMENT LINE*/(1P10E13.4))	5	6190
	WRITE(6,5999) N1,N2	5	6200
5999	FORMAT(*0BODY ATTACHMENT LINE IS*,I3/ *OPANEL ATTACHMENT LINE IS*,I3)	5	6210
	WRITE(6,6000)(GAMT(N1,M),M=1,20)	5	6220
6000	FORMAT(*0BODY GANT AT ATTACHMENT LINE*/(1P10E13.4))	5	6230
	WRITE(6,6001)(GAMTW(N2,M),M=1,20)	5	6240
6001	FORMAT(*OPANEL GANT AT ATTACHMENT LINE*/(1P10E13.4))	5	6250
260	CONTINUE	5	6260
	NTRVP=NTRVP+NVT(NBODS+1)+1	5	6270
	N1PRE=N1	5	6280
300	CONTINUE	5	6290
	RETURN	5	6300
	END	5	6310
		5	6320



SUBROUTINE PANGAM(IP,AK,NL,NL,T,KP,L,NTRP)	5	6330
COMMON/NUMBER/ NPTS(7),NPTS(7),NPTS(7),NLN(7),NLN(7),LNC(7)	5	6340
1 NCT,NB,NBODS,NPA(5),NVL(7),NVL(7),NVI(7),NTAPE,NTAPE,NCIV,NTAPE,JTAPE	5	6350
2 LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)	5	6360
3 LNDIVB(7),LNDIVB(7),NSPP(7),ROOTP(7),OUTER(7),SYMM(7)	5	6370
COMMON/BODY/XV(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)	5	6380
1 YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)	5	6390
2 XCV(20),XCC(20),XLE(20),YLE(20),ZLE(20)	5	6400
3 XTE(20),YTE(20),ZTE(20),SLE(20),XJ(20),YJ(20),ZJ(20)	5	6410
4 ETE(20),XVT(50),YVT(50),ZVT(50),XR(20),YR(20),ZR(20)	5	6420
5 SXMT(1000),SYMT(1000),SZMT(1000),DYS(1000),DZS(1000)	5	6430
6 TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)	5	6440
EQUIVALENCE (B,BP,XVR)	5	6450
DIMENSION BP(13000)	5	6460
DIMENSION B(30000),ALNGTH(5000),BLNGTH(5000),AK(1)	5	6470
EQUIVALENCE(B(20000),ALNGTH),B(25000),BLNGTH)	5	6480
COMMON/SCRAT/DUM(5000),XG(11,151),GAMT(11,151),GAMB(1500)	5	6490
1 DELVH(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)	5	6500
2 XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)	5	6510
3 CP(1320),XGW(50,20),GANTW(50,20)	5	6520
COMMON/PANINF/PANSYM(10)	5	6530
DATA LENGTH/0/	5	6540
LI=0	5	6550
NSPACE = NSPP(IP)	5	6560
SYM=PANSYM(IP)	5	6570
LS=1	5	6580
IF(NBODS) 14,13,14	5	6590
NBVORT=0	5	6600
GO TO 15	5	6610
NBVGRT=NVL(1)*NVT(1)	5	6620
CONTINUE	5	6630
READ(18),BP	5	6640
WRITE(6,6000) IP,NL,NT	5	6650
WRITE(6,6000) KP,L,NTBP	5	6660
FORMAT(2I5,1P2E16.4)	5	6670
DC 305 J=1,NT	5	6680
IF(J.LE.NSPACE) GO TO 305	5	6690
SUM=0.0	5	6700
DC 300 I=1,NL	5	6710

	11=(J-1)*NL+I+KP-1	5	6720
	SUM= SUM+AK(I1)	5	6730
	DUMM(I1)=SUM	5	6740
300	WRITE(6,6000) J,I,I1,SUM,AK(I1)	5	6750
305	AK(I1)=SUM	5	6760
	CONTINUE	5	6770
7007	WRITE(6,7007) (DUMM(I),I=1,100)	5	6780
	FORMAT(*0DUMM*/(IP10E13.4))	5	6790
	NS1=NSPACE+1	5	6800
	N1=NT+1	5	6810
	DO 100 J=1,N1	5	6820
	JMNS=J-NSPACE	5	6830
	JT=J+NTBP	5	6840
	DO 100 I=1,NL	5	6850
	IF(J.EQ.N1) GO TO 45	5	6860
	LENGTH=LENGTH+1	5	6870
	KK=NBVORT+LENGTH	5	6880
	L1=L1+1	5	6890
	ALNGTH(KK)=PLL(L1)	5	6900
	IF(JMNS) 41,41,42	5	6910
41	BLNGTH(KK)=PLT(L1)	5	6920
	GO TO 43	5	6930
42	BLNGTH(KK)=2.0*YSUBV(JMNS)	5	6940
43	CONTINUE	5	6950
	SLOPEC(LENGTH)=TS(LS)	5	6960
45	CONTINUE	5	6970
	IF(JMNS) 11,11,21	5	6980
11	XGW(JT,I)=0.5*(XVR(J,I)+XVR(J,I+1))	5	6990
	GO TO 30	5	7000
21	CONTINUE	5	7010
	IF(J-N1) 22,24,24	5	7020
22	FACT=-0.5	5	7030
	JTR=JMNS	5	7040
	GO TO 251	5	7050
24	LS=NL*(NT-1)*2+1	5	7060
	FACT=+0.5	5	7070
	JTR=JMNS-1	5	7080
251	IF(I-1) 26,26,28	5	7090
26	X1=XVC(JTR,I)+FACT*TS(LS)*SQRT(DYS(LS)**2+DZS(LS)**2)	5	7100

28	GO TO 29	5	7110
29	X1=X2	5	7120
	CONTINUE	5	7130
	IF (I.EQ.NL) GO TO 291	5	7140
	LS2=LS+2	5	7150
	X2=XV0(JTR , I+1)+FACT*TS(LS2)*SQRT(DYS(LS2)**2+DZS(LS2)**2)	5	7160
	GO TO 292	5	7170
291	X2=X1+DELX	5	7180
292	XGV(JT,I)=0.5*(X2+X1)	5	7190
	DELX=X2-X1	5	7200
30	CONTINUE	5	7210
	K1=(J-1)*NL+I+KP-1	5	7220
	LS=LS+2	5	7230
	IF (J-1) 1,1,5	5	7240
1	IF (SYM) 2,9,2	5	7250
2	GAMTW(JT,I)=-AK(K1)	5	7260
	GO TO 10	5	7270
5	IF (J-N1) 5,7,6	5	7280
6	GAMTW(JT,I)= AK(K1-NL)-AK(K1)	5	7290
	GO TO 10	5	7300
7	GAMTW(JT,I)=AK(K1-NL)	5	7310
	GO TO 10	5	7320
9	IF (NBODS.NE.0) GO TO 2	5	7330
	GAMTW(JT,I)=0.0	5	7340
10	CONTINUE	5	7350
	IF (J.EQ.N1) GO TO 40	5	7360
	L=L+1	5	7370
	IF (I-1) 25,20,25	5	7380
20	GAMB(L)=AK(K1)	5	7390
	GO TO 40	5	7400
25	GAMB(L)=AK(K1)-AK(K1-1)	5	7410
40	CONTINUE	5	7420
100	CONTINUE	5	7430
	NTEP=NTBP+NVT(IP+NBODS)+1	5	7440
	RETURN	5	7450
	END	5	7460

SUBROUTINE BODGAM(IR,AK,NL,NT,L)		
COMMON DA(5000)		5 7470
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LNC(7)		5 7480
1 ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NTAPE,JTAPE		5 7490
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)		5 7500
3 ,LNDIVB(7),LTDIVB(7),NSPP(7),ROCTP(7),OUTERP(7),SYMM(7)		5 7510
COMMON/BODY/B(30000)		5 7520
DIMENSION XV(151,31)		5 7530
EQUIVALENCE(XV,B)		5 7540
COMMON/SCRAT/XSOL(5000),XG(11,151),GAMT(11,151),GAMB(1500)		5 7550
1 ,DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)		5 7560
2 ,XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)		5 7570
3 ,CP(1320)		5 7580
DIMENSION AK(1)		5 7590
EQUIVALENCE (SYM,DA(19))		5 7600
READ(18) B		5 7610
N1=NT+1		5 7620
LTDIV=LTDIVB(IB)		5 7630
DO 100 J=1,N1		5 7640
J1=(J-1)*LTDIV+1		5 7650
DO 100 I=1,NL		5 7660
K1=(J-1)*NL+I		5 7670
XG(J,I)=0.5*(XV(I,J1)+XV(I+1,J1))		5 7680
IF(J-1) 1,1,5		5 7690
IF(SYM) 2,4,2		5 7700
GAMT(J,I)=-AK(K1)		5 7710
GO TO 10		5 7720
GAMT(J,I)=0.0		5 7730
GO TO 10		5 7740
IF(J-N1) 6,7,6		5 7750
GAMT(J,I)=AK(K1-NL)-AK(K1)		5 7760
GO TO 10		5 7770
IF(SYM) 8,9,8		5 7780
GAMT(J,I)=AK(K1-NL)		5 7790
GO TO 10		5 7800
GAMT(J,I)=0.0		5 7810
CONTINUE		5 7820
IF(J.EQ.N1) GO TO 40		5 7830
L=L+1		5 7840
		5 7850

20	IF(I-1) 25,20,25	5	7860
	GAMB(L)=AK(K1)	5	7870
	GO TO 40	5	7880
25	GAMB(L)=AK(K1)-AK(K1-1)	5	7890
40	CONTINUE	5	7900
100	CONTINUE	5	7910
	IF(SYM.EQ.0) GO TO 15	5	7920
	DO 14 I=1,NL	5	7930
	GAM1(I,I)=GAMT(I,I)+GAMT(N1,I)	5	7940
14	GAMT(N1,I)=GAMT(I,I)	5	7950
15	CONTINUE	5	7960
	RETURN	5	7970
	END	5	7980

COMMON/BODY/	XAREA(5000),YAREA(5000),ZAREA(5000)	6	0040
1	, XCP(5000), YCP(5000), ZCP(5000), XTL(100)	6	0050
2	, FLNC(150), FLTC(40), XCON(100)	6	0060
COMMON	/COMPRS/ BETAM	6	0070
DIMENSION	YPD(200),ZPD(200),YV(100),ZV(100),PING(10)	6	0080
SEE EQUIVALENCE	IN PANMAT FOR YPD,ZPD	6	0090
EQUIVALENCE	(YPD(1),XTL(1)), (ZPD(1),FLNC(101))	6	0100
EQUIVALENCE	(YV(1),XAREA), (ZV(1),YAREA)	6	0110
COMMON /PANINF/	PSYM(10), DUM11(600), PANREF(10), PCHORD(10)	6	0120
DIMENSION	TBD(10),TPD(10)	6	0130
EQUIVALENCE	(DA(19),BSYM)	6	0140
DIMENSION	AKTB(500),AKTP(500)	6	0150
EQUIVALENCE	(AKTB(1),YCP(4001)), (AKTP(1),YCP(4501))	6	0160
DIMENSION	CP(1320)	6	0170
COMMON/SCRAT/	D(25000)	6	0180
DIMENSION	VM(1500),VT(1500)	6	0190
EQUIVALENCE	(VM(1),D(14323)), (VT(1),D(12823))	6	0200
DIMENSION	CPBODY(1500)	6	0210
EQUIVALENCE	(D(18076),CP), (D(20000),CPBODY)	6	0220
DIMENSION	E(50),XCCO(20),DYI(400),DZI(400)	6	0230
1	,TWIST(20),CPU(400),CPL(400),DSLE(20),XLE(20),YLE(20)	6	0240
2	,ZLE(20),SPCF(40),CHC(20)	6	0250
EQUIVALENCE	(E,D(5001)), (XCCO,D(5101)), (DYI,D(5201)), (DZI,D(5701))	6	0260
1	, (TWIST,D(6201)), (CPU,D(6301)), (CPL,D(6801))	6	0270
2	, (DSLE,D(7301)), (XLE,D(7401)), (YLE,D(7501))	6	0280
3	, (ZLE,D(7601)), (SPCF,D(7701)), (CHC,D(7801))	6	0290
4	, (XCG,DA(71)), (YCG,DA(81)), (ZCG,DA(91)), (PANS,DA(21))	6	0300
5	, (VMU,D(8001)), (VML,D(8201)), (VTU,D(8401)), (VTL,D(8601))	6	0310
DIMENSION	VMU(200),VML(200),VTL(200),VTU(200)	6	0320
COMMON	DA(5000)	6	0330
COMMON/NUMBER/	NVPTS(7),VCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)	6	0340
1	,NCT,NB,NBODS,NPANS,NIVL(7),NVT(7),MTAPE,NTAPE,NICTV,ITAPE,JTAPE	6	0350
2	,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)	6	0360
3	,LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)	6	0370
COMMON/SLOPE/	SIG'AP(500),DZDXT(500),DZDXC(500),TANPI(500)	6	0380
DIMENSION	ZERO(50)	6	0390

	DIMENSION SR(5,8), PS(10,6)	6	0400
	EQUIVALENCE (BR, DA(16)), (PR, PA(3421)), (PSY, DA(3426)),	6	0410
	1 (PCH, DA(3422)), (BCH, DA(17))	6	0420
	COMMON/CONPTS/ XQ(1320), YQ(1320), ZQ(1320)	6	0430
	1 , XN(1320), YN(1320), ZN(1320)	6	0440
	DIMENSION BP(13000)	6	0450
	DIMENSION XCC(20), XCV(20), XSS(1000), YSS(1000), ZSS(1000), ET(20)	6	0460
	1 , TS(1000), SIGMA(1000)	6	0470
	EQUIVALENCE(RP, XAREA), (XCC, RP(2421)), (XCV, BP(2401))	6	0480
	1 , (XSS, BP(8871)), (YSS, RP(9871)), (ZSS, RP(10871)), (TS, RP(7871))	6	0490
	2 , (SIGMA, BP(11871))	6	0500
	DIMENSION CPNET(1000), CDS(100), CDA(100), CDT(100), CDTH(100)	6	0510
	EQUIVALENCE (SPAN, DA(3423)), (PLVI, DA(3431))	6	0520
	EQUIVALENCE(D(23001), CPNET), (D(24101), CDS), (D(24201), CDA)	6	0530
	1 , (D(24301), CDT), (D(24401), CDTH)	6	0540
	COMMON/PANEL/ NPAN, IPSY, IJC, NBVVP, NTVVP, LNCFP, LTCFP, LNCPP, LTCPP	6	0550
	DATA PIND/10*1.0/	6	0560
	NC=0	6	0570
	IF(NBODS.EQ.0) GO TO 50	6	0580
	N2=LTDIVB(1)*NVT(1)+1	6	0590
	READ(12)(YV(I), ZV(I), I=1, N2)	6	0600
50	CONTINUE	6	0610
	NPANS=PANS	6	0620
	IF(PANS.EQ.0.0) GO TO 55	6	0630
	WRITE(6,7002) (I, YPD(I), ZPD(I), I=1, 20)	6	0640
7002	FORMAT(*OPANEL DRAG COORDINATES*/(I5,2F15.5))	6	0650
	WRITE(6,7001) (I, AKTP(I), I=1, 20)	6	0660
7001	FORMAT(*O AKTP*/(I5,1PE20.6))	6	0670
	WRITE(6,6001) NVT(NBODS+1), NVT(NBODS+2)	6	0680
	WRITE(6,6002) PSYM	6	0690
	WRITE(6,6002) PIND	6	0700
55	CONTINUE	6	0710
	NSP=0	6	0720
	DO 101 IS=1, LTCPP	6	0730
	CDS(IS)=0.	6	0740
	CDA(IS)=0.	6	0750
	CDT(IS)=0.	6	0760
101	CDTH(IS)=0.	6	0770
	CDST=0.	6	0780

CDAT=0.	6	0790
CDTT=0.	6	0800
CDTHT=0.	6	0810
IF(DA(3424).NE.1.) GO TO 58	6	0820
IF(NBODS.NE.0) GO TO 58	6	0830
IF(NPANS.NE.1) GO TO 58	6	0840
BETAM=1.0	6	0850
NCPT=NCPTS(1)	6	0860
DO 56 I=1,NCPT	6	0870
IF(ZN(I).NE.1.0) GO TO 58	6	0880
CONTINUE	6	0890
IF(PLVI.NE.0.0) GO TO 58	6	0900
REWIND 18	6	0910
READ(18) BP	6	0920
REWIND 18	6	0930
HSPAN=NTVVP*(YSS(NBVVP*2+1)-YSS(1))	6	0940
DO 41 I=1,LTCPP	6	0950
41 ET(I)=YQ(I-1)*LNCPP+1)/HSPAN	6	0960
DO 59 I=1,NCPT	6	0970
IU=(I-1)*2+1	6	0980
IL=IU+1	6	0990
CPNET(I)=CP(IL)-CP(IU)	6	1000
NVPT=NBVVP*NTVVP*2	6	1010
DO 61 I=1,NVPT	6	1020
YSS(I)=YSS(I)/BETAM	6	1030
ZSS(I)=ZSS(I)/BETAM	6	1040
TS(I)=BETAM*TS(I)	6	1050
SIGMA(I)=SIGMA(I)/BETAM	6	1060
CALL 'FDRAG(NBVVP,NTVVP,LNCPP,LTCPP,XCC,XCV,ET, XSS,YSS,ZSS	6	1070
1 ,TS,SIGMA,CPNET,NSP,CDS,CDA,CNT,CDTH,CNST,CNAT,CNTT,CNTHT)	6	1080
CONTINUE	5	1090
DO 551 I=1,100	6	1100
YV(I)=YV(I)/BETAM	6	1110
ZV(I)=ZV(I)/BETAM	6	1120
DO 552 I=1,200	6	1130
YPD(I)=YPD(I)/BETAM	6	1140
ZPD(I)=ZPD(I)/BETAM	6	1150
CALL 'FDRAG(NBODS,NPANS,YV,ZV,AKTR,N2-1,RSYM,YPD,ZPD,AKTP	6	1160
1 ,NVT(NBODS+1),PGY,PIND,IPD,IPR,ICP)	6	1170



```

IF(NBOPS.EQ.0.) GO TO 100
READ(12) XAREA,YAREA,ZAREA
READ(12) XCP,YCP,ZCP,XTL,XCON,FLNC,FLIC,NBV,NTV,L'PIS,L'PIS
1 LN'DIV,LTDIV,NBV,NTV,CHORD,XCG,YCG,ZCG,ALPHA,PREF
6000 FORMAT( 1P9E12.4)
6001 FORMAT(10I5)
NC=NCPTS(1)
DO 60 I=1,NC
60 CPBODY(I)=CP(I)
6002 FORMAT(*0/(1P10E13.4))
CALL BPRINT(LNPTS,LTPTS,XU,YG,ZO,VM,VT,CP,1)
CALL REFLECT(DA(19), NBV, NTV, XAREA, YAREA, ZAREA, XCP, YCP, ZCP,
1 LNPTS,LTPTS,FLIC,CPBODY,LTDIV)
NPAA = NBV*NTV
DO 30 I=1,NPAA
XAREA(I) = XAREA(I)/BETAM**2
YAREA(I) = YAREA(I)/BETAM
ZAREA(I) = ZAREA(I)/BETAM
YCP(I) = YCP(I)/BETAM
30 ZCP(I) = ZCP(I)/BETAM
CALL BINTES(XAREA,YAREA,ZAREA,XCP,YCP,ZCP,FLNC,LNPTS,FLIC,LTPTS
1 ,CPBODY,LNDIV,LTDIV,XTL,NBV,D(1001),XCON,D(1)
2 ,D(101),D(201),CHORD,D(301),D(401),D(501),D(601),D(701),D(801)
3 ,NBV, NTV, D(951), XCG, YCG, ZCG, ALPHA, BREF,
4 SB(1,1), SB(1,2), SB(1,3), SB(1,4), SB(1,5), SB(1,6),TET(1),1
5)
SB(1,7) = BR
SB(1,8) = BCH
C THE FIRST SUBSCRIPTS OF SB(1,*) SHOULD BE THE BODY(PANEL) NUMBER
100 IF(PANS.EQ.0.) GO TO 200
PANS=PANS
NCPI=NC
NPZ=1
DO 554 I=1,500
DZDXG(I)=DZDXG(I)/DLTA
DZDXT(I)=DZDXT(I)/DLTA
N11=NC+1
554 DO 555 I=11,1320

```

555	YQ(I)=YQ(I)/BETAM	6	1570
	ZQ(I)=ZQ(I)/BETAM	6	1580
	DO 150 K=1,NPANS	6	1590
	READ(12) B,REFA,LTCPP,E, LNCPP,XCCO,DYI,DZI,TWIST,CHC		
1	DO 105 I=1,LTCPP	6	1600
	CHRM,(XLE(I),YLE(I),ZLE(I),I=1,LTCPP),FLTC,SPCF,BS,DSLE	6	1610
	K1=FLTC(I)	6	1620
	DSLE(I)=DSLE(K1)/BETAM	6	1630
	YLE(I)=YLE(I)/BETAM	6	1640
105	ZLE(I)=ZLE(I)/BETAM	6	1650
	NPTS=NCPTS(NBODS+K)	6	1660
	DO 110 I=1,NPTS	6	1670
	IU=(I-1)*2+1+NCPI	6	1680
	IL=IU+1	6	1690
	VMU(I)=VM(IU)	6	1700
	VTU(I)=VT(IU)	6	1710
	VML(I)=VM(IL)	6	1720
	VTL(I)=VT(IL)	6	1730
	CPL(I)=CP(IL)	6	1740
110	CPU(I)=CP(IU)	6	1750
	N1=NC+1	6	1760
	DO 111 I=1,50	6	1770
111	ZERO(I)=0.0	6	1780
	CALL PPRINT(E,XCCO,LTCPP,LNCPP,VMU,VML,VTU,VTL,CPU,CPL,NBODS+K)	6	1790
	B=B/BETAM	6	1800
	BS=B/BETAM	6	1810
	DO 553 I=1,400	6	1820
	DYI(I)=DYI(I)/BETAM	6	1830
553	DZI(I)=DZI(I)/BETAM	6	1840
	CALL PANINT(B,REFA,LTCPP,E, LNCPP,XCCO,DYI,DZI,TWIST,BS,SPCF	6	1850
1	,CPU,CPL,DSLE,XCG,YCG,ZCG,CHC,CHRM,DZDXC(NPDZ),DZDXT(NPDZ)	6	1860
2	,XQ(N1),YQ(N1),ZQ(N1),XLE,YLE,ZLE,D(8000)	6	1870
3	,TPD(K),CDA,CDS,CDTH,CDI,PS(K,1),PS(K,2),PS(K,3),	6	1880
4	PS(K,4), PS(K,5), PS(K,6), K+NBODS, CDAT, COST, COTHT, CDTT)	6	1890
	NC=NC+NPTS	6	1900
	NCPI=NCPI+2*NPTS	6	1910
	NPDZ=NPDZ+NPTS	6	1920
190	CONTINUE	6	1930
200	CONTINUE	6	1940
		6	1950

	CALL CCINTG(	SB(1,1),	SB(1,2),	SB(1,3),	SP(1,4),	SR(1,5),	SR(1,6),	6	1960
1	SB(1,7),	PS(1,1),	PS(1,2),	PS(1,3),	PS(1,4),	PS(1,5),	PS(1,6),	6	1970
2	PANREF,	PSYN,	PCHORD,	SB(1,8)				6	1980
3)								6	1990
								6	2000
								6	2010
								6	2020

STOP 6543  
END

USUBROUTINE CCINTG (CXG, CYG, CZG, CMXB, CMYB, CMZB, AR3, CXP, CYP6  
1, CZP, CMXP, CMYP, CMZP, AP, FS, CP, CB

2030  
2040  
2050  
2060  
2070  
2080  
2090  
2100  
2110  
2120  
2130  
2140  
2150  
2160  
2170  
2180  
2190  
2200  
2210  
2220  
2230  
2240  
2250  
2260  
2270  
2280  
2290  
2300  
2310  
2320  
2330  
2340  
2350  
2360  
2370  
2380  
2390  
2400  
2410

THIS ROUTINE INTEGRATES THE TOTAL CONFIGURATION

CXB CYB CZB -ARRAYS OF C S OF BODIES  
CMXB CMYB CMZB -ARRAYS OF CM S OF BODIES  
ARB -ARRAY OF REFERENCE AREAS OF BODIES  
CXP CYP CZP -ARRAYS OF C S OF PANELS  
CMXP CMYP CMZP -ARRAYS OF CM S OF PANELS  
AP -ARRAY OF PANEL AREAS  
CB -ARRAY OF BODY CHORD LENGTHS  
CP -ARRAY OF PANEL CHORD LENGTHS  
NB -NUMBER OF BODIES  
NP -NUMBER OF PANELS

-TOTAL CONFIGURATION CHORD LENGTH

XCRCG YCRCG ZCRCG-X Y AND Z ( /CR)CG

AR - TOTAL CONFIGURATION REFERENCE AREA

FS - SYMETRY INDICATOR 0.0 IS SYMETRICAL

ODINENSION

1 CMYB(1), CXB(1), CYB(1), CZB(1), CMXB(1),  
CMZB(1), ARB(1),  
2 CXP(1), CYP(1), CZP(1), CMXP(1), CMYP(1),  
CMZP(1), AP(1), FS(1), CP(1), CB(1)  
3 EQUIVALENCE (ARB, CBJC), (CBJC, APJ, AR), (CBJC, CPJC)  
COMMON DA(5100)  
EQUIVALENCE (DA(5), C), (DA(1), BN), (DA(2), PN)  
1, (AR, DA(4)), (XCRCG, DA(7)), (YCRCG, DA(8)), (ZCRCG, DA(9))

NB = BN

NP = PN

PRINT 6

6 FORMAT(1H1,35X,\*TOTAL CONFIGURATION LOADS\*)

CX =0.0

CY =0.0

CZ =0.0

CMX =0.0

CMY =0.0

CMZ =0.0

```

C      SUM OVER BODIES
      IF(NB.LE.0) GOTO 300
      DO 100 J=1, NB
      ARBAR = ARB(J)/AR
      CX = CX + ARBAR * CXB(J)
      CY = CY + ARBAR * CYB(J)
      CZ = CZ + ARBAR * CZB(J)
      CBJC = CB(J)/C * ARBAR
      CMX = CMX + CBJC*CMXB(J)
      CMY = CMY + CBJC*CMYB(J)
      CMZ = CMZ + CBJC*CMZB(J)
      100 CONTINUE
C
C      SUM OVER PANELS
      300 IF(NP.LE.0) GOTO 400
      DO 200 J=1, NP
      FS0 = 1.
      FS2 = 1.
      IF(6S(J)) 102, 101, 102
      101 FS0 = 0.0
      FS2 = 2.0
      102 APJ1R = AP(J)7AR
      CX = CX + APJAR * FS2 * CXP(J)
      CY = CY + APJAR * FS0 * CYP(J)
      CZ = CZ + APJAR * FS2 * CZIP(J)
      CPJC = APJAR * CP(J)/C
      CMX = CMX + CPJC*FS0*CMXP(J)
      CMY = CMY + CPJC*FS2*CMYP(J)
      CMZ = CMZ + CPJC*FS0*CMZP(J)
      200 CONTINUE
      400 CONTINUE
      PRINT 206, CX, CY, CZ, CMX, CMY, CMZ
      206 FORMAT(1H0,8X,*CX*,13X,*CY*,13X,*CZ*,13X,*CMX*,12X,*CMY*,12X,*CMZ*,
1 /1X,6F15.7)
      CPJC = CX*CX + CY*CY + CZ*CZ
      XCR = XCRG + (CY*CMZ - CZ*CMY)/CPJC
      YCR = YCRG + (CZ*CMX - CX*CMZ)/CPJC
      ZCR = ZCRG + (CX*CMY - CY*CMX)/CPJC
      PRINT 216, XCR, YCR, ZCR

```

216	FORMAT(IH0,3X,*(X/CR) CP*,6X,*(Y/CR) CP*,6X,*(Z/CR) CP*/1X,3F15.76	2810
1)		2820
	RETURN	6
	END	6
		2830
		6
		2840

SUBROUTINE PANINT(B, REFA, NETA, ETA, NXOC, XOC, DY, DZ, EPSILO, BS, DA  
 1 CPU, CPL, DSLE, XCG, YCG, ZCG, C, CHORD, DZC, DZT, X, Y, Z  
 2 XLE, YLE, ZLE, DS, CDI, CDT, CDLIC, CTAVRG, CDTICA, CX, CY, CZ, CMX, CMY, CMZ, NC, CDT, CDL, CT, CTDI)  
 3 CODED BY B. D. GAITHER  
 THE GREAT NUMBER OF ARGUMENTS WERE USED TO ALLOW GREATER EASE IN  
 ADDING THIS ROUTINE TO THE OTHERS. AT A LATER DATE THESE SHOULD BE  
 REPLACED BY COMMON REFERENCES.

THIS ROUTINE INTEGRATES PANELS WITH CONTROL SURFACES

B	- SPAN	6	2940
REF1	- REFERENCE AREA	6	2950
NET1	- NUMBER OF ETAS WHERE CPS ARE GIVEN.	6	2960
ETA	- ETAS FOR GIVEN CPS.	6	2970
NXO3	- NUMBER OF X/C STATION FOR GIVEN CPS.	6	2980
XOC	- X/C ARRAY	6	2990
DY	- DELTA Y FOR VORTEX WHERE CP IS GIVEN.	6	3000
DZ	- DELTA Z FOR VORTEX WHERE CP IS GIVEN.	6	3010
EPSILO	- TWIST ARRAY	6	3020
BS	- SURFACE SPAN LENGTH (FOR 2-D SURFACE, BS=B).	6	3030
SPCF	- SPECIAL LONGITUDINAL CONTROL FUNCTIONS	6	3040
CPU	- CP ARRAY FOR UPPER SURFACE.	6	3050
CPL	- CP ARRAY FOR LOWER SURFACE.	6	3060
DSL5	- DELTA S ON LEADING EDGE FOR SPANWISE VORTEX SECTIONS WHERE CPS ARE GIVEN.	6	3070
XCG	- X CENTER OF GRAVITY	6	3080
YCG	- Y CENTER OF GRAVITY	6	3090
ZCG	- Z CENTER OF GRAVITY	6	3100
C	- CHORD ARRAY FOR SPANWISE STATIONS WHERE CPS ARE GIVEN.	6	3110
CHORD	- MEAN AERODYNAMIC CHORD (GIVEN IN INPUT).	6	3120
DZC	- DZ/DX FOR CAMBER.	6	3130
DZT	- DZ/DX FOR THICKNESS.	6	3140
X Y Z	- CO ORDINATES AT CPS	6	3150
XLE	- X ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS.	6	3160
YLE	- Y ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS.	6	3170
ZLE	- Z ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS.	6	3180
DS	- A SCRATCH ARRAY	6	3190
CDTO	(CDT=0 C)/CAVG	6	3200
		6	3210
		6	3220
		6	3230

CDL9C	(CDLIC)/CAVG	6	3240
CTAVRG	(CTC)/CAVG	6	3250
CDTICA	(CDTIC)/CAVG	6	3260
CXCAVG	CXC/CAVG	6	3270
CYCAVG	CYC/CAVG	6	3280
CZCZVG	CZC/CZVG	6	3290
CMX3AV	CMXC/CAVG	6	3300
CMYCAV	CMYC/CAVG	6	3310
CMZCAV	CMZC/CAVG	6	3320
CNCAVG	CNC/CAVG	6	3330
CMLEXC	CMLEXC/CAVG	6	3340
CMLEYC	CMLEYC/CAVG	6	3350
CMLEZC	CMLEZC/CAVG	6	3360
XCPLC	XCP/C LE	6	3370
YCPLC	YCP/C LE	6	3380
ZCPLC	ZCP/C LE	6	3390
T1 2 3 .. 8	SCRATCH ARRAYS NETA LONG WHERE DEPENDENT VARIABLE	6	3400
T1 1 2 .. 8	S TO INTEGRAT ARE STORE	6	3410
CMX CMY CMZ	SCRATCH VARIABLES WHERE VALUES OF INTEGRALS ARE	6	3420
CX CY CZ	STORED	6	3430
XCPC	CMX CMY CMZ	6	3440
YCP3	CX CY CZ	6	3450
ZCPC	XCP/C	6	3460
DS	YCP/C	6	3470
DSLE	ZCP/C	6	3480
DIMENSION DSLE(30), DS(NXOC,NETA),	DELTA S	6	3490
1DZ(NXOC,NETA), CPU(NXOC,NETA), CPL(NXOC,NETA), EPSILO(	DELTA S L.E.	6	3500
2ETA(NETA), XOC(NXOC), X(NXOC,NETA), Y(NXOC,NETA), Z(NXOC,NETA),	DY(NXOC,NETA),	6	3510
3T1(30), T2(30), T3(30), T4(30), T5(30), T6(30), T7(30), T8(30),	NETA),	6	3520
4 CXCAVG(30), CYCAVG(30), CZCAVG(30), CMXCAV(30), CMYCAV(30), CMZCA6	NETA),	6	3530
5V(30), CNCAVG(30), CMLEXC(30), CMLEYC(30), CMLEZC(30), XCPLE(30),	NETA),	6	3540
6YCPLC(30), ZCPLC(30), CDTG(NETA), CDLIC(NETA), CTAVRG(NETA), CDTICA(6	NETA),	6	3550
7 NETA), DA(8,1), XLE(1), YLE(1), ZLE(1)	NETA),	6	3560
DIMENSION DZC(NXOC,NETA), SZT(NXOC,NETA)	NETA),	6	3570
DIMENSION C(1), XSLCS(20), YSLCS(20), ZSLCS(20), NSLCS(20)	NETA),	6	3580
1 , XSMHL(20), YSMHL(20), ZSMHL(20), XSCPCS(20), YSCPCS(20)	NETA),	6	3590
2 , ZSCPCS(20), CMHC(20)	NETA),	6	3600
	NETA),	6	3610
	NETA),	6	3620



```

EQU9VALENCE (CXCAVG,XSLCS), (YSLCS,CYCAVG), 6 3630
(CZCAVG,ZSLCS), (CNCAVG,NSLCS), 6 3640
1 (CHXCAV,XSMAHL), (CMYCAV,YSMAHL), 6 3650
2 (CMZCAV,ZSMAHL), (XCPL,XSCPCS), 6 3660
3 (YCPL,XSCPCS), (ZCPL,XSCPCS), 6 3670
4 (YCPL,XSCPCS), (ZCPL,XSCPCS), (CMLEXC,CMHC) 6 3680
1 RLINT(XI,XO,ETAI,ETAO,ATA) 6 3690
= XI + (XO - XI) * (ATA - ETAI) / (ETAO - ETAI) 6 3700
PRINT 26, NC 6 3710
26 FORMAT(1H1, 36X, *PANEL SECTIONAL LOADS FOR COMPONENT*, I5/ 6 3720
11H0,5X, 3HETA, 7X, 6 3730
2*CXG/CAVG*,5X,*CYC/CAVG*, 5X, *CNC/CAVG* 6 3740
3,4X,*CMLEXC/CAVG*, 2X, *CMLEYC/CAVG*, 2X, *CMLEZC/CAVG* ) 6 3750
C SET UP DELTA S AND DELTA S L.E. 6 3760
BT = B*B 6 3770
AR = BT/REFA 6 3780
DO 20 K=1, NETA 6 3790
DO 10 I=1, NXOC 6 3800
DS(I,K)=SQRT(DY(I,K)**2+DZ(I,K)**2) 6 3810
10 CONTINUE 6 3820
20 CONTINUE 6 3830
ARBSB2 = AR*BS/(BT) 6 3840
STEP SPAN WIZE 6 3850
DO 500 K=1, NETA 6 3860
DYOPIT=DY(I,K)/DS(I,K) 6 3870
DZOPIT=DZ(I,K)/DS(I,K) 6 3880
STEP CHORD WIZE 6 3890
DO 100 I=1, NXOC 6 3900
FILL DEPENDENT VARIABLE ARRAYS 6 3910
DSDSLE=DS(I,K)/DSLE(K) 6 3920
TCON = CPL(I,K)* 6 3930
1TANLFL( DZT(I,K),DZC(I,K),EPSILO( K),DA,ETA(K),X(I,K),1.0) 6 3940
2 -CPU(I,K) * 6 3950
3TANLFL( DZT(I,K),DZC(I,K),EPSILO( K),DA,ETA(K),X(I,K),-1.0) 6 3960
CPLCPU=CPL(I,K)-CPU(I,K) 6 3970
T1(I)=DSDSLE*TCON 6 3980
T2(I)=DSDSLE*CPLCPU 6 3990
YIKYCG=Y(I,K)-YCG 6 4000
ZIKZCG=Z(I,K)-ZCG 6 4010
XIKXCG=X(I,K)-XCG 6 4010

```

T3(9)=DSDSLE*(YIKYCG*DYOPIT+ZIKZCG*DZOPIT)*CPLCPU	6	4020
T4(1)=DSDSLE*(ZIKZCG*TCON-XIKXCG*DYOPIT*CPLCPU)	6	4030
T5(1)=DSDSLE*(-XIKXCG*DZOPIT*CPLCPU-YIKYCG*TCON)	6	4040
XIKXLE= X(I,K) - XLE(K)	6	4050
YIKYLE= Y(I,K) - YLE(K)	6	4060
ZIKZLE= Z(I,K) - ZLE(K)	6	4070
T6(1)=DSDSLE*(YIKYLE*DYOPIT+ZIKZLE*DZOPIT)*CPLCPU	6	4080
T7(1)=DSDSLE*(ZIKZLE*TCON-XIKXLE*DYOPIT*CPLCPU)	6	4090
T8(1)=DSDSLE*(-XIKXLE*DZOPIT*CPLCPU-YIKYLE*TCON)	6	4100
100 CONTINUE	6	4110
C INTEGRATE CHORD WIZE	6	4120
T11=POLINT(XOC,T1,NXOC,0.0,1.0)	6	4130
T12=POLINT(XOC,T2,NXOC,0.0,1.0)	6	4140
T13=POLINT(XOC,T3,NXOC,0.0,1.0)	6	4150
T14=POLINT(XOC,T4,NXOC,0.0,1.0)	6	4160
T15=POLINT(XOC,T5,NXOC,0.0,1.0)	6	4170
T16=POLINT(XOC,T6,NXOC,0.0,1.0)	6	4180
T17=POLINT(XOC,T7,NXOC,0.0,1.0)	6	4190
T18=POLINT(XOC,T8,NXOC,0.0,1.0)	6	4200
CARB9=C(K)*ARBSB2	6	4210
CTAVRG(K)=CTAVRG(K)*ARBSB2	6	4220
CXCAVG(K) = CARB8*(T11)-CTAVRG(K)	6	4230
CYCAVG(K)=-CARB8*DZOPIT*T12	6	4240
CZC1VG(K)=CARB8*DYOPIT*T12	6	4250
CARB8C=CARB8/CHORD	6	4260
CMXCAV(K)=CARB8C*T13	6	4270
CMYCAV(K) = CARB8C*T14-(Z(1,K)-ZCG)*CTAVRG(K)/CHORD	6	4280
CMZCAV(K) = CARB8C*T15+(Y(1,K)-YCG)*CTAVRG(K)/CHORD	6	4290
CNCAVG(K)=-DZOPIT*CYCAVG(K)+DYOPIT*CZCAVG(K)	6	4300
TPIT=CXCAVG(K)**2+CYCAVG(K)**2+CZCAVG(K)**2	6	4310
CMLEXC(K)=ARBSB2*T16	6	4320
CMLEYC(K)=ARBSB2*T17	6	4330
CMLEZC(K)=ARBSB2*T18	6	4340
CDT0(K)=CDT0(K)*ARBSB2	6	4350
CDLIC(K)=CDLIC(K)*ARBSB2	6	4360
CDTICA(K)=CDTICA(K)*ARBSB2	6	4370
IF(ABS(TPIT).GT.1.0E-7) GOTO 131	6	4380
XCPLK(K)=0.0	6	4390
YCPLK(K)=0.0	6	4400

```

ZCPLK(K)=0.0
GOTO 132
131 CONTINUE
  XCPLK(K)=(CYCAVG(K)*CMLEZC(K)-CZCAVG(K)*CMLEYC(K))/TPIT
  YCPLK(K)=(CZCAVG(K)*CMLEXC(K)-CXCAVG(K)*CMLEZC(K))/TPIT
  ZCPLK(K)=(CXCAVG(K)*CMLEYC(K)-CYCAVG(K)*CMLEXC(K))/TPIT
132 CONTINUE
  PRINT 156, ETA(K), CXCAVG(K), CYCAVG(K), CZCAVG(K), CNCAVG(K),
    1CMLEXC(K), CMLEYC(K), CMLEZC(K)
156 FORMAT(1H, F10.6, 7F13.6)
500 CONTINUE
  PRINT 506
506 FORMAT(1H0,5X,3HETA, 7X,
  1*XLE/C CP*, 5X, *YLE/C CP*, 5X, *ZLE/C CP*, 4X, *CDT=0C/CAVG*
  2, 3X, *CDLIC/CAVG*, 4X, *CTC/CAVG*, 4X, *CDTIC/CAVG*)
  PRINT 156, ( ETA(K), XCPLK(K), YCPLK(K), ZCPLK(K), CDT0(K), CDLIC
    1(K), CTAVRG(K), CDTICA(K), K=1, NETA)
  C
  INTEGRATE SPAN WIZE
  CX=POLINT(ETA,CXCAVG,NETA,0.0,1.0)
  CY=POLINT(ETA,CYCAVG,NETA,0.0,1.0)
  CZ=POLINT(ETA,CZCAVG,NETA,0.0,1.0)
  CMX=POLINT(ETA,CMXCAV,NETA,0.0,1.0)
  CMY=POLINT(ETA,CMYCAV,NETA,0.0,1.0)
  CMZ=POLINT(ETA,CMZCAV,NETA,0.0,1.0)
  PRINT 606, CX, CY, CZ, CMX, CMY, CMZ
  CDI = CDI / REFA
  CDT=CDT/REFA
  CDL=CDL/REFA
  CT=CT/REFA
  CTDI=CTDI/REFA
606 FORMAT(//1H0, 44X, *TOTAL PANEL LOADS*/
  1 1H0,7X,*CX*,13X,*CY*,13X,*CZ*,13X,*CMX*,12X,*CMY*,12X,*CMZ*
  2/1H,6F15.6)
  XYZ = CX*CX + CY*CY + CZ*CZ
  IF(ABS(XYZ).GT. 1.0E-7) GOTO 611
  XCPC = 0.0
  YCPC = 0.0
  ZCPC = 0.0
  GOTO 612

```

```

611 CONTINUE
XPCG=XCG/CHORD+(CY*CMZ-CZ*CMY)/XYZ
YPCG=YCG/CHORD+(CZ*CMX-CX*CMZ)/XYZ
ZPCG=ZCG/CHORD+(CX*CMY-CY*CMX)/XYZ
612 CONTINUE
PRINT 616, CDT, CDL, CDI, CT, CTDI, XPCG, YPCG, ZPCG
616 FORMAT(1H0,5X,*CDT=0*,10X,*CDLI*,12X,*CT*,12X,*CTDI*,11X6
I,
1/1H , 5F15.6/1H0, 5X,
2 *N/C CP*, 9X, *Y/C CP*, 9X, *Z/C CP*/1H , 3F15.6)
C
C
C THIS SECTION STEPS THROUGH THE CONTROL SURFACES WHILE SOLVING THE
CONTROL SURFACE EQUATIONS
NCS = 1
709 IF(DA(1,NCS).EQ.0) RETURN
PRINT 706, NCS
706 FORMAT(1H1,35X,15HCONTROL SURFACE, 13,16H SECTIONAL LOADS)
C
C FIN4 THE ETAS AT WHICH THIS CONTROL SURFACE STARTS AND ENDS
DO 710 KS=1, NETA
IF(ETA(KS)-DA(3,NCS)) 710,711,711
710 CONTINUE
711 DO 720 KE=KS, NETA
IF(ETA(KE)-DA(4,NCS)) 720, 721, 722
720 CONTINUE
722 KE = KE - 1
IF(KE.LE. 0) KE = KS
721 NESPN = KE-KS + 1
TFS = (DA(1,NCS)+1.)/2.0
ETAD = DA(4,NCS) - DA (3,NCS)
STEP SPANWISE THROUGH THIS PANEL
DO 900 K=KS, KE
T7(K) = (ETA(K)-DA(3,NCS))/ETAD
DYOPIT= DY(1,K)/DS (K)
DZOPIT= DZ(1,K)/DS (K)
C
FIN4 THE XPCS AT WHICH THIS CONTROL SURFACE STARTS OR ENDS
XKF= RLINT(DA(5,NCS),DA(6,NCS), DA(3,NCS), DA(4,NCS), ETA(K))
XH = RLINT(DA(7,NCS),DA(8,NCS), DA(3,NCS), DA(4,NCS), ETA(K))
XHP =RLINT(DA(7,NCS),DA(8,NCS),DA(3,NCS),DA(4,NCS),ETA(K)+1.5-7)

```

```

XHM = RLINT(DA(7,NC5),DA(8,NC5),CA(3,NC5),CA(4,CS),ETA(K)-1,F-7)
XHP = XHP - XHM
DO 740 ISE=1, NXOC
IF(X(ISE,K)-XKF) 740,741,7422
740 CONTINUE
7422 ISE = ISE - 1
741 IS = 1
IE = ISE
CCS = X(IE,K) - X(1,K)
IF( TFS ) 742, 742, 743
742 IS = IE
IF( X(IS,K) .NE. XKF) IS = IS + 1
IE = NXOC
CCS = X(1,K) + C(K) - X(IS,K)

C FIT FOR Y AND Z OF HING LINE
743 DO 760 I=2, NXOC
IF(X(I,K)-XH) 760,761,761
760 CONTINUE
761 YH = RLINT(Y(I,K), Y(I-1,K), X(I,K), X(I-1,K), XH)
ZH = RLINT(Z(I,K), Z(I-1,K), X(I,K), X(I-1,K), XH)
YHP = RLINT(Y(I,K), Y(I-1,K), X(I,K), X(I-1,K), XHP)
YHM = RLINT(Y(I,K), Y(I-1,K), X(I,K), X(I-1,K), XHM)
YHD = YHP - YH
ZHP = RLINT(Z(I,K), Z(I-1,K), X(I,K), X(I-1,K), XHP)
ZHM = RLINT(Z(I,K), Z(I-1,K), X(I,K), X(I-1,K), XHM)
ZHD = ZHP - ZHM
T = SQRT(YHD*YHD + XHD*XHD + ZHD*ZHD)
HZ = ZHD/T
HY = YHD/T
HX = XHD/T
TT = C(K) / CCS
DO 770 I=IS, IE
LSDSLE= DS(I,K)/DSLE(K)
XIKXH = X(I,K) - XH
YIKYH = Y(I,K) - YH
ZIKZH = Z(I,K) - ZH
T = TAN(DA(2,NC5))
TCCA = CPL(I,K) * (T + DZT(I,K) - LZC(I,K) + _PSILO(K)) /

```

```

1      (1.- (DZT(I,K) - DZC(I,K) + EPSILO(K)) * T)
2      - CPU(I,K) * (T - DZT(I,K) - DZC(I,K) + EPSILO(K)) /
3      (1.-T*(-DZT(I,K) - DZC(I,K) + EPSILO(K)))
      T1(I) = DSDSLE * TCON
      CPLCPU = CFL(I,K) - CPU(I,K)
      T2(I) = DSDSLE*CPLCPU
      T3(I) = DSDSLE*(YIKYH*DYOPIT + ZIKZH*DZOPIT)*CPLCPU
      T4(I) = DSDSLE*(ZIKZH*TCON-XIKXH*DYOPIT*CPLCPU)
      T5(I) = DSDSLE*(XIKXH*DZOPIT*CPLCPU - YIKYH*TCON)
      IF(TFS) 768, 768, 762
768    T8(I) = TT*(XOC(I)+1./TT-1.)
      GOTO 763
762    T8(I) = TT * XOC(I)
763    CONTINUE
770    CONTINUE
      C
      INTEGRATE CHORDSIZE
      NX = IE -IS +1
      T11 = POLINT( T8(IS),T1(IS), NX, 0.0, 1.0)
      T12 = POLINT( T8(IS),T2(IS), NX, 0.0, 1.0)
      T13 = POLINT( T8(IS),T3(IS), NX, 0.0, 1.0)
      T14 = POLINT( T8(IS),T4(IS), NX, 0.0, 1.0)
      T15 = POLINT( T8(IS),T5(IS), NX, 0.0, 1.0)
      CAREB = CCS *ARBSB2
      XSLCS(K) = CAREB*T11 - CTAVRG(K)
      YSLCS(K) = -CAREB*DZOPIT*T12
      ZSLCS(K) = CAREB*DYOPIT*T12
      CAREB = CAREB/CHORD
      NSLCS(K) = -YSLCS(K)*DZOPIT + ZSLCS(K)*DYOPIT
      XSMHL(K) = CAREB*T13
      YSMHL(K) = CAREB*T14-TFS*(Z1,K) - ZH)*CTAVRG(K)/CHORD
      ZSMHL(K) = CAREB*T15-TFS*(Y1,K) - YH)*CTAVRG(K)/CHORD
      T = XSLCS(K)**2 + YSLCS(K)**2 + ZSLCS(K)**2
      T = T * C(K) / CHORD
      XSCPCS(K) = (YSLCS(K)*ZSMHL(K) - ZSLCS(K)*YSMAHL(K))/T
      YSCPCS(K) = (ZSLCS(K)*XSMHL(K) - XSLCS(K)*ZSMHL(K))/T
      ZSCPCS(K) = (XSLCS(K)*YSMAHL(K) - YSLCS(K)*XSMHL(K))/T
      CMHC(K) = XSMHL(K)*HX + YSMHL(K)*HY + ZSMHL(K)*HZ
900    CONTINUE
      CHX = POLINT(T7(KE),XSLCS(KE), NESPN,0.0,1.0)

```

```

CHY = POLINT(T7(KE),YSLCS(KE),NESP,N,0.0,1.0)
CHZ = POLINT(T7(KE),ZSLCS(KE),NESP,N,0.0,1.0)
CVH = POLINT(T7(KE),CMHC(KE),NESP,N,0.0,1.0)
PRINT 806

```

```

806 FORMAT(1H, 7X, 3IETA, 6X, 9HCHXC/CAVG, 3X, 9HCHYC/CAVG, 3X,
1 9HCHZC/CAVG, 2X,

```

```

2 34HCMHXC/CAVG CMHYC/CAVG CMHXC/CAVG, 3X, 9HCMHC/CAVG)
816 FORMAT(///1H, 10X, 27HCONTROL SURFACE TOTAL LOADS/
1 1H, 48H ETA (X/C) CP (Y/C) CP (Z/C) CP CHZ CHM)

```

```

826 FORMAT(47H0 CHX CHY CHZ CHM)
836 FORMAT(1H, 8F12.6)
846 FORMAT(F13.6, 7(12H 0.000000))

```

```

KK = KS-1
IF(KS.EQ. 1) GOTO 841
DO 840 K=1, KK

```

```

840 PRINT 846, ETA(K)
841 DO 850 K=KS, KE

```

```

850 PRINT 836, ETA(5), XSLCS(K), YSLCS(K), ZSLCS(K), XSMHL(K),
1 YSMHL(K), ZSMHL(K), CMHC(K)
IF(KE.EQ.NETA) GO TO 861
KP = KE+1

```

```

DO 860 K=KP, NETA
860 PRINT 846, ETA(K)
861 PRINT 816

```

```

IF(KS.EQ.1) GOTO 871
DO 870 K=1, KK

```

```

870 PRINT 876, ETA(K)
876 FORMAT(F13.6, 4(12H 0.000000))
871 DO 880 K=KS, KE

```

```

880 PRINT 836, ETA(K), XSCPCS(K), YSCPCS(K), ZSCPCS(K)
IF(KE.EQ.NETA) GOTO 891
DO 890 K=KP, NETA

```

```

890 PRINT 876, ETA(K)
891 CONTINUE
NCS = NCS + 1
GOTO 709
END

```

6	5970
6	5980
6	5990
6	6000
6	6010
6	6020
6	6030
6	6040
6	6050
6	6060
6	6070
6	6080
6	6090
6	6100
6	6110
6	6120
6	6130
6	6140
6	6150
6	6160
6	6170
6	6180
6	6190
6	6200
6	6210
6	6220
6	6230
6	6240
6	6250
6	6260
6	6270
6	6280
6	6290
6	6300
6	6310
6	6320
6	6330

```

OSUBROUTINE PPRINT(ETA, XOC, NETA, NXOC, VMU, VML, VTU, VTL, 6 6340
1, CPU, CPL, NC) 6 6350
ODIMENSION ETA(NETA), XOC(NXOC), VMU(NXOC, NETA), VML(NXOC, NETA), 6 6360
1 VTU(NXOC, NETA), VTL(NXOC, NETA), CPU(NXOC, NETA), 6 6370
2 CPL(NXOC, NETA) 6 6380
PRINT 1, NC 6 6390
10FORMAT(1H1,20X,*PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPO6 6400
INENT*, I4 ) 6 6410
DO 10 I=1, NETA 6 6420
PRINT 2, ETA(I) 6 6430
PRINT 3 6 6440
DO 10 J=1, NXOC 6 6450
CP = CPL(J,I) - CPU(J,I) 6 6460
100PRINT 4, XOC(J), VMU(J,I), VML(J,I), VTU(J,I), VTL(J,I), CPU(J,I)6 6470
1, CPL(J,I), CP 6 6480
2 FORMAT(5HCOETA=, F9.6) 6 6490
4 FORMAT(8F12.6) 6 6500
30FORMAT(5X,*X/C*, 7X, *VM/V UP*, 5X, *VM/V LO*, 5X, *VT/V UP*, 5X, 6 6510
1 *VT/V LO*, 6X, *CP UP*, 7X, *CP LO*, 6X, *CP L-U*) 6 6520
RETURN 6 6530
BLAINE D. GAITHER 11/72 6 6540
END 6 6550

```

C



SUBROUTINE BINTEG(DAX, DAY, DAZ, XC, YC, ZC, LONPNA, LONLEN, LATPANS	6560
1, LATLEN, CPD, NSSTAP, NSSECP, XTL, NSTAT, S, X,	6570
2, PANS, SLX, DXC, BCHRD, CX, CY, CZ, CMX, CMY, CMZ	6580
3, NSSTA, NSSEC, LONPAN, XCG, YCG, ZCG, ALPHA, ARBJ,	6590
4, TSXL, TSYL, TSZL, TMX, TMY, TZ, CDI, NC	6600
5)	6610
	6620
	6630
	6640
	6650
DAX DAY DAZ	6660
XC YC ZC	6670
NSSTA	6680
NSSEC	6690
LONPNA	6700
	6710
LATPNA	6720
LONLEN	6730
LATLEN	6740
CPD	6750
	6760
NSSECP	6770
NSSTAP	6780
XTL	6790
NSTAT	6800
BCHRD	6810
ARBJ	6820
X	6830
S SS XX	6840
PANS	6850
SLX	6860
DXC	6870
CX CY CZ	6880
CMX CMY CMZ	6890
LONPAN LATPAN SCRATCH	6900
	6910
	6920
	6930
	6940

DIMENSION DAX(NSSTA, NSSEC), DAY(NSSTA, NSSEC),  
 IDAZ(NSSTA, NSSEC), XC(NSSTA, NSSEC), YC(NSSTA, NSSEC),

```

22C(NSSTA,NSSEC), LONPNA(LONLEN),
3 CPD(LONLEN, LATLEN), X(LONLEN), XX(49), LONPAN(LONLEN), LATPAN
4(LATLEN), XTL(1), SLX(NSSTAT), DXC(NSSTAT),CX(NSSTAT),CY(NSSTAT),
5CZ(NSSTAT), CMX(NSSTAT),CMY(NSSTAT),CMZ(NSSTAT), PANS(NSSTAT),
6 S(NSSEC), SS(49), SLSS(50), SLXX(50)
EQUIVALENCE (SLSS(2),SS), (SLXX(2), XX)
REAL LATPAN, LONPNA
NAMELIST /LA/ SX, SY, SZ /LS/ NSTA,NBOT, NTOP /LC/ LT,LL
1 /LD/ AXI, AYI, AZI, ATX, ATY, ATZ
C CONVERT LATPAN * LONPAN SO THEY POINT TO THE CORRECT SUB-PANEL
C INSTEAD OF PANEL
LT = NSSTAP/ 2 + 1
DO 10 I=1, LONLEN
10 LONPAN(I) = (LONPNA(I)-1)*NSSTAP + LT
SX = 0.
SY = 0.
SZ = 0.
DO 21 J=1,NSSEC
DO 21 I=1,NSSTA
SX = SX + ABS(DAX(I,J))
SY = SY + ABS(DAY(I,J))
21 SZ = SZ + ABS(DAZ(I,J))
SX = SX/2.0
SY = SY/2.0
SZ = SZ/2.0
DO 400 NSTA=1, NSSTAT
NBOT = (NSTA-1)*NSSTAP + 1
NTOP = NBOT + NSSTAP - 1
PANSUM = 0
AXI = 0.
AYI = 0.
AZI = 0.
ATZ = 0.
ATY = 0.
ATX = 0.
START ON THIS STATION
DO 390 NST=NBOT, NTOP
C FIND CPS FOR THIS RING(SUB-STATION)
DO 315 I=1, LATLEN

```

C FIND CP FOR A MERIDIAN

LT = LATPAN(I)

942 FORMAT(1H, I5, G11.4, 5X, 3G11.4, 5X, 3G11.4)

IF(LONLEN.GT. 3) GOTO305

T = XC(NST,LT)

NB = 1

NT = 2

IF(T.LE.X(2).OR.LONLEN.LT.3) GOTO325

NB = 2

NT = 3

325 XX(I) = (T - X(NB)) \* (CPD(NT,I) - CPD(NB,I)) / (X(NT)-X(NB)) +

1 CPD(NB,I)

GOTO 315

305 XX(I) = CODIM1(XC(NST,LT), X, CPD(1,I), LONLEN,-1.0)

315 CONTINUE

941 FORMAT(10G11.4)

31 IF(LATLEN.GT.1) GOTO 32

PANSUM = PANSUM + NSSECP\*XX(1)

DO 500 I=1, NSSEC

AXT = AXT + XX(1)\*DAX(NST,I)

AYT = AYT + XX(1)\*DAY(NST,I)

AZT = AZT + XX(1) \* DAZ(NST,I)

ATX = ATX + ((YC(NST,I)-YCG)\*DAZ(NST,I) - (ZC(NST,I)-ZCG)\*DAY(NST,I)) \* XX(1)

ATY = ATY + ((ZC(NST,I)-ZCG)\*DAX(NST,I) - (XC(NST,I)-XCG)\*DAZ(NST,I)) \* XX(1)

ATZ = ATZ + ((XC(NST,I)-XCG)\*DAY(NST,I) - (YC(NST,I)-YCG)\*DAX(NST,I)) \* XX(1)

GOTO 390

32 LTT = LATPAN(1)

LTP = LTT

IF(LTT.EQ.1) LTP = 2

S(LTP-1) = 0.0

DO 40 I=LTP, NSSEC

S(I) = S(I-1) + SQRT((YC(NST,I) - YC(NST,I-1))\*\*2 + (ZC(NST,I) - ZC(NST,I-1))\*\*2)

TSF = SORT((YC(NSSEC)-YC(1))\*\*2 + (ZC(NSSEC)-ZC(1))\*\*2)

IF(LTT.EQ.1) GOTO 46

S(1) = S(NSSEC) + TSF

6	7340
6	7350
6	7360
6	7370
6	7380
6	7390
6	7400
6	7410
6	7420
6	7430
6	7440
6	7450
6	7460
6	7470
6	7480
6	7490
6	7500
6	7510
6	7520
6	7530
6	7540
6	7550
6	7560
6	7570
6	7580
6	7590
6	7600
6	7610
6	7620
6	7630
6	7640
6	7650
6	7660
6	7670
6	7680
6	7690
6	7700
6	7710
6	7720

```

LTTT = LTT - 1
IF(LTTT.EQ.1) GOTO 46
DO 45 I= 2, LTTT
45 S(I) = S(I-1) + SQRT((YC(NST,I)-YC(NST,I-1))**2 + (ZC(NST,I) -
1 ZC(NST,I-1))**2)
46 XX(LATLEN+1) = XX(1)
DO 50 I=1,LATLEN
LT = LATPAN(I)
50 SS(I) = S(LT)
SS(LATLEN+1) = TSF + S(NSSEC)
IF(LTT.NE.1) SS(LATLEN+1) = S(LTTT) + SQRT((YC(NST,LTTT) - YC(NST,6
1 LTT))**2 + (ZC(NST,LTTT)-ZC(NST,LTT))**2)
SLXX(1) = XX(LATLEN)
SLSS(1) = SS(1) - SS(LATLEN+1) + SS(LATLEN)
DO 60 I=1, NSSEC
T = CODIMI( S(I), SLSS, SLXX, LATLEN+2, 1.0)
PANSUM = PANSUM + T
AXT = AXT + T*DAX(NST,I)
AYT = AYT + T*DAY(NST,I)
AZT = AZT + T*DAZ(NST,I)
ATX = ATX + ((YC(NST,I) - YCG) * DAZ(NST,I) - (ZC(NST,I)-ZCG)*DAY(N
1 ST,I)) * T
ATY = ATY + ((ZC(NST,I)-ZCG)*DAX(NST,I) - (XC(NST,I)-XCG)*DAZ(NST,6
1 I)) * T
60 ATZ = ATZ + ((XC(NST,I)-XCG)*DAY(NST,I) - (YC(NST,I)-YCG)*DAX(NST,6
1 I)) * T
390 CONTINUE
PANS(NSTA) = PANSUM
SLX(NSTA) = (XTL(NSTA)+XTL(NSTA+1))/2.0
T = (XTL(NSTA+1)-XTL(NSTA))/ BCHRDL
DXC(NSTA) = T
TS = T * SY
T = T*SZ
CX(NSTA) = -AXT/T
CY(NSTA) = -AYT/T
CZ(NSTA) = -AZT/T
T = ARBJ * BCHRDL
CMX(NSTA) = -ATX/T
CMY(NSTA) = -ATY/T

```

```

400 CMZ(NSTA)=-ATZ/T
C FIND TOTAL MOMENTS * LOADS
TSXL = 0.
TSYL = 0.
TSZL = 0.
TMX = 0.
TMY = 0.
TMZ = 0.
PRINT 923, NC
923 FORMAT(1H1,35X,*BODY SECTIONAL LOADS FOR COMPONENT*, 15, /1H0,10X,
1 *X/C*,8X, *CXW/WAVG*,7X,*CYH/HAVG*,07X,*CZW/WAVG*)6
DO 410 I=1, NSTAT
SLX(I)=SLX(I)/BCHRD
PRINT 948, SLX(I), CX(I), CY(I), CZ(I)
948 FORMAT(1H ,4F15.6)
TSXL = TSXL + CX(I)*DXC(I)
TSYL = TSYL + CY(I)*DXC(I)
TSZL = TSZL + CZ(I) * DXC(I)
TMX = TMX + CMX(I)*DXC(I)
TMY = TMY + CMY(I)*DXC(I)
TMZ = TMZ + CMZ(I)*DXC(I)
410 TSXL = TSXL * SZ/ARBJ
TSYL = TSYL*SY/ARBJ
TSZL = TSZL*SZ/ARBJ
T = TSXL*TSXL + TSYL*TSYL + TSZL*TSZL
XCPC = XCG / BCHRD + (TSXL*TMZ - TSZL*TMY)/T
YCPC = YCG/BCHRD + (TSZL*TMX-TSXL*TMZ)/T
ZCPC = ZCG/BCHRD + (TSXL*TMY-TSYL*TMX)/T
CL = TSZL*COS(ALPHA) - TSXL*SIN(ALPHA)
CD = TSZL * SIN(ALPHA) + TSXL * COS(ALPHA)
PRINT 906
906 FORMAT(/ /1H0,44X,*TOTAL BODY LOADS*)
CDI=CDI/ARBJ
PRINT 901, TSXL, TSYL, TSZL, TMX, TMY, TMZ, CDI
901 FORMAT(1H0,7X,*CX*,13X,*CY*, 13X,*CZ*,12X,*CMX*,12X,*CMY*,12X,*CMZ*
1*, 12X, *CDI*/1H ,7F15.6)
PRINT 902, XCPC, YCPC, ZCPC, SX, SY, SZ
902 FORMAT(1H0, 5X,*X/C CP*,9X,*Y/C CP*,9X, *Z/C CP*,11X,*SX*,13X,*SY*6
1, 13X, *SZ*/1H , 6F15.6)

```

C  
RETURN  
BLAINE D. GAITHER 9/72  
END

6 8510  
6 8520  
6 8530

```

SUBROUTINE REFLECT(SYM,LO,LA,AX,AY,AZ,X,Y,Z,LON,LAT,LATP,CPD,NSS) 6 8540
DIMENSION AX(LO,LA),AY(LO,LA),AZ(LO,LA),X(LO,LA),Y(LO,LA),Z(LO,LA) 6 8550
1,LATP(LAT),CPD(LON,LAT) 6 8560
REAL LATP 6 8570
LL=NSS/2+1 6 8580
DO 1 I=1,LAT 6 8590
1 LATP(I)=(LATP(I)-1)*NSS+LL 6 8600
IF(SYM.EQ.-1.) RETURN 6 8610
C REFLECT GEOMETRY 6 8620
ILA=LA 6 8630
ILA1=ILA+1 6 8640
K=LA 6 8650
DO 10 J=1,LA 6 8660
JJ=LA+1-J 6 8670
IF(ABS(Y(1,JJ)),LT..0001) GO TO 10 6 8680
K=K+1 6 8690
DO 5 I=1,LO 6 8700
AX(I,K)=AX(I,JJ) 6 8710
AY(I,K)=-AY(I,JJ) 6 8720
AZ(I,K)=AZ(I,JJ) 6 8730
X(I,K)=X(I,JJ) 6 8740
Y(I,K)=-Y(I,JJ) 6 8750
Z(I,K)=Z(I,JJ) 6 8760
10 CONTINUE 6 8770
LA=K 6 8780
IF(SYM.EQ.1.0) RETURN 6 8790
C REFLECT CPS 6 8800
K=LAT 6 8810
DO 20 J=1,LAT 6 8820
JJ=LAT+1-J 6 8830
LT=LATP(JJ) 6 8840
IF(ABS(Y(1,LT)),LT..0001) GO TO 20 6 8850
K=K+1 6 8860
DO 15 I=1,LON 6 8870
15 CPD(I,K)=CPD(I,JJ) 6 8880
DO 25 K1=ILA1,LA 6 8890
IF (Z(1,K1).EQ.Z(1,LT).AND.-Y(1,K1).EQ.Y(1,LT)) GO TO 26 6 8900
25 CONTINUE 6 8910
GO TO 20 6 8920

```

26 LATP(K)=K1  
20 CONTINUE  
LAT=K  
RETURN  
END

6	8930
6	8940
6	8950
6	8960
6	8970



SUBROUTINE	BPRINT(NO, NA, X, Y, Z, VM, VT, CP, NC)	6	8980
DIMENSION	X(NO, NA), Y(NO, NA), Z(NO, NA), VM(NO, NA), VT(NO, NA), CP(NO, NA)	6	8990
	1), CP(NO, NA)	6	9000
	PRINT 1, NC	6	9010
1	FORMAT(1H1, 24X, *BODY VELOCITY AND PRESSURE COEFFICIENTS FOR COMP	6	9020
	1 ONENT*, I3)	6	9030
	DO 10 I=1, NA	6	9040
	PRINT 2, I	6	9050
2	FORMAT(*OLATERAL STATION *, I3)	6	9060
	PRINT 3	6	9070
3	FORMAT(5X, *NO*, 10X, *XQ*, 10X, *YQ*, 10X, *ZQ*, 9X, *VM/V*, 8X,	6	9080
1	*VT/V*, 9X, *CP*)	6	9090
	PRINT 4, ( J, X(J,I), Y(J,I), Z(J,I), VM(J,I), VT(J,I), CP(J,I),	6	9100
1	J=1, NO)	6	9110
4	FORMAT(4X, I3, 5X, 6F12.6)	5	9120
10	PRINT 5	6	9130
5	FORMAT(1H1)	6	9140
	RETURN	6	9150
	BLA9NE D. GAITHER 11/72	6	9160
	END	6	9170

C

```

SUBROUTINE FFDRAG(NB,NP,YB,ZB,XKB,NBE,BSYM,YP,ZP
1 ,XKP,NPE,PSYM,PIND,TBD,TPD,NPS)
DIMENSION YB(1),ZB(1),XKB(1),NBE(1),BSYM(1),YP(1)
1 ,ZP(1),XKP(1),NPE(1),PSYM(1),PIND(1)
2 ,T2D(1),TPD(1)
COMMON/BODY/ DUM(15000),Y(200),Z(200)
1 ,S(200),SYM(200),YN(200),ZN(200),XK(200),D(200),NUMB(200),
2 ,NUP(5),XP(200),DY(200),DZ(200),DS(200),ETA(200),YNP(200)
3 ,ZNP(200)
DATA IONE/17,MAXVOR/2007,NVOR/407
K=0
TNB5=0.
IF(NB.EQ.0) GO TO 20
CALCULATION OF BODY DATA
DO 5 I=1,NB
5 TNBE=TNBE+NBE(I)
L=0
LKB=0
DO 15 I=1,NB
NUB=NBE(I)
DO 10 J=1,NUB
N1=L+J
N2=N1+1
K=K+1
XK(K)=XKB(LKB+J)
DELY=YB(N2)-YB(N1)
DELZ=ZB(N2)-ZB(N1)
S(K)=SQRT(DELY**2+DELZ**2)
EPS=.000001
IF(S(K).GT.EPS) GO TO 2
S(K)=0.
YN(K)=1.
ZN(K)=1.
GO TO 3
2 YN(K)=-DELY/S(K)
ZN(K)=DELY/S(K)
3 SYM(K)=BSYM(I)
Y(K)=.5*(YB(N1)+YB(N2))
10 Z(K)=.5*(ZB(N1)+ZB(N2))

```

```

NUM2(I)=NBE(I)
LKB=LKB+NUB
15 L=L+NUB+1
20 IF(NP.EQ.0) GO TO 50
CALCULATION OF PANEL DATA
INDP=0
IF(NPS.EQ.0) GO TO 150
INDP=1
GO TO 8
150 NPS=NVOR/NPE(I)
DO 4 I=1,NP
4 IF(NVOR/NPE(I).LT.NPS) NPS=NVOR/NPE(I)
6 TNPE=0.
DO 7 I=1,NP
7 TNPE=TNPE+NPS*NPE(I)
IF(TNPE+TNBE.LE.MAXVCR)GO TO 8
NPS=NPS-1
GO TO 6
8 L=L+1
LKP=0
DO 49 I=1,NP
NUP=NPE(I)
SPAN=0.
DO 45 J=L,NUP
N1=L+J
N2=N1+1
DY(J)=YP(N2)-YP(N1)
DZ(J)=ZP(N2)-ZP(N1)
DS(J)=SQRT(DY(J)**2+DZ(J)**2)
YNP(J)=-DZ(J)/DS(J)
ZNP(J)=DY(J)/DS(J)
45 SPAN=SPAN+DS(J)
SUM=0.
DO 46 J=L,NUP
ETA(J)=(SUM+.5*DS(J))/SPAN
46 SUM=SUM+DS(J)
DO 47 J=L,NUP
47 XP(J)=XKP(LKP+J)
NUP=NUP

```

IF(PIND(I),NE.1.) GO TO 44

NUZ=NUP+1

ETA(NUZ)=1.

XP(NUZ)=0.

44 SUM=0.

DO 48 J=1,NUP

N1=L+J

N2=N1+1

DELS=DS(J)/NPS

DO 48 J1=1,NPS

K=K+1

S(K)=DELS

SYM(K)=PSYM(I)

YN(K)=YNP(J)

ZN(K)=ZNP(J)

SUM=SUM+.5\*DELS

ET=SUM/SPAN

SUM=SUM+.5\*DELS

IF(INDP.EQ.1) GO TO 488

CALL CODIM(ETA,XP,NUZ,ET,XK(K),IONE)

488 Y(K)=YP(N1)+(J1-.5)\*DELS\*ZN(K)

48 Z(K)=ZP(N1)+(J1-.5)\*DELS\*(-YN(K))

NUMP(I)=NPS\*NUP

LKP=LKP+NUP

49 L=L NUP+1

CALCULATION OF DRAG ON EACH ELEMENT

50 NV=K

WRITE(6,333)(XK(J),J=1,NV)

333 FORMAT(//('10F12.5'))

DO 58 I=1,NV

V=0.

W=0=

DO 56 J=1,NV

YY=Y(I)-Y(J)

ZZ=Z(I)-Z(J)

CALL HSHOE(YY,ZZ,S(J),YN(J),ZN(J),AV,A..)

IF(SYM(J).EQ.1..OR.SYM(J).EQ.-1.) GO TO 54

YY=-Y(I)-Y(J)

CALL HSHOE(YY,ZZ,S(J),YN(J),ZN(J),DELV,DEL..)

6	9960
6	9970
6	9980
6	9990
6	0000
6	0010
6	0020
6	0030
6	0040
6	0050
6	0060
6	0070
6	0080
6	0090
6	0100
6	0110
6	0120
6	0130
6	0140
6	0150
6	0160
6	0170
6	0180
6	0190
6	0200
6	0210
6	0220
6	0230
6	0240
6	0250
6	0260
6	0270
6	0280
6	0290
6	0300
6	0310
6	0320
6	0330
6	0340

IF(SYM(J).EQ.0.) GO TO 52

DELV=-DELV

DELW=-DELW

52 AV=AV+DELV

AW=AW+DELW

54 V=V AV\*KK(J)

56 W=W+AW\*KK(J)

58 D(I)=-2.\*KK(I)\*S(I)\*(V\*YN(I)+W\*ZN(I))

CALCULATION OF TOTAL DRAG

K=0

IF(NB.EQ.0) GO TO 95

DO 93 I=1,NB

TBD(I)=0.

N1=K+1

N2=K+NUMB(I)

DO 92 J=N1,N2

92 TBD(I)=TBD(I)+D(J)

K=N2

IF(SYM(J).EQ.0..OR.SYM(J).EQ.-2.) TBD(I)=2.\*TBD(I)

93 CONTINUE

95 IF(NP.EQ.0) RETURN

DO 98 I=1,NP

TPD(I)=0.

N1=K+1

N2=K+NUMP(I)

DO 96 J=N1,N2

96 TPD(I)=TPD(I)+D(J)

K=N2

98 CONTINUE

RETURN

END

6 0350

6 0360

6 0370

6 0380

6 0390

6 0400

6 0410

6 0420

6 0430

6 0440

6 0450

6 0460

6 0470

6 0480

6 0490

6 0500

6 0510

6 0520

6 0530

6 0540

6 0550

6 0560

6 0570

6 0580

6 0590

6 0600

6 0610

6 0620

6 0630

6 0640

6 0650

```

SUBROUTINE NFDRAG(NC,NS,NCP,NSP,XOCCP,XOCV,ETACP,XV,YV,ZV
1,TANV,SIGMAV,CPNET,NSPP,CDSCP,CDSCP,CDTCP,CDTH,CDST,CDAT,CDTT,CDTH6
2T)
6
0660
0670
0680
DIMENSION XOCCP(1),XOCV(1),ETACP(1),XV(1),YV(1),ZV(1),TANV(1)
6
0690
1,SIGMAV(1),CPNET(1),CDSCP(1),CDACP(1),CDTCP(1),CDTH(1)
6
0700
DIMENSION DUMI(100),DUMO(100),P(2000),ETAV(100)
6
0710
DIMENSION X(1000,2),Y(1000),Z(1000),TAN(1000,2),CP(2000)
6
0720
1,SIGMA(2000,2),D(2000),CDS(100),CDA(100),CDT(100)
6
0730
DIMENSION XK(200)
6
0740
DIMENSION XOCCP(50)
6
0750
COMMON DA(5000)
6
0760
COMMON/BODY/ B(31550)
6
0770
EQUIVALENCE (Q,DA(13)),(GAM,DA(14)),(YCG,DA(8)),(ZCG,DA(9))
6
0780
1,(CBAR,DA(5)),(SPAN,DA(6))
6
0790
EQUIVALENCE (XK(1),R(16201)),(X(1,1),R(19001)),(Y(1,1),R(17001))
6
0800
1,(Z(1,1),R(18001)),(TAN(1,1),R(15001)),(CP(1),R(21001))
6
0810
2,(SIGMA(1,1),B(23001)),(D(1,1),R(27001)),(CDS(1),R(29001))
6
0820
3,(CDA(1),B(29101)),(CDT(1),B(29201)),(DUMI(1),B(29301))
6
0830
4,(DUMO(1),B(29401)),(P(1),B(29501)),(XOCCP(1),B(31501))
6
0840
333 FORMAT(//('IX,10F12.5'))
6
0850
332 FORMAT(//('IX,10I12'))
6
0860
WRITE(6,332)NC,NS,NCP,NSP,NSPP
6
0870
WRITE(6,333)(XOCCP(I),I=1,NCP)
6
0880
WRITE(6,333)(XOCV(I),I=1,NC)
6
0890
WRITE(6,333)(ETACP(I),I=1,NSP)
6
0900
R4PI=.07957747
6
0910
EPS2=.0075968656
6
0920
EPS3=.00001
6
0930
ISYM=DA(3426)
6
0940
NV1=NS*NC
6
0950
NV2=2*NV1
6
0960
WRITE(6,333)(XV(I),I=1,NV2)
6
0970
WRITE(6,333)(YV(I),I=1,NV2)
6
0980
WRITE(6,333)(ZV(I),I=1,NV2)
6
0990
WRITE(6,333)(TANV(I),I=1,NV2)
6
1000
WRITE(6,333)(SIGMAV(I),I=1,NV2)
6
1010
WRITE(6,333)(CPNET(I),I=1,NV1)
6
1020
WRITE(6,333)(CPNET(I),I=1,NV1)
6
1030
CALCULATE NEW VORTEX COORDINATES
6
1040
NSPP=3

```

```

JK=0
NC2=2*NC
DELY=(YV(NC2+1)-YV(1))/NSPP
DELY1=-DELY*(NSPP/2+1)
DO 100 I=1,NS
DEL=DELY1
DO 100 J=1,NSPP
DEL=DEL+DELY
DO 100 K=1,NC2*2
IK1=(I-1)*NC2+K
IK2=IK1+1
JK=JK+1
X(JK,1)=XV(IK1)+DEL*TANV(IK1)
X(JK,2)=XV(IK2)+DEL*TANV(IK2)
Y(JK)=YV(IK1)+DEL
Z(JK)=ZV(IK1)
TAN(JK,1)=TANV(IK1)
TAN(JK,2)=TANV(IK2)
SIGMA(JK,1)=SIGMAV(IK1)
100 SIGMA(JK,2)=SIGMAV(IK2)
NV=JK
NSS=NS*NSPP
WRITE(6,333)(X(I,1),X(I,2),I=1,NV)
WRITE(6,333)(Y(I),I=1,NV)
WRITE(6,333)(Z(I),I=1,NV)
WRITE(6,333)(TAN(I,1),TAN(I,2),I=1,NV)
WRITE(6,333)(SIGMA(I,1),SIGMA(I,2),I=1,NV)
CALCULATE ETAS OF VORTICES
DELE=1./NSS
SUM=-.5*DELE
DO 110 I=1,NSS
SUM=SUM+DELE
110 ETAV(I)=SUM
WRITE(6,333)(ETAV(I),I=1,NSS)
CALCULATE X/C OF PANEL CENTROIDS
SLOPE=(TANV(2*NC)-TANV(1))/(XV(2*NC)-XV(1))
TANLE=TANV(1)-.5*(XV(2)-XV(1))*SLOPE
TANTE=TANV(2*NC)+.5*(XV(2*NC)-XV(2*NC-1))*SLOPE
DX=CA(3450)+DA(3432)

```

```

XLE=YV(1)*TANLE+DX
XTE=YV(1)*TANTE+DX+DA(3453)
CHORD=XTE-XLE
J=0
DO 1000 I=1,NC2,2
J=J+1
1000 XOCPC(J)=(.5*(XV(I)+XV(I+1))-XLE)/CHORD
CALCULATE CPNET AT VORTICES
DO 120 I=1,NCP
DO 130 J=1,NSP
130 DUMI(J)=CPNET(NCP*(J-1)+I)
CALL POL(ETACP,DUMI,NSP,ETAV,DUMO,NSS)
DO 120 J=1,NSP
NJ=NCP*(J-1)+I
120 P(NJ)=DUMO(J)
DO 140 I=1,NSS
K=NC*(I-1)+1
J=NCP*(I-1)+1
140 CALL POL(XOCCP,P(J),NCP,XOCPC,CP(K),NC)
WRITE(6,333)(CP(I),I=1,NV)
YVV=.5*DELY
EPS1=2.*YVV
NV2=2*NV
IF(ISYM.EQ.1) NSS=2*NSS
DO 1 I=1,NV2
IF(ABS(CP(I)).GT.EPS3) GO TO 3
1 CONTINUE
GO TO 44
CALCULATE DRAG DUE TO LIFT
3 DO 43 II=1,2
WRITE(6,331) YVV
331 FORMAT(1X,10E12.5)
DO 5 JJ=1,NV2
5 D(JJ)=0.
NSKIPI=0
9 DO 20 J=1,NV
IPASS=1
XMF=1.
NSKIP2=0

```

6	1440
6	1450
6	1460
6	1470
6	1480
6	1490
6	1500
6	1510
6	1520
6	1530
6	1540
6	1550
6	1560
6	1570
6	1580
6	1590
6	1600
6	1610
6	1620
6	1630
6	1640
6	1650
6	1660
6	1670
6	1680
6	1690
6	1700
6	1710
6	1720
6	1730
6	1740
6	1750
6	1760
6	1770
6	1780
6	1790
6	1800
6	1810
6	1820



527

```

30 KK=K+NSKIP2
   U=W+XMF*CP(KK)*(X(K,2)-X(K,1))*F
   IF(IPASS.EQ.2.OR.ISYM.EQ.-1) GO TO 31
   IPASS=2
   DELY=-YY-Y(K)
   IF(ISYM.EQ.0) GO TO 10
   IF(ISYM.EQ.1) GO TO 11
   XMF=-1.
   GO TO 10
11 NSKIP2=NV
   GO TO 10
31 IPASS=1
   XMF=1.
22 NSKIP2=0
   KK=J+NSKIP1
20 D(KK)=2.*CP(KK)*(X(J,2)-X(J,1))*W*R4PI
   IF(ISYM.NE.1.OR.NSKIP1.NE.0) GO TO 21
   NSKIP1=NV
   GO TO 9
21 IF(II.EQ.2) GO TO 40
   DO 41 I=1,NSS
   CDS(I)=0.
   DO 41 J=1,NC
   K=NC*(I-1)+J
41 CDS(I)=CDS(I)+D(K)
   GO TO 43
40 DO 42 I=1,NSS
   CDA(I)=0.
   DO 42 J=1,NC
   K=NC*(I-1)+J
42 CDA(I)=CDA(I)+D(K)
43 CONTINUE
44 DO 45 I=1,NV2
   DO 45 J=1,2
   IF(ABS(SIGMA(I,J)).GT.EPS3) GO TO 39
45 CONTINUE
   GO TO 500
CALCULATE DRAG DUE TO THICKNESS
39 DO 46 I=1,NV2

```

6	2220
6	2230
6	2240
6	2250
6	2260
6	2270
6	2280
6	2290
6	2300
6	2310
6	2320
6	2330
6	2340
6	2350
6	2360
6	2370
6	2380
6	2390
6	2400
6	2410
6	2420
6	2430
6	2440
6	2450
6	2460
6	2470
6	2480
6	2490
6	2500
6	2510
6	2520
6	2530
6	2540
6	2550
6	2560
6	2570
6	2580
6	2590
6	2600

46	D(I)=0.	6	2610
	NSKIP1=0	6	2620
47	DO 50 I=1,NV	6	2630
	DO 60 J=1,2	6	2640
	IPASS=1	6	2650
	NSKIP2=0	6	2660
	U=0.	6	2670
	DO 70 K=1,NV	6	2680
	DO 70 L=1,2	6	2690
	DELX=X(I,J)-X(K,L)	6	2700
	YY=Y(I)	6	2710
	IF(1SYM.EQ.1.AND.NSKIP1.NE.0) YY=--YY	6	2720
	DELY=YY-Y(K)	6	2730
	DELZ=Z(I)-Z(K)	6	2740
	T=TAN(K,L)	6	2750
	XP=DELX+T*YVV	6	2760
	XN=DELX-T*YVV	6	2770
	YP=DELY+YVV	6	2780
62	YN=DELY-YVV	6	2790
	XY=DELX-T*DELY	6	2800
	TSQ1=1.+T**2	6	2810
	PP=SQRT(TSQ1)	6	2820
	R2SQ=XY**2+DELZ**2*TSQ1	6	2830
	TEST1=XN**2+YN**2	6	2840
	TEST2=XP**2+YP**2	6	2850
	R4=SQRT(TEST1+DELZ**2)	6	2860
	R5=SQRT(TEST2+DELZ**2)	6	2870
	YX=DELY+T*DELX	6	2880
	YVT=YVV*TSQ1	6	2890
	TERM4=1./R4-1./R5	6	2900
	XYSQ=XY**2	6	2910
	CHECK1=XYSQ/TEST1	6	2920
	CHECK2=XYSQ/TEST2	6	2930
	IF(DELZ.LT.EPS1.AND.CHECK1.LT.EPS2.AND.CHECK2.LT.EPS2.AND.ABS(DELY	6	2940
	1)*GT.YVV) GO TO 48	6	2950
	IF(DELZ.LT.EPS1.AND.ABS(DELY).LE.YVV.AND.ABS(XY).LT.ABS(.5*(X(K,2)	6	2960
	1-X(K,1))) GO TO 49	6	2970
	XI2=(YX+YVT)/R5	6	2980
	XI3=-((YX-YVT)/R4	6	2990



CDAT=CDAT*2.*YVV	6	3390
CDTT=CDTT*2.*YVV	6	3400
CDTH=CDAT-CDST	6	3410
CALCULATE SPANWISE DISTRIBUTION OF DRAG		
CALL POL(ETAV,CDS,NSS,ETACP,CDSCP,NSP)	6	3420
CALL POL(ETAV,CDA,NSS,ETACP,CDACP,NSP)	6	3430
CALL POL(ETAV,CDT,NSS,ETACP,CDTCP,NSP)	6	3440
DO 190 I=1,NSP	6	3450
190 CDTH(I)=CDACP(I)-CDSCP(I)	6	3460
WRITE(6,333)(CDS(I),I=1,NSS)	6	3470
WRITE(6,333)(CDA(I),I=1,NSS)	6	3480
WRITE(6,333)(CDT(I),I=1,NSS)	6	3490
WRITE(6,333)(CDSCP(I),I=1,NSS)	6	3500
WRITE(6,333)(CDACP(I),I=1,NSS)	6	3510
WRITE(6,333)(CDTCP(I),I=1,NSS)	6	3520
WRITE(6,333)(CDTH(I),I=1,NSS)	6	3530
WRITE(6,333)(CDST,CDAT,CDTT,CDTH)	6	3540
WRITE(6,333) CDST,CDAT,CDTT,CDTH	6	3550
CALCULATE TRAILING EDGE K VALUES	6	3560
DO 180 I=1,NSS	6	3570
XK(I)=0.	6	3580
DO 180 J=1,NC	6	3590
K=NC*(I-1)+J	6	3600
180 XK(I)=XK(I)+CP(K)*(X(K,2)-X(K,1))	6	3610
WRITE(6,333)(XK(I),I=1,NSS)	6	3620
RETURN	6	3630
END	6	3640

```

C
C
C
C
C
C
C
      FUNCTION POLINT(X,Y,NPTS,RLLIM,RULIM)
C
C      X      - INDEPENDENT VARIABLES
C      Y      - DEPENDENT VARIABLES
C      NPTS   - NUMBER OF DEPENDENT AND INDEPENDANT VARIABLES
C      O .LT. RLLIM .LT. RULIM .LT. 1
C
      DIMENSION X(NPTS), Y(NPTS), PX(22), PY(22)
      IF(NPTS-20) 11, 11, 1
      1 PRINT 6
      6 FORMAT(*000 MANY POINTS GIVEN TO POLINT*)
      STOP
      11 PX(1) = ACOS(1.0-2.0*RLLIM)
      PY(1) = 0.0
      PX(NPTS+2) = ACOS(1.0-2.0*RULIM)
      PY(NPTS+2) = 0.0
      DO 20 I=1, NPTS
      PHI = ACOS(1.0-2.0*X(I))
      PX(I+1) = PHI
      PY(I+1) = Y(I)*.5*SIN(PHI)
      20 CONTINUE
      CALL INTNC(PX(1),PX(NPTS+2), NPTS+2, PX, PY, POLINT, IFERR)
      RETURN
      END

```

6	3650
6	3660
6	3670
6	3680
6	3690
6	3700
6	3710
6	3720
6	3730
6	3740
6	3750
6	3760
6	3770
6	3780
6	3790
6	3800
6	3810
6	3820
6	3830
6	3840
6	3850
6	3860
6	3870
6	3880

	FUNCTION TANALF(	DZT,DZC,EPSILO,DA,ETA,X,TB)	
C			6 3890
C			6 3900
C			6 3910
C	DZT - DELTA X T		6 3920
C	DZC - DDELTA X C		6 3930
C	ESILO - EPSILON		6 3940
C	X - X VALUE AT THIS POINT		6 3950
C	TB - 1.FOR BOTTOM SURFACE	-1.FOR TOP SURFACE	6 3960
C	DA - ARRAY OF CONTROL SURFACE INFO.		6 3970
C	DIMENSION DA(1)		6 3980
C	IS THERE A CONTROL SURFACE AT THIS POINT		6 3990
C	IJ = 1		6 4000
C	IS THIS CONTROL POINT AT OUR ETA		6 4010
	1 IF(DA(IJ).EQ.0.0) GOTO 150		6 4020
	IF(DA(IJ+2).LE.ETA .AND. DA(IJ+3).GE. ETA) GOTO 30		6 4030
	2 IJ = IJ + 8		6 4040
	IF(IJ.LT.80) GOTO 1		6 4050
	PRINT 156, IJ, DA		6 4060
	156 FORMAT(I3,8G14.6)		6 4070
	STOP 445		6 4080
	30 T3= DA(IJ+4)+(DA(IJ+5)*DA(IJ+4))/(DA(IJ+3)-DA(IJ+2))*(ETA-DA(IJ+2))		6 4090
	1)		6 4100
	IF( DA(IJ) .EQ.1..AND. T3 .GE. X)GOTO 100		6 4110
	IF( DA(IJ) .EQ.-1..AND.T3 .LE. X)GOTO 75		6 4120
	GOTO 2		6 4130
	75 TA = DA(IJ+1)		6 4140
	GOTO 101		6 4150
	100 TA =-DA(IJ+1)		6 4160
	101 T =(TB*DZT - DZC) + EPSILO		6 4170
	TA = TAN(TA)		6 4180
	TANILF =(TA+T)/(1.0-T*TA)		6 4190
	RETURN		6 4200
	150 TANALF = (TB*DZT-DZC) + EPSILO		6 4210
	RETURN		6 4220
	END		6 4230

```

SUBROUTINE HSHOE(Y,Z,S,YN,ZN,V,W)
DATA R4PI/.07957747/
DATA EPS/.000001/
IF(S.GT.EPS) GO TO 5
V=0.
W=0.
RETURN
5 HS=.5*S
YP=Y*ZN-Z*YN
ZP=Z*ZN+Y*YN
YPP=Y*YP+HS
YPN=YP-HS
RP=YPP**2+ZP**2
RN=YPN**2+ZP**2
EV=ZP*(1./RP-1./RN)*R4PI
EW=(YPN/RN-YPN/RP)*R4PI
V=EV*ZN+EW*YN
W=EW*ZN-EV*YN
RETURN
END

```

6	4240
6	4250
6	4260
6	4270
6	4280
6	4290
6	4300
6	4310
6	4320
6	4330
6	4340
6	4350
6	4360
6	4370
6	4380
6	4390
6	4400
6	4410
6	4420
6	4430



```

SUBROUTINE POL(XI,YI,NI,XO,YO,NO)
DIMENSION XI(1),YI(1),XO(1),YO(1)
DIMENSION TI(50),FI(50),TO(100)
DATA PI/3.1415927/
NI1=NI+1
NI2=NI+2
TI(1)=0.
DO 10 I=2,NI1
  TI(I)=ACOS(1.-2.*XI(I-1))
  FI(I)=YI(I-1)*2.*SQRT(XI(I-1)*(1.-XI(I-1)))
  FI(1)=0.
  TI(NI2)=PI
  FI(NI2)=0.
DO 15 I=1,NO
  TO(I)=ACOS(1.-2.*XO(I))
  CALL CODIM(TI,FI,NI2,TO,YO,NO)
DO 20 I=1,NO
  YO(I)=YO(I)/(2.*SQRT(XO(I)*(1.-XO(I))))
RETURN
END

```

C	SUBROUTINE INTNC (XL, XU, NX, X, Y, Z, IERR )	6	4640
C	F R AN4ERSON 056 291 072 BLDG. 2 STATION 15	6	4650
C	INTEGRATION ROUTINE, NEWTON-COTES METHOD	6	4660
C	CALL INTNC (XL, XU, NX, X, Y, Z, IERR)	6	4670
C	XL LOWER LIMIT OF INTEGRATION	6	4680
C	XU UPPER LIMIT OF INTEGRATION	6	4690
C	NX NUMBER OF ITEMS IN X AND Y ARRAYS	6	4700
C	X ARRAY OF ABSCISSAS (INDEPENDENT VARIABLE)	6	4710
C	Y ARRAY OF ORDINATES (DEPENDENT VARIABLE)	6	4720
C	Z VALUE OF DEFINITE INTEGRAL	6	4730
C	XI ARRAY OF ABSCISSAS USED IN INTEGRATION	6	4740
C	YI ARRAY OF ORDINATES USED IN INTERPOLATION	6	4750
C	IERR ERROR INDICATOR, ZERO VALUE INDICATES PROPER	6	4760
C	SOLUTION, NONZERO INDICATES ERROR	6	4770
C	IXU INDICATOR FOR UPPER LIMIT	6	4780
C	DIMENSION X(1), Y(1), XI(5), YI(5)	6	4790
C	IXU = 0	6	4800
C	IERR = 0	6	4810
C	TEST TO BE WITHIN RANGE	6	4820
C	1 IF (X(1) - XL)10,10,5	6	4830
C	5 WRITE ( 6, 7) XL, X(1)	6	4840
C	7 FORMAT ( 1H0 E15.8, 66H, LOWER LIMIT OF INTEGRATION IS LESS THAN F6	6	4850
C	1 FIRST ABSCISSA OF ARRAY, E17.8 )	6	4860
C	GO TO 110	6	4870
C	10 IF (XU - X(NX)) 15,15,12	6	4880
C	12 WRITE ( 6, 14) XU, X(NX)	6	4890
C	14 FORMAT (1H0 E15.8, 68H, UPPER LIMIT OF INTEGRATION IS GREATER THAN6	6	4900
C	1 LAST ABSCISSA OF ARRAY, E17.8)	6	4910
C	GO TO 110	6	4920
C	INITIALIZE	6	4930
C	15 Z = 0.0	6	4940
C	XK = 1.0	6	4950
C	FIND FIRST INTERVAL	6	4960
C	DO 30 I = 1, NX	6	4970
C	IF (X(I) - XL) 30,45,40	6	4980
C	30 CONTINUE	6	4990
C	SETUP FOR FIRST INTERVAL	6	5000
C	40 DX = X(I) - XL	6	5010
C	XI(1) = XL	6	5020

XI(5) = X(I)  
YI(5) = Y(I)

NI = 4

K1 = 1

GO TO 60

LOWER LIMIT SAME AS A TABULAR VALUE

45 I = I + 1

SETUP FOR SUCCEEDING INTERVALS

50 XI(1) = X(I-1)

YI(1) = Y(I-1)

XI(5) = X(I)

YI(5) = Y(I)

NI = 3

K1 = 2

DX = X(I) - X(I-1)

INTERMEDIATE POINTS IN INTERVAL

60 XI(2) = XI(1) + .25 \* DX

XI(3) = XI(1) + .5 \* DX

XI(4) = XI(1) + .75 \* DX

OBTAIN INTERPOLATED ORDINATES

70 CALL CODIS1 (NI, XI(K1), YI(K1), X, Y, NX, XK)

NEWTON-COTES FIVE POINT FORMULA

80 Z = Z + (DX / 90.0) \* (7.0 \* (YI(1) + YI(5)) + 32.0 \* (YI(2) + YI(4)) + 12.0 \* YI(3))

PREPARE FOR NEXT INTERVAL

90 I = I + 1

TEST FOR LAST TABULAR VALUE

IF (NX - I) 100, 94, 94

TEST FOR LAST INTERVAL

94 IF (XU - X(I)) 120, 120, 50

110 IERR = 1

TEST FOR COMPLETION

100 RETURN

120 IF (IXU) 100, 130, 100

SETUP FOR LAST INTERVAL

130 IXU = 1

XI(5) = XU

XI(1) = X(I-1)

YI(1) = Y(I-1)

6 5030

6 5040

6 5050

6 5060

6 5070

6 5080

6 5090

6 5100

6 5110

6 5120

6 5130

6 5140

6 5150

6 5160

6 5170

6 5180

6 5190

6 5200

6 5210

6 5220

6 5230

6 5240

6 5250

6 5260

6 5270

6 5280

6 5290

6 5300

6 5310

6 5320

6 5330

6 5340

6 5350

6 5360

6 5370

6 5380

6 5390

6 5400

6 5410

NI = 4  
K1 = 2  
DX = XU - X(I-1)  
GO TO 60  
END

6 5420  
6 5430  
6 5440  
6 5450  
6 5460

SUBROUTINE DUMP  
PRINT 6  
FORMAT(\*0SUBROUTINE DUMP\*)  
STOP  
END

6 5470  
6 5480  
6 5490  
6 5500  
6 5510

C	F	R	ANDERSON 056 291 072	BLDG. 21	STATION 15	6	5520	
C			CONTROLLED DEVIATION	INTERPOLATION SUBROUTINE...	CODIS1	6	5530	
C						6	5540	
C			CALLING SEQUENCE			6	5550	
C						6	5560	
C						6	5570	
C			CALL CODIS1(N1,X,Y,XI,YI,N2,XK)			6	5580	
C			N1 = NO. OF ARGUMENTS X			6	5590	
C			X = ARGUMENTS-ABSCISSA VALUES.			6	5600	
C			Y = INTERPOLATED ORDINATES.			6	5610	
C			XI = ARRAY OF THE ABSCISSAE.			6	5620	
C			YI = ARRAY OF THE ORDINATES.			6	5630	
C			N2 = NO. OF POINTS ON CURVE.			6	5640	
C			XK = END INTERVAL CONTROL CONSTANT ( 0 TO 1.0 )			6	5650	
C						6	5660	
			DIMENSION X(1),Y(1),XI(1),YI(1), D(2),A(2),B(2),C(2)				6	5670
			100 IN = 0			6	5680	
			DO 800 N = 1,N1			6	5690	
			IF(N2-2)110,115,120			6	5700	
			110 Y(N) = YI(N2)			6	5710	
			GO TO 800			6	5720	
			115 Y(N) = (YI(2)-YI(1))/(XI(2)-XI(1))* (X(N)-XI(1))+YI(1)			6	5730	
			GO TO 800			6	5740	
			120 J = 1			6	5750	
			125 IF(XI(J)-X(N))130,140,150			6	5760	
			130 J = J+1			6	5770	
			IF(J-N2)125,125,145			6	5780	
			140 Y(N) = YI(J)			6	5790	
			GO TO 800			6	5800	
			145 Y(N) = (YI(N2)-YI(N2-1))/(XI(N2)-XI(N2-1))*(X(N)-XI(N2-1))+YI(N2-1)			6	5810	
			GO TO 800			6	5820	
			150 IF(J-2)115,155,160			6	5830	
			155 K = 2			6	5840	
			JJ = 1			6	5850	
			GO TO 185			6	5860	
			160 IF(J-N2)170,165,145			6	5870	
			165 K = N2-1			6	5880	
			JJ = 2			6	5890	
			GO TO 185			6	5900	

```

170 IF(J-IN)180,300,180
180 JJ = 3
    K = J
185 DO 200 M = 1,2
    X1 = XI(K-1)-XI(K)
    X2 = XI(K)-XI(K-2)
    X3 = XI(K-2)-XI(K-1)
    Y1 = YI(K-1)-YI(K)
    Y2 = YI(K)-YI(K-2)
    Y3 = YI(K-2)-YI(K-1)
    XX1 = XI(K-2)**2
    XX2 = XI(K-1)**2
    XX3 = XI(K)**2
    D(M) = XX1*X1 + XX2*X2 + XX3*X3
    A(M) = (YI(K-2)*X1 + YI(K-1)*X2 + YI(K)*X3)/D(M)
    B(M) = (XX1*Y1 + XX2*Y2 + XX3*Y3)/D(M)
    C(M) = YI(K-2) - A(M)*XX1 - B(M)*XI(K-2)
    200 K = K+1
    300 P1 = X(N)*(A(1)*X(N)+B(1)) + C(1)
    P2 = X(N)*(A(2)*X(N)+B(2)) + C(2)
    AL = (X(N)-XI(J-1))/(XI(J)-XI(J-1))
    S = YI(J)*AL + YI(J-1)*(1.0-AL)
    GO TO (320,330,350),JJ
    320 P2 = P1
    AL = (X(N)-XI(1))/(XI(2)-XI(1))
    S = AL*YI(2) + (1.0-AL)*YI(1)
    P1 = S + XK*(P2-S)
    GO TO 350
    330 P1 = P2
    AL = (X(N)-XI(N2-1))/(XI(N2)-XI(N2-1))
    S = AL*YI(N2) + (1.0-AL)*YI(N2-1)
    P2 = S + XK*(P1-S)
    350 E1 = ABS(P1-S)
    E2 = ABS(P2-S)
    IN = J
    IF(E1+E2)700,750,750
    700 Y(N) = S
    GO TO 800
    750 BT = (E1*AL)/(E1*AL+(1.0-AL)*E2)

```

Y(N)=BT\*P2+(1.0-BT)\*P1  
800 CONTINUE  
900 RETURN  
END

6 6300  
6 6310  
6 6320  
6 6330

```

OVERLAY(OVL,07,0)
PROGRAM TRAIL
C MAIN PROGRAM FOR FREE TRAILING VORTICES
  DIMENSION XTV(1000),YTV(1000),ZTV(1000),XTVI( 500),YTVI(500)
1    ,ZTVI(500),UTV(1000),VTV(1000),WTV(1000),V1(100),V2(100)
2    ,V3(100),NIP(100),NTVE(100)
  EQUIVALENCE(B(15001),UTV), (B(16001),VTV), (R(17001),YTVI)
1    , (B(18001),XTVI), (R(19001),YTVI), (B(20001),ZTVI)
2    , (B(21001),XTVI), (R(21501),YTVI), (B(22001),ZTVI)
3    , (B(22501),V1), (R(22601),V2), (R(22701),V3)
4    , (B(22801),NIP), (B(22901),NTVE)
5    , (B(23001),NV)
  EQUIVALENCE(XVS,B(12871))
  DIMENSION XVS(100)
  COMMON/BODY/ B(25000)
  COMMON/SCRAT/ XQ(1000),YQ(1000),ZQ(1000)
  COMMON DA(5000)
  EQUIVALENCE (DA(7), XCG), (DA(8),YCG), (DA(9),ZCG), (DA(10),ALPHA)
1    , (DA(11),BETA), (DA(12),PSTAR), (DA(13),QSTAR), (DA(14),RSTAR)
C
C COMPUTE FREE TRAILING VORTEX POINTS, INCLUDING INITIAL POINTS.
C
  XMIN=XVS(100)
  XMAX=XTVI(1)
  WRITE(6,70) XMIN,XMAX
70  FORMAT(6HCTRAIL/(1P3E20.6))
  WRITE(6,65) (NIP(J),J=1,NV)
65  FORMAT(11HONIP ARRAY=1015)
  K=0
  DO 75 J=1,NV
  NN=NIP(J)
  DO 75 I=1,NN
  K=K+1
  WRITE(6,70) XTVI(K),YTVI(K),ZTVI(K)
  IF(XMAX.GT.XTVI(K)) GO TO 75
  XMAX=XTVI(K)
  CONTINUE
75  DXX=XMAX-XMIN
  DELX=DXX/20.0

```



	XMAX=XMIN+1.25*DXX	8	0390
	WRITE(6,5) XMAX,DXX,XMIN,DELX	8	0400
5	FORMAT(20H0XMAX,DXX,XMIN,DELX=1P4E20.6)	8	0410
	K=0	8	0420
	DO 50 I=1,NV	8	0430
	N1=0	8	0440
	IF(I-1) 15,15,10	8	0450
10	IM1=I-1	8	0460
	DO 11 II=1,IM1	8	0470
11	N1=NIP(II)+M1	8	0480
15	NP = NIP(II)	8	0490
	DO 20 J=1,NP	8	0500
	K=K+1	8	0510
	IJ=N1+J	8	0520
	XIV(K)=XIVI(NJ)	8	0530
	YIV(K)=YIVI(NJ)	8	0540
20	ZIV(K)=ZIVI(NJ)	8	0550
	DO 30 J=1,20	8	0560
	J1=J	8	0570
	K=K+1	8	0580
	XIV(K)=XIV(K-1)+DFLX	8	0590
	YIV(K)=YIV(K-1)	8	0600
	ZIV(K)=ZIV(K-1)	8	0610
	IF(XIV(K).GT.XMAX) GO TO 35	8	0620
30	CONTINUE	8	0630
35	CONTINUE	8	0640
50	NIVE(II)=NIP(II)+J1	8	0650
	WRITE(6,80)(XIV(I),YIV(I),ZIV(I),I=1,K)	8	0660
80	FORMAT(19H0TRAIL, XIV,YIV,ZIV/(1P3E20.6))	8	0670
C		8	0680
	COMPUTE POINTS AT WHICH VELOCITIES WILL BE FOUND IN SUB•TRAVEL.	8	0690
C		8	0700
	K=0	8	0710
	DO 100 II=1,NV	8	0720
	M=NIVE(II)	8	0730
	DO 90 J=1,M	8	0740
	K=K+1	8	0750
	K1=K+1	8	0760
	IF(J.EQ.M) GO TO 85	8	0770

	XQ(K)=0.5*(XTV(K1)+XTV(K))	8	0780
	YQ(K)=0.5*(YTV(K1)+YTV(K))	8	0790
	ZQ(K)=0.5*(ZTV(K1)+ZTV(K))	8	0800
	GO TO 90	8	0810
85	XQ(K)=XTV(K)	8	0820
	YQ(K)=YTV(K)	8	0830
	ZQ(K)=ZTV(K)	8	0840
90	CONTINUE	8	0850
100	CONTINUE	8	0860
	CALL TRAVEL(XQ,YQ,ZQ,UTV,VTV,WTV,K)	8	0870
	WRITE(6,105)(XQ(I),YQ(I),ZQ(I),UTV(I),VTV(I),WTV(I),I=1,K)	8	0880
105	FORMAT(28H0TRAIL. XQ,YQ,ZQ,UTV,VTV,WTV/(1P6E20.6))	8	0890
C	TEMPORARY CBAR,BREF. *****	8	0900
	CBAR=10.0	8	0910
	BREF=1.0	8	0920
C		8	0930
	DO 110 I=1,K	8	0940
	XX=XQ(I)-XCG	8	0950
	YY=YQ(I)-YCG	8	0960
	ZZ=ZQ(I)-ZCG	8	0970
	VX=1.0-2.0*(QSTAR*ZZ/CBAR-RSTAR*YY/BREF)	8	0980
	VY=-BETA-2.0*(PSTAR*ZZ-RSTAR*XX)/BREF	8	0990
	VZ=ALPHA+2.0*(PSTAR*YY/BREF+QSTAR*XX/CBAR)	8	1000
109	WRITE(6,109) I,VX,VY,VZ	8	1010
	FORMAT(15,1P3E16.4)	8	1020
	UTV(I)=UTV(I)+VX	8	1030
	VTV(I)=VTV(I)+VY	8	1040
110	WTV(I)=WTV(I)+VZ	8	1050
	CALL INTEG(XTV,YTV,ZTV,UTV,VTV,WTV,NV,NTVE)	8	1060
	WRITE(6,106)(XTV(I),YTV(I),ZTV(I),I=1,K)	8	1070
106	FORMAT(23H0INTEGRATED XTV,YTV,ZTV/(1P3E20.6))	8	1080
	END	8	1090
	SUBROUTINE INTEG(X,Y,Z,U,V,W,NTV,NTVE)	8	1100
	DIMENSION X(1),Y(1),Z(1),U(1),V(1),W(1),NTVE(1)	8	1110
	K=0	8	1120
	DO 10 I=1,NTV	8	1130
	NE=NTVE(I)	8	1140
	DO 5 J=2,NE	8	1150
	L2=J+K	8	1160

```

L1=L2-1
DELX=(X(L2)-X(L1))/U(L1)
Y(L2)=Y(L1)+DELX*V(L1)
Z(L2)=Z(L1)+DELX*W(L1)
5 CONTINUE
10 K=K+NE
RETURN
END
SUBROUTINE TRAVEL(XI,YI,ZI,UI,VI,WI,NPTS)
DIMENSION XI(1),YI(1),ZI(1),UI(1),VI(1),WI(1)
EQUIVALENCE(B(1),XV), (B(5001),YV), (B(1001),ZV)
DIMENSION XV(5000),YV(5000),ZV(5000)
COMMON/INDEX/ X,Y,Z,II1,II2,IF1,IF2
COMMON DA(5000)
COMMON/MATRIX/ NDUM(1000),XSOL(200)
COMMON/BODY/ B(25000)
EQUIVALENCE (B(1),XVR), (B(201),YVR), (B(401),ZVR), (B(601),XVO)
1 ,B(801),YVO), (B(1001),ZVO), (B(2871),SXMT)
2 ,B(3871),SYMT), (B(4871),SZMT), (B(5871),DYS)
3 ,B(6871),DZS), (B(7871),TS), (B(8871),XSS)
4 ,B(9871),YSS), (B(10871),ZSS)
5 ,B(11871),SIGNA), (B(12871),XVS), (B(12971),YVS)
6 ,B(13071),ZVS)
DIMENSION XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1 ,YVO(10,20),ZVO(10,20),SXMT(1000),SYMT(1000),SZMT(1000)
2 ,DYS(1000),DZS(1000),TS(1000),XSS(1000),YSS(1000)
3 ,ZSS(1000),SIGMA(1000),XVS(100),YVS(100),ZVS(100)
DIMENSION BODYG(15000),PANELG(13170)
EQUIVALENCE(B(5),BODYG,PANELG)
COMMON/SCRAT/ XQ(1000),YQ(1000),ZQ(1000),AXB(200),AYB(200)
1 ,SUM(3),TRSUMX(100),TRSUMY(100),TRSUMZ(100),AZB(200)
2 ,SUM1(3)
COMMON/NUMBER/ NVPTS(5),NCPTS(5),NLN(5),NLT(5),LNC(5)
1 ,NCT,NB,NBODS,NVL(5),NVT(5),NTAPE,NTAPE,NCTV,ITAPE,JTAPE
2 ,LSEG(5),TSEG(5),LFUNC(5),TFUNC(5)
3 ,LNDIVB(5),LTDIVB(5),NSPP(5),ROOTP(5),OUTERP(5)
4 ,SYM1(5)
COMMON/PANINF/PANSYM(10)
EQUIVALENCE(SYMP,DA(3426)),(SYMB,DA(19))

```

```

EQUIVALENCE(DA(2),PANS)
PI=3.1415926
PI4=12.5663704
NPANS=PANS
REWIND 18
LNDIV=1
DO 2 I=1,NPTS
  UI(I)=0.0
  VI(I)=0.0
  WI(I)=0.0
  IF(NBODS.EQ.0) GO TO 190
  DO 65 KK=1,NBODS
    READ(18) BODYG
    SYMB=SYMM(KK)
    LTDIV=LTDIVB(KK)
    NTVV=NVTT(KK)
    NBVV=NVL(KK)
    DO 63 M=1,NPTS
      X=XI(M)
      Y=YI(M)
      Z=ZI(M)
      U=0.0
      V=0.0
      W=0.0
      N=0
      DO 60 J=1,NTVV
        DO 60 I=1,NBVV
          N=N+1
          TOT1=0.0
          TOT2=0.0
          TOT3=0.0
          DO 55 K=1,4
            DO 8 KS=1,3
              SUM(KS)=0.0
              GO TO (10,20,30,39),K
            IF(I.EQ.1) GO TO 12
            DO 11 KS=1,3
              SUM(KS)=-SUM1(KS)
              GO TO 50

```

```

8 1560
8 1570
8 1580
8 1590
8 1600
8 1610
8 1620
8 1630
8 1640
8 1650
8 1660
8 1670
8 1680
8 1690
8 1700
8 1710
8 1720
8 1730
8 1740
8 1750
8 1760
8 1770
8 1780
8 1790
8 1800
8 1810
8 1820
8 1830
8 1840
8 1850
8 1860
8 1870
8 1880
8 1890
8 1900
8 1910
8 1920
8 1930
8 1940

```

12	III=1	8	1950
	IF1=1	8	1960
	II2=(J-1)*LTDIV	8	1970
	DO 14 L=1,LTDIV	9	1980
	II2=II2+1	8	1990
	IF2=II2+1	8	2000
14	CALL VORTEX(SUM)	8	2010
	GO TO 50	8	2020
20	II2=1+J*LTDIV	8	2030
	IF2=II2	8	2040
	II1=(I-1)*LNDIV	8	2050
	DO 24 L=1,LNDIV	8	2060
	II1=II1+1	8	2070
	IF1=II1+1	8	2080
24	CALL VORTEX(SUM)	8	2090
	SX=SUM(1)	8	2100
	SY=SUM(2)	8	2110
	SZ=SUM(3)	8	2120
	GO TO 50	8	2130
30	II1=IF1	8	2140
	II2=II2+1	8	2150
	DO 34 L=1,LTDIV	8	2160
	II2=II2-1	8	2170
	IF2=II2-1	8	2180
	IF(I.NE.NBVV) GO TO 33	8	2190
	DO 32 KS=1,3	8	2200
32	SUM(KS)=0.0	8	2210
	GO TO 34	8	2220
33	CALL VORTEX(SUM)	8	2230
34	CONTINUE	8	2240
	DO 35 KS=1,3	8	2250
35	SUM1(KS)=SUM(KS)	8	2260
	GO TO 50	8	2270
39	IF(J.EQ.1) GO TO 42	8	2280
	SUM(1)=-TRSUMX(I)	8	2290
	SUM(2)=-TRSUMY(I)	8	2300
	SUM(3)=-TRSUMZ(I)	8	2310
	GO TO 50	8	2320
42	II2=IF2	8	2330

	III=III+1	8	2340
	DO 44 L=1,LNDIV	8	2350
	III=III-1	8	2360
	IF1=III-1	8	2370
44	CALL VORTEX(SUM)	8	2380
50	TOT1=TOT1+SUM(1)	8	2390
	TOT2=TOT2+SUM(2)	8	2400
	TOT3=TOT3+SUM(3)	8	2410
55	CONTINUE	8	2420
	TRSUMX(I)=SX	8	2430
	TRSUMY(I)=SY	8	2440
	TRSUMZ(I)=SZ	8	2450
	AXB(N)=TOT1/PI4	8	2460
	AYB(N)=TOT2/PI4	8	2470
	AZB(N)=TOT3/PI4	8	2480
60	CONTINUE	8	2490
	DO 62 II=1,N	8	2500
	U=U+AXB(II)*XSOL(II)	8	2510
	V=V+AYB(II)*XSOL(II)	8	2520
62	W=W+AZB(II)*XSOL(II)	8	2530
	UI(M)=U+UI(M)	8	2540
	VI(M)=V+VI(M)	8	2550
63	WI(M)=W+WI(M)	8	2560
65	CONTINUE	8	2570
190	IF(NPANS.EQ.0) GO TO 405	8	2580
	DO 200 KK=1,NPANS	8	2590
	READ(18) PANELG	8	2600
	SYMP=PANSYM(KK)	8	2610
	NSPACE=NSPP(KK)	8	2620
	NTVV=NVT(NBODS+KK)	8	2630
	N3VV=NVL(NBODS+KK)	8	2640
	IF(KK-1) 191,191,192	8	2650
191	NPPV=0	8	2660
	GO TO 193	8	2670
192	KK1=K-1	8	2680
	NB1=NVL(NBODS+KK1)	8	2690
	NT1=NVT(NBODS+KK1)	8	2700
	NPPV=NT1*NB1	8	2710
193	CONTINUE	8	2720

DO 198 M=1,NPTS

X=XI(M)

Y=YI(M)

Z=ZI(M)

U=J.0

V=0.0

W=0.0

KVOR=0

DO 195 I=1,NTIV

IS=I-NSPACE

DO 195 J=1,NBVV

J1=J+1

KVOR=KVOR+1

IF (IS.LE.0) GO TO 75

IT=(I-1)\*NBVV\*2

IT1=IT+1+(J-1)\*2

IT12=IT1+2

T1=TS(IT12)

T2=TS(IT1)

CONTINUE

DO 5 K1=1,3

SUM(K1)=0.0

DO 41 K=1,4

IF (J.EQ.NBVV.AND.K.EQ.4) GO TO 41

IF (K.GT.1) GO TO 21

IF (IS.LE.0) GO TO 76

X2=XVO(IS,J) -T2\*SQR(DYS(IT1)\*\*2+DZS(IT1)\*\*2)\*0.5

Y2=YVO(IS,J) -0.5\*DYS(IT1)

Z2=ZVO(IS,J) -0.5\*DZS(IT1)

IF (J.EQ.NBVV) GO TO 305

X1=XVO(IS,J1)-T1\*SQR(DYS(IT12)\*\*2+DZS(IT12)\*\*2)\*0.5

Y1=YVO(IS,J1)-0.5\*DYS(IT12)

Z1=ZVO(IS,J1)-0.5\*DZS(IT12)

X3=X1

Y3=Y1

Z3=Z1

GO TO 40

X1=XVS(IS)

Y1=YVS(IS)

8 2730

8 2740

8 2750

8 2760

8 2770

8 2780

8 2790

8 2800

8 2810

8 2820

8 2830

8 2840

8 2850

8 2860

8 2870

8 2880

8 2890

8 2900

8 2910

8 2920

8 2930

8 2940

8 2950

8 2960

8 2970

8 2980

8 2990

8 3000

8 3010

8 3020

8 3030

8 3040

8 3050

8 3060

8 3070

8 3080

8 3090

8 3100

8 3110

75

5

305

	Z1=ZVS(IS)	8	3120
	X3=X1	8	3130
	Y3=Y1	8	3140
	Z3=Z1	8	3150
	GO TO 40	8	3160
76	CONTINUE	8	3170
	X1=XVR(I,J+1)	8	3180
	Y1=YVR(I,J+1)	8	3190
	Z1=ZVR(I,J+1)	8	3200
	X2=XVR(I,J)	8	3210
	Y2=YVR(I,J)	8	3220
	Z2=ZVR(I,J)	8	3230
	GO TO 40	8	3240
21	X1=X2	8	3250
	Y1=Y2	8	3260
	Z1=Z2	8	3270
	IF(K-3) 25,36,38	8	3280
25	IF(I.GT.NSPACE) GO TO 251	8	3290
	X2=XVR(I+1,J)	8	3300
	Y2=YVR(I+1,J)	8	3310
	Z2=ZVR(I+1,J)	8	3320
	GO TO 40	8	3330
251	X2=XVO(IS,J)+0.5*SQR(DYS(IT1)**2+DZS(IT1)**2)*T2	8	3340
	Y2=YVO(IS,J)+0.5*DYS(IT1)	8	3350
	Z2=ZVO(IS,J)+0.5*DZS(IT1)	8	3360
	GO TO 40	8	3370
36	IF(I.GT.NSPACE) GO TO 301	8	3380
	X2=XVR(I+1,J+1)	8	3390
	Y2=YVR(I+1,J+1)	8	3400
	Z2=ZVR(I+1,J+1)	8	3410
	GO TO 40	8	3420
301	IF(J.EQ.NBV) GO TO 303	8	3430
	X2=XVO(IS,J1)+0.5*SQR(DYS(IT12)**2+DZS(IT12)**2)*T1	8	3440
	Y2=YVO(IS,J1)+0.5*DYS(IT12)	8	3450
	Z2=ZVO(IS,J1)+0.5*DZS(IT12)	8	3460
	GO TO 40	8	3470
303	X2=XVS(IS+1)	8	3480
	Y2=YVS(IS+1)	8	3490
	Z2=ZVS(IS+1)	8	3500



38	GO TO 40	8	3510
	IF(I.GT.NSPACE) GO TO 351	8	3520
	X2=XVR(I,J+1)	9	3530
	Y2=YVR(I,J+1)	8	3540
	Z2=ZVR(I,J+1)	8	3550
	GO TO 40	8	3560
351	X2=X3	8	3570
	Y2=Y3	8	3580
	Z2=Z3	8	3590
40	IF(J.EQ.NEUV.AND.K.EQ.4) GO TO 4111	8	3600
401	CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,X,Y,Z)	8	3610
41	CONTINUE	8	3620
	GO TO 417	8	3630
4111	CONTINUE	8	3640
	Y2H=Y2	8	3650
	Y1H=Y1	8	3660
	SYMPAS=-1.0	8	3670
C	LEFT SEMI-INFINITE VORTEX LINE.	8	3680
410	T1=SQR((Y2-Y)**2+(Z2-Z)**2)	8	3690
	IF(T1-0.00001) 412,411,411	8	3700
411	CONTINUE	8	3710
	T2=(X2-X)/SQR((X2-X)**2+(Y2-Y)**2+(Z2-Z)**2)	8	3720
	QT=0.25*(1.0-T2)/(PI*T1)	8	3730
	SUM(2)=SUM(2)+SYMPAS*QT*(Z2-Z)/T1	8	3740
	SUM(3)=SUM(3)-SYMPAS*QT*(Y2-Y)/T1	8	3750
C	RIGHT SEMI-INFINITE VORTEX LINE.	8	3760
412	CONTINUE	8	3770
	T1=SQR((Y1-Y)**2+(Z1-Z)**2)	8	3780
	IF(T1-0.00001) 414,413,413	8	3790
413	CONTINUE	8	3800
	T2=(X1-X)/SQR((X1-X)**2+(Y1-Y)**2+(Z1-Z)**2)	8	3810
	QT=0.25*(1.0-T2)/(PI*T1)	8	3820
	SUM(2)=SUM(2)-SYMPAS*QT*(Z1-Z)/T1	8	3830
	SUM(3)=SUM(3)+SYMPAS*QT*(Y1-Y)/T1	8	3840
414	CONTINUE	8	3850
	IF(SYMPAS.EQ.-1.0.AND.SYMP.EQ.0) GO TO 415	8	3860
	GO TO 416	8	3870
415	Y1=-Y1	8	3880
	Y2=-Y2	8	3890

416	SYMPAS=1.0	8	3900
	GO TO 410	8	3910
	Y1=Y1H	8	3920
	Y2=Y2H	8	3930
417	IF (IS.GT.0) GO TO 46	8	3940
	TERM=XSOL(KVOR+NPPV)	8	3950
	GO TO 48	8	3960
46	TERM=0.0	8	3970
	K1=(I-1)*NBVV	8	3980
	DO 47 I2=1,J	8	3990
47	TERM=TERM+XSOL(K1+I2+NPPV)	8	4000
48	U=U+SUM(1)*TERM	8	4010
	V=V+SUM(2)*TERM	8	4020
195	W=W+SUM(3)*TERM	8	4030
	UI(M)=UI(M)+U	8	4040
	VI(N)=VI(N)+V	8	4050
198	WI(M)=WI(M)+W	8	4060
200	CONTINUE	8	4070
	RETURN	8	4080
	DO 400 K=1,NPTS	8	4090
	X=XI(K)	8	4100
	Y=YI(K)	8	4110
	Z=ZI(K)	8	4120
	U=U.0	8	4130
	V=V.0	8	4140
	W=W.0	8	4150
	MA=0	8	4160
	MS=0	8	4170
	DO 300 I=1,NTVV	8	4180
	DO 300 J=1,NBVV	8	4190
	DO 300 IS=1,2	8	4200
	MS=MS+1	8	4210
	KSOL=1	8	4220
	T=TS(MS)	8	4230
	DZ=DZS(MS)	8	4240
	DY=DYS(MS)	8	4250
300	CALL PVSK(T,DY,DZ,X,Y,Z,TS,MA,I,J,KSOL)	8	4260
	DC 250 KV=1,MS	8	4270
	U=U+XXMT(KV)*SIGMA(KV)	8	4280

350	V=V+SYMT(KV)*SIGMA(KV)	8	4290
	W=W+SZMT(KV)*SIGMA(KV)	8	4300
	UI(K)=UI(K)+U	8	4310
400	VI(K)=VI(K)+V	8	4320
405	WI(K)=WI(K)+W	8	4330
	CONTINUE	8	4340
	RETURN	8	4350
	END	8	4360
	SUBROUTINE VORTEX(SUM)	8	4370
	COMMON/BODY/ XV(151,31),YV(151,31),ZV(151,31)	8	4380
	COMMON/COMPTS/ XQ(1320),YQ(1320),ZQ(1320)	8	4390
	COMMON DA(5000)	8	4400
	EQUIVALENCE (DA(19),SYM)	8	4410
	COMMON/INDEX/ X,Y,Z,I1,I2,I12,IF1,IF2	8	4420
	DIMENSION SUM(1)	8	4430
	XI=XV(I1,I12)	8	4440
	YI=YV(I1,I12)	8	4450
	ZI=ZV(I1,I12)	8	4460
	XF=XV(IF1,IF2)	8	4470
	YF=YV(IF1,IF2)	8	4480
	ZF=ZV(IF1,IF2)	8	4490
	XFQ=XF-X	8	4500
	ZFQ=ZF-Z	8	4510
	XIQ=XI-X	8	4520
	ZIQ=ZI-Z	8	4530
	SYMLOO=1.0	8	4540
10	YFQ=YF-Y	8	4550
	YIQ=YI-Y	8	4560
	DELX=XF-XI	8	4570
	DELY=YF-YI	8	4580
	DELZ=ZF-ZI	8	4590
	RXS1=YFQ*DELZ-ZFQ*DELY	8	4600
	RXS2=ZFQ*DELX-XFQ*DELZ	8	4610
	RXS3=XFQ*DELY-YFQ*DELX	8	4620
	RXS =SQRT(RXS1**2+RXS2**2+RXS3**2)	8	4630
	TERM1=SQRT(DELX**2+DELY**2+DELZ**2)	8	4640
	TERM2=SQRT(XFQ**2+YFQ**2+ZFQ**2)	8	4650
	TERM3=SQRT(XIQ**2+YIQ**2+ZIQ**2)	8	4660
	TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ	8	4670

RATIO = TERM4/TERM1**2	8	4680
COSA=(DELX*XIQ+DELY*YIQ+DELZ*ZIQ)/(TERM1*TERM3)	8	4690
COSB = TERM4/(TERM1*TERM2)	8	4700
CC=COSB-COSA	8	4710
HX=XFQ-RATIO*DELX	8	4720
HY=YFQ-RATIO*DELY	8	4730
HZ=ZFQ-RATIO*DELZ	8	4740
H=SQRT(HX*HX+HY*HY+HZ*HZ)	8	4750
IF(H-0.00001) 11,12,12	8	4760
COEF=0.0	8	4770
GO TO 13	8	4780
CONTINUE	8	4790
HRXS=H*RXS	8	4800
COEF=SYML00*CC/HRXS	8	4810
CONTINUE	8	4820
SUM(1)=COEF*RXS1 + SUM(1)	8	4830
SUM(2)=COEF*RXS2 + SUM(2)	8	4840
SUM(3)=COEF*RXS3 + SUM(3)	8	4850
IF(SYML00.EQ.-1.0.OR.SYM.NE.0.0) RETURN	8	4860
SYML00=-1.0	8	4870
YI=-YI	8	4880
YF=-YF	8	4890
GO TO 10	8	4900
END	8	4910
SUBROUTINE VORPAN(SUM,XI,YI,ZI,XF,YF,ZF,X,Y,Z)	8	4920
COMMON DA(5000)	8	4930
EQUIVALENCE(DA(3426),SYM)	8	4940
DIMENSION SUM(1)	8	4950
R4PI=0.07957747	8	4960
YIH=YI	8	4970
YFH=YF	8	4980
XFQ=XF-X	8	4990
ZFQ=ZF-Z	8	5000
XIQ=XI-X	8	5010
ZIQ=ZI-Z	8	5020
SYML00=1.0	8	5030
YFQ=YF-Y	8	5040
YIQ=YI-Y	8	5050
DELX=XF-XI	8	5060

DELY=YF-YI	8	5070
DELZ=ZF-ZI	8	5080
RXS1=YFQ*DELZ-ZFQ*DELY	8	5090
RXS2=ZFQ*DELX-XFQ*DELZ	8	5100
RXS3=XFQ*DELY-YFQ*DELX	8	5110
RXS =SQRTRXS1**2+RXS2**2+RXS3**2)	8	5120
TERM1=SQRTERM1**2+DELY**2+DELZ**2)	8	5130
TERM2=SQRTERM2**2+YFQ**2+ZFQ**2)	8	5140
TERM3=SQRTERM3**2+YIQ**2+ZIQ**2)	8	5150
TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ	8	5160
RATIO = TERM4/TERM1**2	8	5170
COSA=(DELX*YIQ+DELY*YIQ+DELZ*ZIQ)/(TERM1*TERM3)	8	5180
COSB = TERM4/(TERM1*TERM2)	8	5190
CC=COSB-COSA	8	5200
HX=XFQ-RATIO*DELX	8	5210
HY=YFQ-RATIO*DELY	8	5220
HZ=ZFQ-RATIO*DELZ	8	5230
H=SQRTHX*HX+HY*HY+HZ*HZ)	8	5240
IF(H-0.00001) 11,12,12	8	5250
COEF=0.0	8	5260
GO TO 13	8	5270
CONTINUE	8	5280
HRXS=H*RXS	8	5290
COEF=R4PI*SYML00*CC/HRXS	8	5300
CONTINUE	8	5310
SUM(1)=COEF*RXS1 + SUM(1)	8	5320
SUM(2)=COEF*RXS2 + SUM(2)	8	5330
SUM(3)=COEF*RXS3 + SUM(3)	8	5340
YI=YIH	8	5350
YF=YFH	8	5360
IF(SYML00.EQ.-1.0.OR.SYM.NE.0.0) RETURN	8	5370
SYML00=-1.0	8	5380
YI=-YI	8	5390
YF=-YF	8	5400
GO TO 10	8	5410
END	8	5420
SUBROUTINE PVSK(T,DYY,DZZ,XC,YC,ZC,MS,MA,IV,JV,KSOL)	8	5430
COMMON DA(5000)	8	5440
EQUIVALENCE(DA(3426),SYM)	8	5450

COMMON/PANINF/ PAUSYM(10)	8	5460
COMMON/SCRAT/XSOL(5000),ROUND(5000),AK(5000),AY(5000),AZ(5000)	8	5470
COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)	8	5480
1 ,YVC(10,20),ZVO(10,20),PLL(500),PLT(500),YSURV(100),CHCRD(100)	8	5490
2 ,XCVO( 20),XCCO( 20),XLE( 20),YLE( 20),ZLE( 20)	8	5500
3 ,XTE( 20),YTE( 20),ZTE( 20),SLE( 20),XJ( 20),YJ( 20),ZJ( 20)	3	5510
4 ,ETLE( 20),XVT( 50),YVT( 50),ZVT( 50),XR( 20),YR( 20),ZR( 20)	8	5520
5 ,SXIT(1000),SYIT(1000),SZIT(1000),DYS(1000),DZS(1000)	8	5530
6 ,TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)	8	5540
COMMON/PANEL/ NPAN,IPSY,IPC,NBVVP,NTVVP,LNCFP,LTCFP,LNCP,LTCP	8	5550
1 ,NPERPT,NSPACE,NATCH,NTRATT,NPRCLN,NPRCLT,NACTXC,NWCTET,NTHXC	8	5560
2 ,NTHET,NTIP,CHTIP,ROOT,OUTER,NNATT	8	5570
3 ,MPI,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10	8	5580
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LAC(7)	8	5590
1 ,NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,JIAPE	8	5600
2 ,LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)	8	5610
3 ,LNDIVB(7),LNDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)	8	5620
	8	5630
	8	5640
	8	5650
	8	5660
	8	5670
	8	5680
	8	5690
	8	5700
	8	5710
	8	5720
	8	5730
	8	5740
	8	5750
	8	5760
	8	5770
	8	5780
	8	5790
	8	5800
	8	5810
	8	5820
	8	5830
	8	5840

C KSOL=1 FOR SOURCE PTS. ONLY  
C KSOL=2 FOR BOTH SOURCE PTS. AND VORTEX POINTS.  
C MS=SUBSCRIPT OF SOURCE PT.  
C XC,YC,ZC --- CONTROL PT.  
C IV = LATERAL VORTEX SUBSCRIPT.  
C JV = LONGITUDINAL VORTEX SUBSCRIPT.  
C INSERT SPECIFICATION STATEMENTS HERE.  
REAL I1,I2,I3,I4  
YVV=0.5\*SORT(DYY\*\*2+DZZ\*\*2)  
YV2=2.0\*YVV  
PI=3.141592654  
YV=YVV  
GO TO (10,20),KSOL  
YK=YSS(MS)  
ZK=ZSS(MS)  
XK=XSS(MS)  
GO TO 25  
IF(MS-2\*(MS/2).EQ.0) GO TO 10  
YK=YVO(IV,JV)  
ZK=ZVO(IV,JV)  
XK=XVO(IV,JV)

25

CONTINUE

SUMX=0.0 8 5850

SUMY=0.0 8 5860

SUMZ=0.0 8 5870

UTSUM=0.0 8 5880

VTSUM=0.0 8 5890

WTSUM=0.0 8 5900

SIGN=1.0 8 5910

DZ=ZC-ZK 8 5920

X=XC-XK 8 5930

50

CONTINUE

DY=YC-SIGN\*YK

RY=DYY/YV2

RZ=DZZ/YV2

Y=RY\*DY+RZ\*DZ

Z=-RZ\*DY+RY\*DZ

R12=(Y+YV)\*\*2+Z\*\*2

R22=(X-T\*Y)\*\*2+Z\*\*2\*(1.0+T\*T)

R32=(Y-YV)\*\*2+Z\*\*2

R4= SQRT((X-T\*YV)\*\*2+(Y-YV)\*\*2+Z\*\*2)

R5= SQRT((X+T\*YV)\*\*2+(Y+YV)\*\*2+Z\*\*2)

I1=(X+T\*YV)/R5

I2=(Y+T\*X+YV\*(1.0+T\*\*2))/R5

I3=- (Y+T\*X-YV\*(1.0+T\*\*2))/R4

I4=(X-T\*YV)/R4

IF (ABS(Z).GT.YV2) GO TO 42

Z=0.0

R6D=(X+T\*YV)\*\*2+(Y+YV)\*\*2

R7D=(X-T\*YV)\*\*2+(Y-YV)\*\*2

RXTY=(X-T\*Y)\*\*2

R6=RXTY/R6D

R7=RXTY/R7D

IF (R6.GE.0.0075968656) GO TO 41

IF (R7.GE.0.0075968656) GO TO 41

IF (ABS(Y).LE.YV) GO TO 41

TERM1=ABS(1.0/R7D-1.0/R6D)\*0.5/P

GO TO 43

IF (ABS(Y).GT.YV) GO TO 42

IF (ABS(X-T\*Y).GE.0.25\*ABS(PLL(MA))) GO TO 42

41

5850

5860

5870

5880

5890

5900

5910

5920

5930

5940

5950

5960

5970

5980

5990

6000

6010

6020

6030

6040

6050

6060

6070

6080

6090

6100

6110

6120

6130

6140

6150

6160

6170

6180

6190

6200

6210

6220

6230

42	TERM1=0.0	8	6240
43	GO TO 43	8	6250
	TERM1=(I2+I3)/R22	8	6260
	CONTINUE	8	6270
	TERM2=(I1+I.0)/R12	8	6280
	TERM3=(I4+I.0)/R32	8	6290
	TERM4=1.0/R4-1.0/R5	8	6300
	P=SQRT(1.0+T*T)	8	6310
	EUS=(T*TERM4+(X-T*Y)*TERM1)/P	8	6320
	EVS=(TERM4-T*(X-T*Y)*TERM1)/P	8	6330
	EWS=P*Z*TERM1	8	6340
	US=0.25*EUS/PI	8	6350
	VS=0.25*EVS/PI	8	6360
	WS=0.25*EWS/PI	8	6370
	UT=US	8	6380
	VT=VS*RY-WS*RZ	8	6390
	WT=VS*RZ+WS*RY	8	6400
	UTSUM=UT+UTSUM	8	6410
	VTSUM=VT+VTSUM	8	6420
	WTSUM=WT+WTSUM	8	6430
	IF(KSOL.EQ.1) GO TO 45	8	6440
	EU=Z*TERM1	8	6450
	EV=Z*(-T*TERM1+TERM2-TERM3)	8	6460
	EW=-X-T*Y)*TERM1-(Y+YV)*TERM2+(Y-YV)*TERM3	8	6470
	UV=0.25*EU/PI	8	6480
	VV=0.25*EV/PI	8	6490
	WV=0.25*EW/PI	8	6500
	UI=UV	8	6510
	VI=RY*VV-RZ*WV	8	6520
	WI=RZ*VV+RY*WV	8	6530
	SUMX=UI+SUMX	8	6540
	SUMY=VI+SUMY	8	6550
	SUMZ=WI+SUMZ	8	6560
45	CONTINUE	8	6570
C	FOR SYMMETRY, GET IMAGE CONTRIBUTION.	8	6580
	IF(SYM.NE.0.0) GO TO 60	8	6590
	IF(SIGN.LT.0.0) GO TO 60	8	6600
	SIGN=-1.0	8	6610
	DZZ=-DZZ	8	6620



```

T=-T
GO TO 50
CONTINUE
SXMT(MS)=UTSUM
SYMT(MS)=VTSUM
SZMT(MS)=WTSUM
IF(KSOL.EQ.1) RETURN
AX(MA)=SUNX
AY(MA)=SUNY
AZ(MA)=SUNZ
RETURN
END

```

8	6630
8	6640
8	6650
8	6660
8	6670
8	6680
8	6690
8	6700
8	6710
8	6720
8	6730
8	6740

```

OVERLAY(OVL,10,0)
PROGRAM HARDAP
COMMON/SCRAT/A(20),DA(5000),ATTACH(5,10)
INTEGER UNIT,UNIT2
DATA UNIT2/21/
DATA UNIT/10/
REWIND UNIT
REWIND UNIT2
PRINT 3
READ 5, A
IF(A(1).EQ.4H .AND.A(2).EQ.4H .AND.A(3).EQ.4H 1) GOTO 60
CALL OUTIN (A)
GOTO 99
1 READ 5, A
60 IF(EOF, 5) 2, 372
372 PRINT 6, A
WRITE(UNIT, 5) A
GOTO 1
2 ENDFILE UNIT
REWIND UNIT
CALL DECRD(DA, UNIT)
NB = DA(1)
IF(NB.EQ.0) GOTO 20
DO 10 I=1, NB
10 CALL DECRD(DA, UNIT)
20 NP = DA(2)
IF(NP.EQ.0) GOTO 40
DO 30 I=1, NP
CALL DECRD(DA, UNIT)
ATTACH(1,I) = DA(3420)
ATTACH(2,I) = DA(3444)
ATTACH(3,I)=DA(3445)
ATTACH(4,I)=DA(3446)
30 ATTACH(5,I)=DA(3447)
40 REWIND UNIT
WRITE(UNIT2) ATTACH
WRITE(6,51) UNIT2
51 FORMAT(*,UNIT2=*13)
REWIND UNIT2

```

99 CONTINUE  
5 FORMAT(20A4)  
5 FORMAT(1H,20A4)  
3 FORMAT(1H1,30X,\*LISTING OF INPUT DATA CARDS\*/1H0)  
BLAINE D. GAITHER 11/72  
END

C

```

SUBROUTINE DECHRT(DA,IARG,MAX,IUNIT)
DIMENSION DA(1)
ISTART = IARG
444 IF(MAX - ISTART - 4) 1, 1, 2
1 MSTART = -ISTART
WRITE(IUNIT, 111) MSTART, (DA(I), I=ISTART, MAX)
PRINT 111, MSTART, (DA(I), I=ISTART, MAX)
111 FORMAT(112,5G12.5)
RETURN
2 MSTART = ISTART + 4
IF(CA(ISTART).EQ.0.0.AND.DA(ISTART+1).EQ.0.0.AND.DA(ISTART+2).EQ.0.0)
1.0.AND.DA(ISTART+3).EQ.0.0.AND.DA(ISTART+4).EQ.0.0) GOTO 3
WRITE(IUNIT, 111) ISTART, (DA(I), I=ISTART, MSTART)
PRINT 111, ISTART, (DA(I), I=ISTART, MSTART)
3 ISTART = ISTART + 5
GOTO 444
END

```

OUT 6710
OUT 6720
OUT 6730
OUT 6740
OUT 6750
OUT 6760
OUT 6770
OUT 6780
OUT 6790
OUT 6800
OUT 6810
OUT 6820
OUT 6830
OUT 6840
OUT 6850
OUT 6860
OUT 6870

SUBROUTINE OUTIN(TITLE)

CODED BY B. D. GAITHER

AT SOME LATER DATE ALL TAPE I/O SHOULD BE DONE BY BUFFER IN AND  
OUT, THEN ALL ZEROING LOOPS AND CALLS TO DECRO AND DECRRT COULD  
BE REMOVED. AT PRESENT THE MAIN ROUTINES EXPECT DECRO STYLE INPUT

INTEGER TITLE(20), NRADX(4), NFORX(4), OUNIT, OUNIT2, SCRICH

REAL DA(5000), XAF(30), XOAFLF(20), YOAFLF(20), ZOAFLF(20),

IAFSWCL(20), TZORD(30), WAFORD(410), XVXFXF(120), ZLCCS(120) OUT 0170

2, FCSEA(120), YOH5(30,120), ZOH5(30,120)

3,XOP, YOP, ZOP, PX(30), PODRAD(30), FINXL, OUT 0190

5FINYL, FINZL, LCHORD, FINXH, FINYH, FINZH, OUT 0200

6 HCHORD, XLC, YLC, ZLC, CONCRD, XHC, YHC, ZHC OUT 0210

7, CONCOH, CANPCL(10), CANPLN(10), CANS(10) OUT 0220

8,PCHORD(10), FAFHT(10), ATA(29), ZC(440),PPD(150) OUT 0230

9, PANARE(1580),BODARE(3405) OUT 0240

A, ATACH(5,10), NPPDP, NITXCS

B,NXSBOT,BCT(50),LCT(570),R3(800)

C,TT(29),ETA(29),NATA

D,BMFCXS(49)

E,ZLCCT(120),SR(30),S(39),ZCSA(39),SZ(120,30),SY(120,30) OUT 0290

DOUBLE PRECISION PI, RAD, RAD 1 DG

COMMON /PATACH/ ATACH

EQUIVALENCE(REFA,DA(16)),(PSI,DA(9))

1,(ATA(1),DA(3631)),(ANAS,DA(3630)),(ZC(1),DA(3660))

2,(XAF(1),DA(3601)),(XCNUM,DA(3600))

3,(REFAF,DA(3421))

4,(XPO,DA(3432)),(YPO,DA(3433)),(ZPO,DA(3434))

5,(AFSIM,DA(3426))

6,(XAF(1),PCHORD(1)),(WAFORD(1),DA(4190)),(WAFORD(1),FAFHT(1))

7, (PANARE(1),DA(3420)), (BODARE(1), DA(15)), (XCL,FINXL),

8 (YCL,FINYL), (ZCL,FINZL), (CONCRD, LCHORD), (XHC,FINXH),

9 (YHC,FINYH), (ZHC,FINZH), (CONCOH, HCHORD), (CANPCL(1), PCHORD(1))

A1), (CANPLN(1), FAFHT(1)), (PPD(1), DA(3450))

B, (NPPDP, DA(3443)), (NITXCS, DA(4130))

OUT 0400

OUT 0410

OUT 0420

OUT 0430

OUT 0440

OUT 0450

```

C,(ROX,DA(27)),(BOY,DA(28)),(BOZ,DA(29)),(IXSROT,DA(40)),
D(LCT(1),DA(130)),(RA(1),DA(800)),(PCT(1),DA(41))
E,(TI(1),DA(4131)),(ETA(1),DA(4161)),(NATA,DA(4160))
F,(BMFCXN,DA(1600)),(BMFCXS(1),DA(1601))
G,(RCTII,DA( 20)),(SN,DA(133)),(ZLCC(1),DA(1800)),
H(S(1),DA(131)),(BSYM,DA(19)),(PCONT,DA(3429))

DATA IUNIT/5/,CUNIT/10/,CUNIT2/20/,DA/5000*-0.0/,SCRATCH/18/
1,PI/ 3.1415 92653 58979 32384 62643 D+0/,
2 RAD 1 DG/.0174 53292 51994 32957 69237 D+07
3, ZLCCS/120*-0.0/

NAMELIST /JLIST/ J0, J1, J2, J3, J4, J5, J6, NNAF, NNAFOR, NFUS,
1RADX,NFORX,NP, NPQDOR, NF, NFINOR, NCAN, NCANOR
2/WLIST/ XAF, XQAFLE, YQAFLE, ZOAFLE, AFSWCL, SPAN, ATA, PPD
3,WAFORD
4/BLIST/ XVXFXF, ZLCCS, FCSC
5/PLIST/ XOP, YOP, ZOP, PX, PODRAD
6/FLIST/ FINXL, FINYL, FINZL, LCHORD, FINXH, FINYH, FINZH, HCHORD,
7 PCHORD, FAFHT
8/PLOTLT/ PHI,THETA, PSI
9/CLIST/ XLC, YLC, ZLC, CONCRD, XHC, YHC, ZHC, CONCHO, CANPCL,
ACANPLN, CANS
E/ATTACH/ATTACH

READ IDENTIFICATION CARD AND ECHO
PRINT 6
5 FORMAT(1H1, 30X,*BEGIN CONVERSION OF LANGLEY DATA*)
PRINT 2, TITLE
2 FORMAT(1X,20A4)

READ CONTROL INTEGERS
READ 3, J0, J1, J2, J3, J4, J5, J6, NNAF, NNAFOR, NFUS, (NRADX(1),OUT 0780
1NFORX(1), I=1, 4, 1), NP, NPQDOR, NF, NFINOR, NCAN, NCANOR
3 FORMAT(24I3)
PRINT JLIST

READ REFERENCE AREA CARD
IF(J0.EQ.1) READ 4, REFA

```



```

DO 102 I=2, NNAF
  ATA(I)=SQRT((YOAFL(I)-YOAFL(I-1))**2+(ZOAFL(I)-ZOAFL(I-1))**2)
  SPAN = ATA(I) + SPAN
102  ATA(I)=SPAN
C
C
  LOAD(AIRFOIL LENGTH / SPAN) INTO ARRAY ATA
DO 104 I=1, NNAF
  ATA(I)=ATA(I)/SPAN
104  ETA(I)=ATA(I)
C
C
  READ DELTA Z VALUES AND CONVERT TO Z/ CHORD LENGTH
  IF(J1.NE.1) GOTO 222
  I=1
DO 31 J=1, NNAF
  IF(AFSWCL(J).NE.0) GOTO 517
  PRINT 228, J
228  FORMAT('THE CHORD LENGTH OF WING AIRFOIL',I3,' IS ZERO, IT HAS BECOME
1EN SET TO .01*//')
  AFSWCL(J)=.01
517  READ 4, (TZORD(I),I=1, NNAF)
  PRINT 400, (TZORD(I), I=1, NNAF)
400  FORMAT('TZORD',3(/10G12.5))
DO 31 IT = 1, NNAF
  ZC(I)=TZORD(IT)/AFSWCL(J)
31  I=I+1
C
C
  PANEL (WING) ORIGIN
222  XPO = 0.0
  YPO = 0.0
  ZPO = 0.0
C
C
  PANEL PERIMETER DESCRIPTION (LEADING EDGE)
  J=1
DO 150 I=1, NNAF
  PPD(J)=XOAFL(I)
  PPD(J+1)=YOAFL(I)
  PPD(J+2)=ZOAFL(I)
  PPD(J+3) = AFSWCL(I)
150  J = J + 4

```



```
C C SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS  
C C NPPDP=NWAF OUT 1600  
C C OUT 1610  
C C OUT 1620  
C C OUT 1630  
C C SET NUMBER OF X/C STATIONS ENTRIES IN THE THICKNESS TABLE PER AIRFOIL  
C C NTTXCS=NWAFOR OUT 1640  
C C OUT 1650  
C C OUT 1660  
C C READ WING AIRFOIL ORIGINATE CARDS (HALF THICKNESS)  
C C JJ = 0 OUT 1670  
C C DO 22 J=1, NWAF OUT 1680  
C C IT = JJ+1 OUT 1690  
C C JJ = NWAFOR+JJ OUT 1700  
C C 22 READ 4, (WAFORD(I), I=IT, JJ) OUT 1710  
C C DO 516 I=1, JJ OUT 1720  
C C 516 WAFORD(I)=WAFORD(I)/100. OUT 1730  
C C PRINT WLST OUT 1740  
C C OUT 1750  
C C OUT 1760  
C C ***** OUT 1770  
C C START FUSELAGE DATA CARDS OUT 1780  
C C OUT 1790  
C C OUT 1800  
C C 101 IF(J2.EQ.0 .OR. NFUS.EQ.0) GOTO 171 OUT 1810  
C C OUT 1820  
C C SET BODY COORDINATE TABLE INPUT INDICATOR TO INDICATE A CIRCULAR OUT 1830  
C C BODY OR ARBITRARY BODY OUT 1840  
C C BCTII = 0.0  
C C BSYM = 1.0  
C C IF(J2.EQ.1) BCTII = 1.0 OUT 1860  
C C LLL = 0  
C C LL = 1  
C C DO 100 IX=1, NFUS  
C C NFO = NFORX(IX)  
C C LLL=LLL+NFO  
C C READ X STATIONS  
C C READ 4, (XVXF(I), I=LL, LLL)  
C C XVXF(LL) = XVXF(LL)+1.0E-7  
C C READ CAMBER(IF ANY)  
C C OUT 1910  
C C OUT 1920  
C C OUT 1930  
C C OUT 1940  
C C OUT 1950  
C C OUT 1960  
C C OUT 1970  
C C OUT 1980
```

```

IF(J2.EQ.-1.AND.J6.EQ.0) READ 4, (ZLCCS(I), I=LL, LLL)
IF(J2.NE.-1)GOTO 200
C
C
READ CROSS SECTIONAL AREAS(IF ANY)
READ 4, (FCSA(I), I=LL, LLL)
GOTO 100
200 IF (J2.NE.1) GOTO 100
C
C
READ Y S AND Z S OF ARBITRARY BODY
NFF=NRADX(IX)
DO 7044 M=LL, LLL
READ 4, (YOHs(I,M), I=1, NFF)
READ 4, (ZOHs(I,M), I=1, NFF)
PRINT 401, (YOHs(I,M), I=1, NFF)
PRINT 402, (ZOHs(I,M), I=1, NFF)
7044 CONTINUE
401 FORMAT(*YOHs*,3(/10G12.5))
402 FORMAT(5H0ZOHs,3(/10G12.5))
100 LL=LL+NFO
C
C
FIND AND SET TOTAL NUMBER OF X STATIONS
J=NFORX(1)+NFORX(2)+NFORX(3)+NFORX(4)
NXS = J
C
C
PUNCH FUSELAGE AND WING
CALL DECWRT( DA, 15, 3419, OUNIT)
PRINT BLIST
171 IF(IABS(J1).NE.1) GOTO 715
CALL DECWRT( DA, 3420, 5000,OUNIT)
C
C
ZERO BODY AREA OF STORAGE
715 DO 229 I=1, 3405
229 BODARE(I)=-0.0
C
C
ZERO PANEL SECTION OF STORAGE
DO 226 I=1, 1580
226 PANARE(I) =-0.0
C
C
*****
OUT 1990
OUT 2000
OUT 2010
OUT 2020
OUT 2030
OUT 2040
OUT 2050
OUT 2060
OUT 2070
OUT 2080
OUT 2090
OUT 2150
OUT 2160
OUT 2260
OUT 2270
OUT 2280
OUT 2290
OUT 2500
OUT 2510
OUT 2520
OUT 2530
OUT 2540
OUT 2550
OUT 2560
OUT 2570
OUT 2580
OUT 2590
OUT 2600
OUT 2610
OUT 2620
OUT 2630
OUT 2640
OUT 2650

```

C	READ POD DATA	OUT 2660
C		OUT 2670
C		OUT 2680
C	IF (J3.NE.1.OR.NP.EQ.0) GOTO 300	OUT 2690
C		OUT 2700
C	REWIND SCRATCH UNIT TO PUT PODS ON	OUT 2710
	REWIND SCRATCH	OUT 2720
	DO 290 IX=1, NP	OUT 2730
	READ 4, XOP, YOP, ZOP	OUT 2740
	READ 4, (PX(I), I=1,NPODOR)	OUT 2750
	READ 4, (PODRAD(I), I=1,NPODOR)	OUT 2760
	PRINT PLIST	OUT 2770
C	IS THIS A POD OR A NACELLE	OUT 2780
C	IF(PODRAD(1).LE.0)GOTO 123	OUT 2790
		OUT 2800
C	ITS A NACELLE	OUT 2810
C	SET REFERENCE AREA CARDS	OUT 2820
C	REFAF=REFAA	OUT 2830
C		OUT 2840
C	SET PANEL CONTOUR INDICATOR	
C	PCONT = 1.0	
C	SET PANEL ORIGIN	OUT 2850
C	XPO=XOP	OUT 2860
	YPO=YOP	OUT 2870
	ZPO=ZOP	OUT 2880
C		OUT 2890
C	FIND CHORD LENGTHS	OUT 2900
C	CHORD=PX(NPODOR)	OUT 2910
C		OUT 2920
C	PANEL PERIMETER DESCRIPTION(STEP BY TEN DEGREES)	OUT 2930
	J=1	OUT 2940
	DO 764 K=1, 361, 10	OUT 2950
	RAD = RAD 1 DG * FLOAT(K-1)	OUT 2960
	PPD(J)=PX(1)	OUT 2970
	PPD(J+1)=DSIN(RAD)*PODRAD(1)	OUT 2980
	PPD(J+2)=DCOS(RAD)*PODRAD(1)	OUT 2990
	PPD(J+3)=CHORD	OUT 3000
		OUT 3010

764 J=J + 4	OUT 3020
C	OUT 3030
C SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS	OUT 3040
NPPDP=37	OUT 3050
C	OUT 3060
C PERCENT CHORD LOCATIONS(X/CHORD LENGTH)	OUT 3070
XCNUM=NPODOR	OUT 3080
DO 746 I=1, NPODOR	OUT 3090
746 PCHORD(I)=PX(I)/CHORD	OUT 3100
C	OUT 3110
C SET ETA(ATA) STATIONS (AS WELL AS THEIR NUMBER)	OUT 3120
ANAS=2	OUT 3130
ATA(1)=0.0	OUT 3140
ATA( 2)=1.0	OUT 3150
C	OUT 3160
C CAMBER LINE = DELTA Z S (X S) / AIRFOIL STREAMWISE CHORD LENGTH	OUT 3170
DO 476 I=1, NPODOR	OUT 3180
ZC(I)=PODRAD(I)/CHORD	OUT 3190
476 ZC(NPODOR+I)=ZC(I)	OUT 3200
C	OUT 3210
C PUNCH PANEL	OUT 3220
CALL DECMRT(04,3420,5000,0UNIT)	OUT 3230
ZERO PANEL AREA	OUT 3240
DO 227 I=1, 1580	OUT 3250
227 PANARE(I)=-0.0	OUT 3260
GOTO 290	OUT 3270
C	OUT 3280
C ITS A POD	OUT 3290
C	OUT 3300
C SET BODY ORIGIN	OUT 3310
123 BOX=XOP	OUT 3320
BOY=YOP	OUT 3330
BOZ=ZOP	OUT 3340
C	OUT 3350
C SET BODY COORDINATE TABLE INPUT INDICATOR	OUT 3360
BCTII = 0.0	
BSYH =1.	
C	
C SET NUMBER OF X STATIONS IN BODY CO - ORDINATE TABLE	OUT 3380
C	OUT 3390

NXSBOT = NPODOR  
BMFCXN = NXSBOT

FILL IN BODY CO - ORDINATE TABLE

DO 321 I=1, NPODOR

BMFCXS(I)=PX(I)

321 BCT(I)=PX(I)

K=1

DO 456 I=1, NPODOR

FILL IN LATERAL CO - ORDINATE TABLE

SET RADII

RB(K)=PODRAD(I)

RB(K+1)=PODRAD(I)

456 K=K+2

LCT(1) = 2.0

LCT(2)=0.0

LCT(3)=180.0

SET REFERENCE AREA

REFA=REFAA

PUNCH BODY(POD)

CALL DECURT(14,15,3419,SCRATCH)

ZERO BODY AREA

DO 722 I=1, 3405

722 BODARE(I)=-0.0

290 CONTINUE

READ FIN DATA

300 IF(J4.NE.1.OR.NF.EQ.0) GOTO390

DO 380 IX=1, NF

READ 4, FINXL, FINYL, FINZL, LCHORD, FINXH,

1FINYH, FINZH, HCHORD

READ 4,(PCHORD(I), I=1, NFINOR)

OUT 3420

OUT 3430

OUT 3440

OUT 3450

OUT 3460

OUT 3480

OUT 3490

OUT 3500

OUT 3510

OUT 3560

OUT 3570

OUT 3580

OUT 3590

OUT 3600

OUT 3610

OUT 3620

OUT 3630

OUT 3640

OUT 3650

OUT 3660

OUT 3670

OUT 3680

OUT 3690

OUT 3700

OUT 3710

OUT 3720

OUT 3730

OUT 3740

OUT 3750

OUT 3760

OUT 3770

OUT 3780

OUT 3790

OUT 3800



```

C      SET CAMBER
DO 1052 I=1, J
1052 ZC(I)=(CANPLN(I)-CANS(I))/2.0
C
C      ADJUST THICKNESS TO BE RELATIVE TO CAMBER LINE
DO 1050 I=1, J
1050 CANPLN(I)=(CANPLN(I)+CANS(I))/2.0
C
C      PERCENT CHORD LOCATIONS(NUMBER OF ORDINATES USED TO DESCRIBE EACH
SECTION(THICKNESS TABLE STATIONS))
7094 XCNUM= J
NTTXCS=XCNUM
C      ZERO UNIVERSAL DATA STORAGE
DO 7093 I=1, J
7093 TT(I)=CANPCL(I)
PRINT CLIST
ASSIGN 500 TO LABEL
GOTO 1000
500 CONTINUE
C
C      *****
C      READ PLOT DATA
C
999 READ 7,PHI,THETA,PSII
7 FORMAT(7X,3F5.0)
PRINT PLOTIT
C
C      REWIND OUTPUT FILE FROM COMPUTER CONVERSION
C      REWIND OUNIT
C      ENDFILE SCRITCH
C      REWIND SCRITCH
C
C      *****
C      USER EDIT SECTION
C      EDIT UNIVERSAL INFO

```

```

OUT 4180
OUT 4190
OUT 4200
OUT 4210
OUT 4220
OUT 4230
OUT 4240
OUT 4250
OUT 4260
OUT 4270
OUT 4280
OUT 4290
OUT 4300
OUT 4310
OUT 4320
OUT 4330
OUT 4340
OUT 4350
OUT 4360
OUT 4370
OUT 4380
OUT 4390
OUT 4400
OUT 4410
OUT 4420
OUT 4430
OUT 4440
OUT 4450
OUT 4460
OUT 4470
OUT 4480
OUT 4490
OUT 4500
OUT 4510
OUT 4520
OUT 4530
OUT 4540
OUT 4550
OUT 4560

```

```

C      ZERO UNIVERSAL AREA OF STORAGE
      PRINT 8
      8  FORMAT(1H1,30X,*START EDIT*)
      DO 710 I=1, 14
710  DA(I)=-0.0
      CALL DECRD(DA,OUNIT)
      CALL DECRD(DA,IUNIT)
      CALL DECRD(DA,1,14,OUNIT2)
      ZERO BODY AREA STORAGE
      DO 711 I=1, 3405
711  BODARE(I)=-0.0
C
C      EDIT BODIES IF ANY
      NPOD = 0
      IF(J2.EQ. 0) GOTO 505
      CALL DECRD(DA,OUNIT)
      CALL DECRD(DA,IUNIT)
      J = NXS
      IF( J .GT. 44) GOTO 201

```

```

OUT 4570
OUT 4580
OUT 4590
OUT 4600
OUT 4610
OUT 4620
OUT 4630
OUT 4640
OUT 4650
OUT 4660
OUT 4670
OUT 4680
OUT 4690
OUT 4700
OUT 4710
OUT 4720
OUT 4730

```

```

C
C      FILL IN DATA FOR BODY THAT FITS AS IS
      NXSBOT = NXS
      BMFCXN = NXSBOT
      DO 7030 I=1, J
7030  BCT(I) = XVXFXF(I)
      ZLCCT(I) = ZLCCS(I)
      BMFCXS(I) = BCT(I)
      IF( J2 .NE. -1) GOTO 202
C
C      SET ANGLE OF CIRCULAR BODY
      LCT(1)=2.0
      LCT(2)=0.0
      LCT(3)=180.0
      L=1
      DO 7040 I=1, J
7040  RB(I) = DSORT(FCSA(I)/PI)
      RB(L+1) = RB(L)

```

```

C      FIND RADIUS OF CIRCULAR BODY
      RB(L) = DSORT(FCSA(I)/PI)
      RB(L+1) = RB(L)

```



7040 L = L + 2

GOTO 205

202 NM = 1

N = 0

LLL = 0

LL = 1

DO 110 IX=1, NFUS

NFF = NRADX(IX)

NZ = NFF

NFO = NFORX(IX)

LLL = LLL + NFO

DO 10 M=LL, LLL

LCT(MM)= NFF

DO 7090 I=1, NFF

C

SET Y AND Z VALUES

RB(N+I) = ZOHS(NZ,M)

LCT(MM+I) = YOHS(NZ,M)

7090 NZ = NZ - 1

N = N + NFF

10 MM = MM + 1 + NFF

110 LL = LL + NFO

GOTO 205

C

INTERPOLATE FOR OVER SPECIFIED DATA

201 J = NXSBOT

BMFCXN = NXSBOT

DO 217 I=1, J

217 BMFCXS(I) = BCT(I)

IF( J2 .NE. -1) GOTO 204

C

SET ANGLE OF CIRCULAR BODY

LCT(1) = 2.0

LCT(2) = 0.0

LCT(3) = 180.0

J=NXSBOT

L=1

C

FIND RADIUS OF CIRCULAR BODY

C

```

DO 203 I=1, J
  RB(L) = DSQRT(CODIM1(BCT(I), XVXFXF, FC5A, NX5, 0.0) / PI)
  RB(L+1) = RB(L)
C
C  FIND CAMBER
  ZLCCT(I) = CODIM1(BCT(I), XVXFXF, ZLCCS, NX5, 0.0)
  203 L = L + 2
  GOTO 205
  204 CONTINUE
C
C  FIND S BAR(CIRCUMFERENCE)
  LL = 1
  LLL = 0
  N = SN
  DO 206 IX=1, NFUS
    NFO = NFORX(IX)
    LLL = LLL + NFO
    NFF = NRADX(IX)
C
C  FILL AND NON DIMENSIONALIZE S MATRIX
  DO 208 M=LL, LLL
    SE(1) = 0.0
    DO 207 I=2, NFF
      207055(I) = SE(I-1) + SQRT((YCHS(I,M) - YCHS(I-1,I))**2 +
1      (ZCHS(I,M) - ZCHS(I-1,I))**2)
2      + 1.0E-7
      T = SE(NFF)
    DO 209 I=2, NFF
      209 SE(I) = SE(I) / T
    DO 210 I=1, N
      FC5A(I) = CODIM1(S(I), 50, YCHS(1,M), NFF, 0.0)
      210 ZC5A(I) = CODIM1(S(I), 50, ZCHS(1,M), NFF, 0.0)
      IN = N
    DO 215 I=1, N
      YCHS(I,H) = FC5A(IN)
      ZCHS(I,H) = ZC5A(IN)
      215 IN = IN - 1
    206 CONTINUE
    206 LL = LL + NFO

```

```

DO 211 I=1, N
DO 211 N=1, NXS
SZ(M,I) = ZOHS(I,M)
211 SY(M,I) = YOHS(I,M)
J = NXSBOT
L = 1
K = 0
DO 212 I=1, J
LCF(L) = N
DO 213 M=1, N
LCF(L+M) = CODIM1(BCT(I), XVXFXF, SY(1,M), NXS, 0.0)
213 RB(K+M) = CODIM1(BCT(I), XVXFXF, SZ(1,M), NXS, 0.0)
L = L + 1 + N
212 K = K + N
DO 214 I=1, J
214 ZLCCT(I) = CODIM1(ECT(I), XVXFXF, ZLCCS, NXS, 0.0)
205 CONTINUE
CALL DECWRT(DA,15,3419,OUNIT2)
ZERO BODY AREA STORAGE
DO 721 I=1, 3405
720 BODARE(I)=-0.0
721 EDIT PODS( WITH ZERO RADIUS OF NOSE)
505 CALL DECRD(DA, SCRTCH)
IF(SCRTCH) 501,501,499
499 CALL DECRD(DA, IUNIT)
CALL DECWRT(DA,15,3419,OUNIT2)
NPOD = NPOD + 1
GOTO 720
C
C
C EDIT PANELS (IF ANY)
501 CALL DECRD(DA,OUNIT)
IF(OUNIT) 502, 502, 503
503 CALL DECRD(DA, IUNIT)
CALL DECWRT(DA, 3420, 5000, OUNIT2)
ZERO PANEL AREA OF STORAGE
DO 713 I=1, 1580
713 PANARE(I)=-0.0
GOTO 501
C
C

```

```

OUT 4740
OUT 4750
OUT 4760
OUT 4770
OUT 4780
OUT 4790
OUT 4800
OUT 4810
OUT 4820
OUT 4830
OUT 4840
OUT 4850
OUT 4860
OUT 4870
OUT 4880
OUT 4890
OUT 4900
OUT 4910
OUT 4920
OUT 4930
OUT 4940
OUT 4950

```

```

C      REWIND FILES
502 ounit = iabs(ounit)
      REWIND ounit
      REWIND ounit2
      *****
      ADDITION SECTION
      COPY PAST OLD BODY (IF ANY)
      PRINT 9
9  FORMAT(1H1, 3UX,*START ADDITIONS*)
      DO 714 I=1, 14
714  DA(I)=-0.0
      CALL DECRD(DA, ounit2)
      CALL DECRD(DA, 1, 14, ounit)
      I = DA(I)
      IF (J2.EQ. 0) GOTO 507
      I = I - 1
      CALL DECRD(DA, ounit2)
      CALL DECRD(DA, 15, 3419, ounit)
      ZERO BODY AREA STORAGE
513  DO 716 J=1, 3400
716  BODARE(J)=-0.0
507  IF(NPOD) 504, 504, 508
508  CALL DECRD(DA, ounit2)
      CALL DECRD(DA, 15, 3419, ounit)
      I=I-1
      NPOD = NPOD - 1
      GOTO 513
C      ADD NEW BODIES (IF ANY)
504  IF(I) 518, 518, 505
506  DO 509 J=1, I
      CALL DECRD(DA, ounit)
      CALL DECRD(DA, 15, 3419, ounit)
      ZERO BODY AREA STORAGE
      DO 717 K=1, 3400
717  BODARE(K)=-0.0

```

```

OUT 4960
OUT 4970
OUT 4980
OUT 4990
OUT 5000
OUT 5010
OUT 5020
OUT 5030
OUT 5040
OUT 5050
OUT 5060
OUT 5080
OUT 5090
OUT 5100
OUT 5110
OUT 5120
OUT 5130
OUT 5140
OUT 5150
OUT 5160
OUT 5170
OUT 5180
OUT 5190
OUT 5200
OUT 5210
OUT 5220
OUT 5230
OUT 5240
OUT 5250
OUT 5260
OUT 5270
OUT 5280
OUT 5290
OUT 5300
OUT 5310
OUT 5320
OUT 5330
OUT 5340

```

509	CONTINUE	OUT 5350
C		OUT 5360
C	COPY REST OF PANELS	OUT 5370
518	IF(DA(2).EQ.0) GOTO 514	OUT 5380
	I = DA(2)	OUT 5390
	DO 510 J=1,I	OUT 5400
C	ZERO PANEL AREA OF STORAGE	OUT 5410
	DO 718 K=1, 1580	OUT 5420
718	PANARE(K)=-0.0	OUT 5430
	CALL DECRD(DA,OUNIT2)	OUT 5440
	IF (OUNIT2) 512, 512, 511	OUT 5450
511	CALL SECURT(DA, 3420, 5000, OUNIT)	OUT 5460
	ATACH(1,J) = DA(3420)	OUT 5470
	ATACH(2,J) = DA(3444)	OUT 5480
	ATACH(3,J) = DA(3445)	OUT 5490
	ATACH(4,J) = DA(3446)	
	ATACH(5,J) = DA(3447)	
510	CONTINUE	OUT 5500
C		OUT 5510
C	OUNIT NOW CONTAINS FULLY UPDATED DATA, READY FOR PRED	OUT 5520
C	OUNIT2 IS FREE	OUT 5530
514	REWIND OUNIT	OUT 5540
	REWIND OUNIT2	OUT 5550
	PRINT ATTACH	OUT 5560
	RETURN	
C		OUT 5580
C	ADD NEW PANELS (IF ANY)	OUT 5590
512	OUNIT2 = IABS(OUNIT2)	OUT 5600
	DO 515 K=J, I	OUT 5610
C	ZERO PANEL AREA OF STORAGE	OUT 5620
	DO 719 L=1, 1580	OUT 5630
719	PANARE(L)=-0.0	OUT 5640
	CALL DECRD(DA, IUNIT)	OUT 5650
	CALL SECURT(DA, 3420, 5000, OUNIT)	OUT 5660
	ATACH(1,K) = DA(3420)	OUT 5670
	ATACH(2,K)=DA(3444)	
	ATACH(3,K) = DA(3445)	
	ATACH(4,K) = DA(3446)	
	ATACH(5,K) = DA(3447)	

OUT 5700  
OUT 5710  
OUT 5720

50T 5730

OUT 5750

OUT 5790

COG-17C

STG LHC

OUT 5-23-68

OUT 5340

OUT

OUT 870

OUT

0635 CUT 4890

5900  
5900

OUT 5010

OUT 5920

OUT 5920  
OUT 5930

0765 11:00  
0765 11:00

OUT 5940  
OUT 5950

OUT 5970

OUT 5980

OUT 6910

OUT 6020

OUT 6030

OUT 6050

OUT 6060

OUT 6070

0.1 6980  
0.1 6980

ATA(2)=1.0

C

PUNCH PANEL

C

CALL DECRIT(DA,3420,5000,OUNIT)

C

ZERO PANEL SECTION OF STORAGE

C

DO 224 I=1,1500

224 PANARE(I)=-0.0

C

RETURN TO CALLER

C

GOTO LABEL,(380,500)

END

OUT 6090

OUT 6100

OUT 6110

OUT 6120

OUT 6130

OUT 6140

OUT 6150

OUT 6160

OUT 6170

OUT 6180

OUT 6190

OUT 6200

## REFERENCES

1. Thwaites, B., Incompressible Aerodynamics.  
Oxford: Clarendon Press (1960)
2. Kuchemann, D. and J. Weber, The Subsonic Flow Past Swept Wings at Zero Lift Without and With Body, R&M No. 2908, (1956)
3. Weber, J., The Calculation of the Pressure Distribution Over the Surface of Two-Dimensional and Swept Wings Symmetrical Aerofoil Sections, R&M No. 2918, (1956)
4. Brehner, G. G., The Calculation of the Loading and Pressure Distribution on Cranked Wings, R&M No. 2947, (1953)
5. Hunton, L. W. and J. K. Dew, The Effects of Camber and Twist on the Aerodynamic Loading and Stalling Characteristics of a Large-Scale 45° Swept-Back Wing, NACA RM A50J24, (1951)
6. Method for Predicting the Pressure Distribution on Swept Wings With Subsonic Attached Flow, British Transonic Data Memorandum 6312, (1963)
7. Liepmann, H. W. and A. Roshko, Elements of Gasdynamics, New York: John Wiley and Sons (1957)
8. Tulinius, J., Theoretical Prediction of Wing-Fuselage Aerodynamic Characteristics at Subsonic Speeds, NA-69-789, (To be published)
9. Kojima, J. Instructions for Use of the Douglas-Neumann Potential Flow Program for Axisymmetric Flow, NA-65-385, (1965)
10. Weber, J. and G. G. Brehner, Low-Speed Tests of 45-deg. Swept-Back Wings, R&M No. 2882, (1958)
11. Tulinius, J., Theoretical Prediction of Symmetrical Thick Wing Surface Pressures for Zero Angle of Attack and Subsonic Speeds, NA-70-374, (1970)
12. Blenk, J., Der Endecker Als Tragende Wirbelfläche, Z.A.M.M., Vol. 5, (1925)
13. Betz, A., The Theory of Aerofoils, Proceedings of the German Scientific Society for Aeronautics, No. II, (1920)



14. Birnbaum, W., Die Tragende Wirbelfläche Als Hilfsmittel Zur Behandlung Des Zehenen Problems Der Tragflugeltheorie, Z.A.M.M., Vol. III, (1923)
15. Wagner, S., On the Singularity Method of Subsonic Lifting-Surface Theory, A.I.A.A. Paper No. 69-37, (1969)
16. Falkner, V. M., The Calculation of Aerodynamic Loading on Surfaces of any Shape, R&M, (1910)
17. Kuchemann, D., The Distribution of Lift over the Surface of Swept Wings, The Aeronautical Quarterly, Vol. IV, (1955)
18. Goldstein, W., Approximate Two-Dimensional Aerofoil Theory, A.R.C. Technical Report, C.P. No. 68, (1951)
19. Tulinius, J., Subsonic Aerodynamic Load Theories for Wings of Polygonal Planform, NA-66-781, (1966)
20. Der, E. S., and M. Nagai, A Controlled Deviation Interpolation Method (Codim), NA-60-1604, (1960)
21. Householder, A. S., Unitary Triangularization of a Nonsymmetric Matrix, J. Assoc. Comp. Mach. 5, (1958)
22. Kuchemann, D., Progress in Aeronautical Sciences, Pergamon Press, Vol. 8, (1967)
23. Tulinius, J., Theoretical Prediction of Thick Wing Aerodynamic Characteristics at Subsonic Speeds, NA-70-104, (1970)
24. Tulinius, J., Derivation of Thick Wing and Pylon-Fanpod-Nacelle Aerodynamic Influence Equations, NA-71-23, (1971)
25. James, R. M., On the Remarkable Accuracy of the Vortex Lattice Discretization in Thin Wing Theory, DAC 67211, (1969)
26. Tulinius, J. R., Theoretical Prediction of Thick Wing and Pylon-Fanpod-Nacelle Aerodynamic Characteristics at Subcritical Speeds, NA-71-447, (1971)
27. Tulinius, J. R., Unified Subsonic, Transonic, and Supersonic NAR Vortex Lattice, TFD-72-523, (1972)
28. Tulinius, J. R., Aerodynamic Design Philosophy, TFD-71-1145, (1971)

29. Tulinius, J. R., Aerodynamic Design Procedure, TFD-71-1150, (1971)
30. Tulinius, J. R., Derivation of Design Net Pressure Expression, TFD-71-1156, (1971)
31. Tulinius, J. R., Spanwise Variation of Potential Pressure Drag, TFD-72-524, (1972)
32. Tulinius, J. R., Development of a Fortran Program to Compute the Twist, Camber, and Angles of Attack Necessary to Produce a Desired Net Pressure Distribution on a Wing of Arbitrary Planform, NA-65-872, (1965)
33. Tulinius, J. R., Proposed Program for Stability Derivative Determination, TFD-70-585, (1970)
34. Margason, R. J. and J. E. Lamar, Vortex-Lattice Fortran Program for Estimating Subsonic Aerodynamic Characteristics of Complex Planforms, TN-D-6142, (1971)
35. Milne-Thomson, J. M., Theoretical Aerodynamics, The MacMillan Company, (1960)
36. Rubbert, P. E., G. R. Saaris, M. B. Scholey, N. M. Standen, R. E. Wallace, A General Method for Determining the Aerodynamic Characteristics of Fan-in-Wing Configurations, TR-67-61A, (1967)
37. Hess, J. L. and A. M. O. Smith, Calculation of Non-Lifting Potential Flow About Arbitrary Three-Dimensional Bodies, E. S. 40622, (1962)
38. Giesing, J. P., Potential Flow About Two-Dimensional Airfoils (Part I), A Summary of Two-Dimensional Airfoil Methods (Part II), Solution of the Flow Field About One or More Airfoils of Arbitrary Shape in Uniform or Nonuniform Flows by the Douglas Neumann Method, LB 31946, (1965)
39. Giesing, J. P., T. P. Kalman, and W. P. Rodden, Subsonic Unsteady Aerodynamics for General Configurations, AIAA Paper 72-26, (1972)
40. Woodward, F. A., Analysis and Design of Wing-Body Combinations at Subsonic and Supersonic Speeds, Journal of Aircraft, Volume 6, Number 6., (1968)

41. Tulinius, J. R., Development of a Computer Program to Define the Camber Shape Necessary to Support a Desired Pressure Distribution Over a Two-Dimensional Airfoil, NA-65-614, (1965)
42. Tulinius, J. R., Development of a Subsonic Aerodynamic Analysis for Wings of Arbitrary Twist, Camber, and Planform - Extension of Falkner's Vortex Lattice Theory, NA-65-871, (1966)
43. Lamar, J. E., A Modified Multhopp Approach for Predicting Lifting Pressures and Camber Shape for Composite Planforms in Subsonic Flow, TN-D-4427, (1968)
44. Multhopp, H., Methods for Calculating the Lift Distribution of Wings (Subsonic Lifting - Surface Theory), R&M 2884, (1950)
45. Rubbert, P. E., Theoretical Characteristics of Arbitrary Wings by a Non-Planar Vortex Lattice Method, D6-9244, (1964)
46. Dulmovits, J., A Lifting Surface Method for Calculating Load Distributions and the Aerodynamic Influence Coefficient Matrix for Wings in Subsonic Flow, ADR 61-02-64.1, (1964)
47. Ashley, H. and M. T. Landahl, Aerodynamics of Wings and Bodies, Addison-Wesley Publishing Company, Inc., (1965)
48. Rubbert, P. E., and G. R. Saaris, Review and Evaluation of a Three-Dimensional Lifting Potential Flow Analysis Method for Arbitrary Configurations, A.I.A.A. Paper 72-188, (1972)
49. Nielsen, J. N., Missile Aerodynamics, McGraw Hill Book Company, (1960)
50. Wagner, S., Beitrag Zum Singularitaetenverfahren Der Tragflaechentheorie Bei Inkompressibler Stroemung, Ph. D. Thesis, Technical University, Munich, (1967)
51. Jones, R. T., Subsonic Flow Over Thin Oblique Airfoils at Zero Lift, TN 1340, (1947)
52. Craidon, C. B., Description of a Digital Computer Program for Airplane Configuration Plots, NASA TM X-2074, (1970)
53. Kuchemann, D. and J. Weber, On the Chordwise Lift Distribution at the Centre of Swept Wings, The Aerodynamic Quarterly, Vol. II, (1950)

54. Pope, A., Basic Wing and Airfoil Theory, McGraw-Hill, (1951)
55. Woodward, F. A. and J. W. Larsen, A Method of Optimizing Camber Surfaces for Wing-Body Combinations at Supersonic Speeds - Theory and Application, Part I, Boeing Report, D6-1074-1, (1965)
56. Kalman, T. P., Rodden, W. P., and Giesing, J. P., Application of the Doublet-Lattice Method to Nonplanar Configurations in Subsonic Flow, Journal of Aircraft, Vol. 8, No. 6, (1971)
57. Lopez, M. L. and C. C. Shen, Recent Developments in Jet Flap Theory and its Application to STOL Aerodynamic Analysis, AIAA Paper No. 71-578, (1971)
58. Garner, H. C., Some Remarks on Vortex Drag and its Spanwise Distribution in Incompressible Flow, NPL Aero Note 1048, (1966)
59. Aerodynamic Dimensional Data for the X-15 Airplane, NA-56-810, (1956)
60. Pressure Distributions on the .667-Scale Pressure - Heat Transfer Model of the X-15 Airplane at Mach Number from 0.60 to 7.0, NA-59-1362, (1959)
61. Basic Aerodynamic Data Manual, BDL-57-18, (1957)